“Remittance, Output, and Exchange Rate Regimes: Theory with an Application to Latin America”

Technical Appendix

A.1. System of Equations

(A1) \[ w_t A_t = \frac{x_{1-t} \Phi(H_t + \Omega_t)^{\Phi-1}}{(c_t - x_t(H_t + \Omega_t)^{\Phi})^{\Phi}} \]

(A2) \[ A_t = \beta E_t \left[ \frac{\Delta t_{t+1} s_{t+1}}{\pi_{t+1} s_t} (1 + i_{t+1}) \right] \]

(A3) \[ \pi_t = \beta E_t \left[ \frac{\Delta t_{t+1} s_{t+1}}{\pi_{t+1} s_t} (1 + i_{t+1}) \right] \]

(A4) \[ w_t A_t \pi_t (\Delta M^c_t - \theta) + A_t = \beta E_t \left[ \frac{1 - \chi(\Pi_{t+1} + \Omega_{t+1})^\Phi y_{t+1}^\gamma x_t^\gamma}{\pi_{t+1} (c_t - x_t(H_t + \Omega_t)^{\Phi})^{\Phi} x_t^{\gamma}} \right] + \]

(A5) \[ \Delta M^c_t = \frac{m_{t+1} \pi_t}{m_t} \]

(A6) \[ \pi_t = \frac{s_t}{s_{t-1}} \pi_t^* \]

(A7) \[ Y_t = e^{z_t} K^\alpha H_t^{1-\alpha} \]

(A8) \[ l_t = K_{t+1} - (1 - \delta) K_t \]

(A9) \[ w_t = (1 - \alpha) \frac{y_t}{H_t} \]

(A10) \[ (1 + i_t) + v(K_{t+1} - K_t) = \beta E_t \left[ \frac{\Delta t_{t+1} s_{t+1}}{\pi_{t+1} s_t} (1 - \delta) (1 + i_{t+1}) + \right] \]

(A11) \[ m_{t+1} = \theta_t \frac{m_t}{\pi_t} \]

(A12) \[ m_t = m_t^b + m_t^c \]

(A13) \[ \pi_t l_t = m_t^b + (\theta_t - 1) m_t + (1 - \eta) \Gamma_t \]

(A14) \[ \pi_t^c C_t = m_t^c + \eta \Gamma_t \]

(A15) \[ i + \tau = i^w - \phi b_t \]

(A16) \[ b_{t+1} = \frac{s_{t+1}}{s_{t-1}} (1 + i_t) \frac{b_t}{\pi_t} = Y_t - C_t - I_t + [1 - (1 + i_t) (1 - \eta)] \frac{\Gamma_t}{\pi_t} - \frac{u}{2} (K_{t+1} - K_t)^2 \]

(A17) \[ \Gamma_t = E_t [\psi(Y_{t}^{SS} \pi_t Y_{t}^{-\gamma}) e^t \gamma] \]

(A18) \[ \frac{1 + i_t}{1 + i} = \left( \frac{1 + i_{t+1}}{1 + i_t} \right) \frac{x_t}{\pi_t} \frac{x_t}{\gamma} \frac{x_t}{\pi} \frac{x_t}{\gamma} \frac{x_t}{\pi} \frac{x_t}{\gamma} \frac{x_t}{\gamma} e^{t \gamma} \]

(A19) \[ RER_t = \frac{s_t}{s_{t-1} \pi_t} \]

(A20) \[ X_t = C_t^\gamma X_{t-1}^{1-\gamma} \]

(A21) \[ \log(\theta_{t+1}) = (1 - \rho_\theta) \log(\bar{\theta}) + \rho_\theta \log(\theta_t) + \epsilon_{\theta t+1} \]

(A22) \[ \log(\gamma_{t+1}) = (1 - \rho_\gamma) \log(\bar{\gamma}) + \rho_\gamma \log(\gamma_t) + \epsilon_{\gamma t+1} \]

(A23) \[ \log(\zeta_{t+1}) = (1 - \rho_\zeta) \log(\bar{\zeta}) + \rho_\zeta \log(\zeta_t) + \epsilon_{\zeta t+1} \]
A.2. The log-linearized system of equations is given by

\[ (B1) \quad 0 = -\hat{\omega}_t - \hat{\lambda}_t + \left( (\Phi - 1) + \frac{\sigma(h)\phi_X \Phi}{c - \chi(h)\phi_X} \right) \hat{H}_t - \frac{\sigma c}{c - \chi(h)\phi_X} \hat{C}_t + \left( 1 + \frac{\sigma \chi(h)\phi_X}{c - \chi(h)\phi_X} \right) \hat{X}_t \]

