

Math Club Discussions

August 29, 2019

1 8/28/19: On Plane Curves

Using Lecture 12 of *The Mathematical Omnibus*.

1. Draw an exciting closed curve that is smooth. (Draw our own on the board.)
2. Smooth: No sharp corners.
3. Wiggling argument
 - (a) Define a double point: Points of self-intersection.
 - (b) What is a triple point? We don't want it on our curve, so wiggle it.
 - (c) Define a double tangent: tangent at two **distinct** points.
 - (d) what is a self-tangency: The two points coincide. We don't want it, so wiggle it.
 - (e) What is a triple tangent? We don't want it on our curve, so wiggle it.
4. Question: What kinds of double tangents are there?
 - (a) outer
 - (b) inner
5. Count the number of your outer and inner double tangents and your double points. Call the number of inner double tangents T_- and the number of outer double tangents T_+ and the number of double points D .
6. Inflection points
 - (a) Inflection Point: moving along the curve, one is turning left or right. the inflection points are where the direction of rotation changes.
 - (b) Can you think of a smooth closed curve with exactly one inflection point.
 - (c) What can you say in general? Ans: Number of inflection points is even.

7. Fabricius-Bjerre formula: $T_+ - T_- - \frac{1}{2}I = D$
- (a) Check $T_+ - T_- - \frac{1}{2}I$ if it is less than or greater than D .
 - (b) (check with the curve drawn on the board)
8. Why is this true?
- (a) Choose a starting point and a walking direction. Shoot a laser in front of you and count the number of intersections of this laser with the curve in front of you. Let N be this number.
 - (b) Start walking. The rest on our 3 problems (week 13) file.