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A Classification Approach for Selecting Forecasting Techniques for Intermittent Demand

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Abstract

The intermittent demand forecasting problem involves the forecasting of demand series that are characterized by the time between demands being significantly larger than the unit of time used for the forecast period. This causes the time series associated with the demand to have a large percentage of periods for which there are no demands. These types of series are often found in spare parts inventory management systems. This paper examines the intermittency of a demand series by relating the lag-1 correlation coefficient of non-zero demand, squared coefficient of variation of non-zero demand and probability of zero of the demand series to the error properties of various forecasting techniques. A classification method is presented by which a time series can be characterized in terms of key parameters related to intermittency and through this relationship the best of a set of forecasting techniques can be recommended. The method is illustrated on both real intermittent demand series and randomly generated time series in order to understand the efficacy of the procedure to improve overall forecasting effectiveness.

Keywords
Intermittent demand forecasting, classification, inventory control

1. Introduction and Motivation

The Naval Aviation Maintenance Program (NAMP) of US Navy is a multi-echelon supply network with 3 levels. At the lower level the demand for a reparable spare part arrives when the part fails. The demand at the higher levels occurs when the lower levels are unable to repair the failed spare part. The repair cycle may include shipping time, processing time, repair time, waiting time, and delivery time. Because of the repair cycle as well as the failure cycle, the demand for repair and spare parts is often intermittent in nature. Intermittent demand is characterized by demand data that has many time periods with zero demands. However, other definitions can be found in the literature [11] [1] [6], for intermittent demand. Intermittent demand is hard to model using conventional distributions and is hard to forecast. There have been several intermittent demand forecasting techniques proposed in the literature.

The selection of the best forecasting technique for a given demand series can be approached in several ways. The most common approach is to select a forecasting technique that minimizes the forecast error using the available demand history. Another approach is to forecast based on several forecasting techniques and subsequently combine the forecasted values into a single forecast. Armstrong [2] recommends this as an appropriate approach instead of selecting a single forecasting technique. This paper considers a demand categorization approach to choosing an appropriate forecasting technique. We investigate this approach's usability in an intermittent demand scenario. In this demand categorization scheme, the demand scenarios are categorized based on the forecast errors (across several forecasting techniques). The forecast error or difficulty to forecast is related with the demand attributes lag-1 correlation coefficient of non-zero demand, squared coefficient of variation of non-zero demand and probability of zero demand for the demand series of each demand scenario. The degree of intermittency of a demand scenario can be viewed as the difficulty to forecast the demand scenario or the forecast error associated with the demand scenario. Each demand category (high intermittency, medium intermittency and low intermittency) can be mapped to its best forecasting technique. Once the most appropriate forecasting technique has been identified, it is used to make forecasts for the demand series. The article uses the following notations for the demand attributes.

- $\phi_{1,NZ}$: lag-1 correlation coefficient of non-zero demand
- CV_{NZ}^2: squared coefficient of variation of non-zero demand

- π_Z: probability of zero demand for the demand series

Through the perspective of forecast error 80 demand scenarios were ranked, and categorized in terms of high intermittency, mild intermittency and low intermittency. An artificial demand generator created in Varghese [15] was used to simulate the demand scenarios of each of the categories. A comparative experimental run of the various intermittent demand forecasting techniques upon these 80 demand scenarios yielded a classification scheme. The classification scheme holds the demand classifications and its attributes and the best forecasting technique associated with it. This research considered the forecasting techniques: moving average, simple exponential smoothing, Croston's method [3], Syntetos' approximation method [13], and a cumulative average (CA) forecasting method. We evaluated the efficacy of the classification scheme using randomly selected demand series of the items from a US Navy inventory system. The classification scheme proposes the best forecasting technique for each demand series and subsequently its forecast error can be computed. The recommended technique was then compared with the forecast error associated with the existing forecasting technique (CA) for the demand series, We saw considerable reduction in error in using this categorization scheme to choose the best forecasting technique.

1.1. Intermittent Demand and Categorization Scheme

The literature refers to the "hard to forecast" demand scenarios as intermittent demand, lumpy demand, erratic demand, sporadic demand, slow-moving demand etc. and often these words are used interchangeably which amounts to much confusion. As previously discussed, the demands are generally characterized by the attributes: intermittence (or sporadicity) and lumpiness. Usually, intermittent demand (or sporadic demand) is defined as demand occurring randomly with many time periods with zero demands. However, this limits the definition to the attribute of intermittence or sporadicity. Silver [11] proposed a definition for intermittent demand as "*infrequent in the sense that the average time between consecutive transactions is considerably larger than the unit time period, the latter being the interval of forecast updating*." Smart [1] defined intermittent demand as a demand series with at least 30 % of zero demand. Representative US Navy [10] inventory managers consider those demand series with less than or equal to 60 - 70 % non-zero demands as intermittent. Johnston et al. [6] proposed that if the mean interval between non-zero demands is 1.25 times greater than the inventory review period, the demand series can be considered as intermittent. Most of the definitions of intermittent demand (or sporadic demand) do not include the demand attribute: lumpiness. Slow demands are usually defined as those with infrequent demands, which occur in very few units [7] [13] [19]. Slow demands are usually intermittent demands. Meanwhile erratic (or irregular) demand is described as in [13] as patterns with high variability in non-zero demands. Syntetos [13] based his definition on the demand size and excluded demand incidence and so did Silver [11]. Syntetos [13] defined lumpy demand as those demand patterns with some zero-demands and with non-zero demand having high variability. He considered all lumpy demands as intermittent demands; however not all intermittent demand is lumpy demand. Ward [17] also used intermittent demand and lumpy demand interchangeably. These types of demand scenarios overlap with similar characterizations of intermittent demand. In this paper we view these demand scenarios by the difficulty to forecast or the error associated with the demand scenario.