\[ (B2) \quad 0 = E_t \left[ -\hat{\lambda}_t + \hat{\lambda}_{t+1} - \hat{\pi}_{t+1} + \hat{s}_{t+1} - \hat{s}_t + \frac{i^*}{1+i^*} t_{t+1} \right] \]

\[ (B3) \quad 0 = E_t \left[ -\hat{\lambda}_t + \hat{\lambda}_{t+1} - \hat{\pi}_{t+1} + \frac{i}{1+i} t_{t+1} \right] \]

\[ (B4) \quad 0 = E_t \left[ -\hat{\lambda}_t - w \frac{\xi}{m^c} \pi \Delta M^c \Delta M^c + \beta w \frac{\xi}{m^c} (\Delta M^c)^2 \Delta M^c - \hat{\pi}_{t+1} + \beta \frac{\xi}{m^c} (\Delta M^c)^2 \Delta M^c \right] \]

\[ \hat{X}_{t+1} - \beta (1-\gamma) \chi(h)\phi_X \Phi \chi_{t+1} + \frac{\beta \chi(h)\phi_X \Phi \chi_{t+1} - \hat{\pi}_{t+1}}{\Lambda \pi (c - \chi(h)\phi_X)} \hat{G}_{t+1} + \left( \frac{\sigma \chi(h)\phi_X \Phi \chi_{t+1}}{\Lambda \pi (c - \chi(h)\phi_X)} \right) \hat{H}_{t+1} \]

\[ (B5) \quad 0 = -\Delta M^c + \hat{q}_t + \hat{t}_t - \hat{m}_t \]

\[ (B6) \quad 0 = -\hat{q}_t + \hat{s}_t - \hat{s}_{t-1} \]

\[ (B7) \quad 0 = \hat{q}_t + \alpha \hat{R}_t + (1 - \alpha) \hat{H}_t + \hat{z}_t \]

\[ (B8) \quad 0 = -\frac{\hat{t}}{K} \hat{t}_t + \hat{R}_{t+1} - (1 - \delta) \hat{R}_t \]

\[ (B9) \quad 0 = -\hat{w}_t + \hat{Y}_t - \hat{H}_t \]

\[ (B10) \quad 0 = E_t \left[ (1-\gamma) (1-\delta) (1+i) \hat{t}_{t+1} + \frac{\alpha \beta \Phi}{\kappa} \hat{G}_{t+1} - \left( \frac{\alpha \beta \Phi}{\kappa} + \beta (1-\delta) (1+i) \right) \hat{H}_t + \left( \frac{\alpha \beta \Phi}{\kappa} + \beta (1-\delta) (1+i) \right) \hat{L}_{t+1} + \frac{\alpha \beta \Phi}{\kappa} \hat{H}_t - \left( \frac{\alpha \beta \Phi}{\kappa} + \beta (1-\delta) (1+i) \right) \hat{L}_{t+1} \right] \]

\[ (B11) \quad 0 = -\hat{m}_{t+1} + \hat{m}_t - \hat{r}_t + \hat{Q}_t \]

\[ (B12) \quad 0 = -(\hat{m}) \hat{m}_t + (\hat{m}) \hat{m}_t + (\hat{m}) \hat{m}_t \]

\[ (B13) \quad 0 = -\hat{r}_t - \frac{\hat{r}}{\hat{m}} + \frac{m^b}{\hat{m}} \frac{\hat{m}}{\hat{m}} + \frac{m^b}{\hat{m}} (\theta - 1) \hat{m}_t + \frac{m^b}{\hat{m}} \hat{m}_t + \frac{m^b}{\hat{m}} \hat{m}_t + \frac{m^b}{\hat{m}} \hat{m}_t \]

\[ (B14) \quad 0 = -\hat{r}_t - \frac{\hat{r}}{\hat{m}} + \frac{m^b}{\hat{m}} \frac{\hat{m}}{\hat{m}} + \frac{m^b}{\hat{m}} \hat{m}_t \]

\[ (B15) \quad 0 = \frac{\hat{r}_t}{\hat{m}} + \frac{\hat{r}}{\hat{m}} \hat{b}_t \]

\[ (B16) \quad 0 = -\hat{b}_{t+1} + (1+i^*) \hat{t}_t - \frac{(1+i^*)}{\pi^2} \hat{t}_{t+1} + \frac{(1+i^*)}{\pi^2} \hat{t}_{t+1} + \frac{i^*}{i^*} \hat{t}_t + \frac{t}{b} \hat{t}_t - \frac{c}{b} \hat{t}_t - \frac{b}{b} \hat{t}_t \]

