Math Club's [π] Problems Week 12*
π Day

$\pi/2019$

1. (Basel Problem.) This problem was posed by Pietro Mengoli in 1650 and solved by Leonhard Euler in 1734. The problem asks to evaluate

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

(Hint: Use Weierstrass factorization theorem for the Taylor expansion of $\sin(x)$ function, and stare at x^2 term.)

2. ($(\frac{1}{2})!$) The gamma function is defined as

$$\Gamma(z) = \int_0^{\infty} x^{z-1} e^{-x} dx.$$

Find $\Gamma(\frac{1}{2})$. (Hint: Use change of variable and try to evaluate the Gaussian integral $\int_0^{\infty} e^{-y^2} dy$ using a polar coordinates trick.)

3. (Barbier's Theorem.) Prove that the perimeter of any compact convex subset of $\mathbb{E}^2$ of constant width 1 is equal to π.

(Relevant Definitions: Let D be a compact convex subset of $\mathbb{E}^2$. A supporting line l is a line that intersects the boundary of D but does not intersect the interior of it, i.e. $\partial D \cap l \neq \emptyset$ and $int(D) \cap l = \emptyset$. The width of D in the direction of l is defined as the distance of two supporting lines that are perpendicular to l. D is said to be of constant width if the width in all directions is the same.)

*Email your solutions to us at sbumathclub@gmail.com (and/or) present it during our GBM on the next Wed. (at 7pm at Math Tower p-131).