William's categorization scheme [19] is one of the earliest ones of its kind and is based on a concept called variance partitioning, in which the variance of the lead time demand is split, to classify the demand. His classification represented intermittence, by how often the demand occurs during the lead time. He also considered the variance of non-zero demand, commonly called as lumpiness. Figure 1 shows the categorization scheme proposed by Williams [19]. The Category D2 in the scheme indicates highly sporadic demand characterized by high intermittence and high lumpiness. The Category B represents a slow-moving inventory and others are classified as smooth demand patterns. The cut-off value was assigned based on the inventory system that William observed for his 1984 study. Hence, the applicability of this scheme is problematic. Syntetos et al. [13] [14] in their research on intermittent demand forecasting techniques, proposed a demand categorization scheme with recommendations for an appropriate cut-off value for squared coefficient of variation and mean interval between non-zero demands. They compared the mean square error of the forecasting technique Simple Exponential Smoothing (or Exponentially Weighted Moving Average), and Croston's approach [3], with that of the forecasting technique proposed by Syntetos [13]. Figure 2 illustrates their categorization scheme, where Region 1 indicates erratic demand, Region 2 indicates lumpy demand, Region 3 indicates smooth demand and Region 4 indicates intermittent demand. In this scheme also, intermittency is associated with a high percentage of zero demands.

The research literature on demand classification reveals that most of the schemes consider only intermittence and lumpiness. In this paper we consider dependence (through $\phi_{1,NZ}$) in addition to intermittence and lumpiness. The demand classification and the resulting technique are based on forecast error (across a wide range of forecasting techniques). The application of the classification scheme will be in the mapping of the demand classes to its best forecasting technique. Using this mapping, we can predict the best forecasting technique for a given demand scenario.

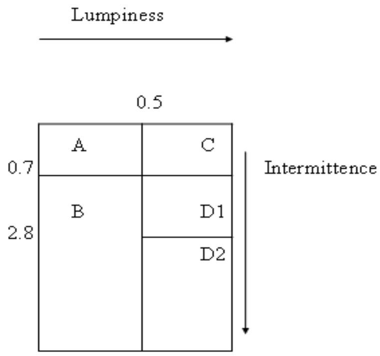

Figure 1 William's Categorization Scheme

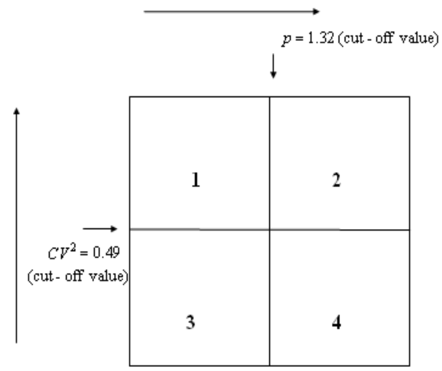

Figure 2 Syntetos Categorization Scheme

1.3 Intermittent Demand Forecasting

The simpler traditional forecasting methods like simple exponential smoothing and moving average are often unsuitable in intermittent demand scenarios. There are several forecasting techniques relevant for intermittent demand. These techniques are discussed in detail by Varghese [15] and by the authors in another paper presented in this conference [16]. Croston's [3] approach and its variant Syntetos [13] are two of the primary techniques. In addition to these approaches, Willemain [18] developed a non-parametric bootstrapping approach forecasting especially intermittent demand. Meanwhile, Snyder [12] proposed a parametric bootstrapping to forecasting slow demand. The performance of forecasting techniques can be measured by the forecast error's mean absolute deviation (MAD), mean square error (MSE) and mean absolute percentage error (MAPE). Though MAD, MSE and MAPE are sufficient to compare between errors associated with each of the demand scenarios, when it comes to identifying the best forecasting technique the winners may be different across each of these error metrics. Because of multiple measures for error, we apply a multi-criteria approach when analyzing which forecasting technique has the best performance. We refer the reader to [4] or [9] for further information on these techniques. The objective is to find the best forecasting technique with the metrics low MAD, low MSE and low MAPE. Given the MAD, MSE and MAPE related to each forecasting technique, they can be scaled to a 0 to 1 scale by the normalizing method discussed in [4]. Now by the weighted arithmetic mean method, the scaled MAD, scaled MSE and scaled MAPE can be weighted to form an overall objective. The weighted mean gives performance taking into account all the metrics according to its weighted priority. Our technique chooses the forecasting technique with least weighted error as the winner. This winner is specific to the priority given by the inventory manager on each of the metrics. In this paper, we consider equal error-weighing policies whose weighted averages are computed as in Equation 1.