\[ (B17) \quad 0 = E_t \left[ -\hat{t}_t + \hat{t}_t - \tau \hat{Y}_t + \hat{G}_t \right] \]

\[ (A18) \quad 0 = -\frac{i}{i^*} \hat{t}_t + \frac{i}{i^*} \hat{t}_t + \frac{i}{i^*} \hat{t}_t + \frac{i}{i^*} \hat{t}_t + \frac{i}{i^*} \hat{t}_t + \frac{i}{i^*} \hat{t}_t + \frac{i}{i^*} \hat{t}_t + \frac{i}{i^*} \hat{t}_t + \frac{i}{i^*} \hat{t}_t \]

\[ (A19) \quad 0 = -\hat{R}_{t+1} + \hat{s}_t - \hat{s}_{t-1} - \hat{t}_t \]

\[ (B20) \quad 0 = -\hat{R}_t + \gamma \hat{C}_t + (1 - \gamma) \hat{X}_t \]

\[ (B21) \quad \hat{s}_{t+1} = \rho_{irs} \hat{s}_{st} + \hat{e}_{irt+1} \]

\[ (B22) \quad \hat{g}_{t+1} = \rho_{g}\hat{g}_t + \hat{e}_{gt+1} \]

\[ (B23) \quad \hat{z}_{t+1} = \rho_{z}\hat{z}_t + \hat{e}_{zt+1} \]
A3. Steady State Derivations

**Given H.** We assume that the domestic gross inflation rate is given by the gross money growth rate so that \( \pi = \theta \). Equation (A3) gives the steady state for the domestic nominal interest rate

\[
i = \frac{\pi}{\beta} - 1
\]

The uncovered interest rate parity condition – equation (A3) equal to equation (A2) – implies that the domestic and the interest rates on foreign bonds are equal \( (i = i^*) \).

We can derive the steady state level of remittances from equation (A17) as

\[
\Gamma = \phi \pi
\]

From the definition of the change in money balances (equation (A5) we get that

\[
\Delta M^c = \pi
\]

To find the steady state capital/output ratio (denoted \( \kappa \)) we get, from the stationarity of equation (A10):

\[
\kappa \equiv \frac{K}{Y} = \left[ \frac{\alpha \beta}{(1 + i) - (1 - \delta)(1 + i)\beta} \right]
\]

Then from the production function (A7) we can solve for the output

\[
Y = H\kappa^{\frac{\alpha}{1 - \alpha}}
\]

which can be used in equation (A9) to solve for the real wage

\[
w = (1 - \alpha)\frac{Y}{H}
\]

Then the steady state physical capital stock will be given by \( K = \kappa Y \), and steady state investment A8 by \( I = \delta K \).

The right-hand side of equation (A14) defines the adjusted trade balance, \( TB = Y - C - I + (1 - (1 + i)(1 - \eta))\frac{C}{H} \), which when divided by output allows us to use the calibration \( v = TB/Y \). The steady state stock of foreign assets in real terms is derived from the balance
of payments equilibrium (equation (A16)), so the household’s stock of foreign assets in real terms is

\[ b = \frac{\nu Y}{1 - \frac{1 + i^*}{\pi}} \]

By reinserting in A16, we can solve for consumption

\[ C = Y - I - b \left( 1 - \frac{1 + i^*}{\pi} \right) + \left[ 1 - (1 + i)(1 - \eta) \right] \frac{\Gamma}{\pi} \]

From the CIA constraint A14, steady state real money-cash balances are:

\[ m^c = \pi C - \eta \Gamma \]

Then using equation (A13) and the definition of money (equation (A16)), the household’s steady state real money deposits is

\[ m^b = \frac{I \pi - (1 - \eta) \Gamma - (\theta - 1)m^c}{\theta} \]

Given the definition of real money balances, then its steady state level is:

\[ m = m^b + m^c \]

From A20:

\[ X = C \]

Back in equation (A4) and (A1), solving for \( \Lambda \) and setting them equal we can solve for

\[ \chi = \frac{w \beta}{w \beta (H)^{\phi} \gamma C^{\gamma-1} X^{1-\gamma} + \pi \chi \Phi (H)^{\phi}} \]

And back in (A4)

\[ \Lambda = \frac{\chi \Phi X (H)^{\phi}}{w (C - \chi (H)^{\phi} X)^{\sigma}} \]
A.4. Monetary and Technology Shocks

Figure A1: Dynamic response to a 1 standard deviation (0.35%) monetary shock
Remittances 5% of GDP and 80% going towards Consumption
Figure A2: Dynamic response to a 1 standard deviation (1.5%) technological shock
Remittances 5% of GDP and 80% going towards Consumption