$$Weighted\ Error = \frac{1}{3}MAD_{scaled} + \frac{1}{3}MSE_{scaled} + \frac{1}{3}MAPE_{scaled} \quad (1)$$

2. Experiment and Results

The experiments require the generation of various demand scenarios based on the demand attributes π_Z, CV_{NZ}^2 and $\phi_{1,NZ}$. We create the demand scenario and, for each time index, we make a forecast according to its kind, gather the demand, and compute the error measures associated with the forecasting technique. Subsequently, we have to identify highly intermittent demand sources configured by attributes π_Z, CV_{NZ}^2 and $\phi_{1,NZ}$ based on forecast error (across several forecasting techniques). Assessing the cause-effect interaction, with the error measures being the effect, we cataloged the factors that are relevant to the experiment. The error varies with the forecasting technique and hence will be one of the factors. This qualitative variable will have the forecasting techniques that we have selected, as various levels. The forecasting methods that we examined are Simple Exponential Smoothing, Moving Average, Croston's approach, Syntetos' and Boylan's Approximation method and Cumulative Average. The levels are: Simple Exponential Smoothing with alpha value 0.1 and 0.2, Moving Average with N value 19 and 9, Croston with alpha value 0.1 and 0.2, Syntetos' and Boylan's Approximation method with alpha value 0.1and 0.2 and Cumulative average. Comparison across demand scenarios with distinct attribute values is required in order to create the demand classification scheme. We fix the levels of these factors such that, they range across representative intermittent demand patterns. For the experiments, the levels listed in Table 1 were used. Each design point was replicated 50 times. Note that the demand attributes yields 80 demand scenarios, which will be compared in order to create the categorization scheme. Varghese [17] explains all the details for how the experiments are laid out and executed.

Table 1 Experimental Design

Factors	Levels
Forecasting Technique	SES(0.1), SES(0.2), MA(19), MA(9), Croston(0.1), Croston(0.2), Syntetos(0.1), Syntetos(0.2) and CA
Squared coefficient of variance of demand	0.25, 0.5, 0.75, 0.95
Probability of zero of demand series	0.1, 0.3, 0.52, 0.75
Lag 1 correlation coefficient of demand	-0.8, -0.2, 0.0, +0.2, +0.8

2.1 MCB Technique across demand scenarios

The comparison of each of the 80 demand scenario is based on MAD, MSE and weighted error using Hsu's Multiple Comparison with Best (MCB) to determine the most intermittent series. At each of these levels, the demand scenarios with high error were identified, ranked, and categorized. Hsu's approach compares each scenario with the best of the remaining scenarios [5] [8]. When compared with other multiple comparison techniques, in Hsu's MCB approach, the comparison procedure is implemented in a single-stage. In addition, the relative performance of each demand scenario can be estimated. As an illustration, the confidence intervals for the MCB statistics of Syntetos (0.2) at $\phi_{1,NZ} = -0.8$ are included in Table 2. The scenario with the lower interval 0 is the best case. In the table, the scenario with $CV_{NZ}^2 = 0.95$ and $\pi_Z = 0.095238$ is the best scenario. In other words, the high error implies a highly intermittent scenario. The relative performance of other scenarios can be also interpreted. For example, the scenario with $CV_{NZ}^2 = 0.95$ and $\pi_Z = 0.297872$ with the lower interval at -1.004 can be interpreted as a demand scenario that cannot be 1.004 more intermittent than the best demand scenario.

Table 2 Confidence Interval of Hsu's MCB Procedure Syntetos (0.2) at$\phi_{1,NZ} = -0.8$

Level of $CV_{NZ}^2$and π_Z	Lower	Center	Upper	Level of $CV_{NZ}^2$and π_Z	Lower	Center	Upper
0.25,0.095238	-14.114	-13.882	0	0.75,0.095238	-11.988	-11.757	0
0.25,0.297872	-13.979	-13.747	0	0.75,0.297872	-11.978	-11.747	0
0.25,0.518519	-13.962	-13.731	0	0.75,0.518519	-12.348	-12.117	0
0.25,0.750000	-14.112	-13.881	0	0.75,0.750000	-13.038	-12.807	0
0.50,0.095238	-13.523	-13.292	0	0.95,0.095238	0	0.773	1.004
0.50,0.297872	-13.466	-13.234	0	0.95,0.297872	-1.004	-0.773	0
0.50,0.518519	-13.568	-13.337	0	0.95,0.518519	-2.74	-2.509	0
0.50,0.750000	-13.884	-13.653	0	0.95,0.750000	-6.944	-6.712	0

Table 2 identifies the 4 demand scenarios when $CV_{NZ}^2$of 0.95 as the one with highest forecast error. Followed by these scenarios are the demand scenarios with CV_{NZ}^2: 0.75, 0.5 and 0.25 ranked in the descending order of forecast error. In addition, it should be noted that for CV_{NZ}^2=0.95, as the π_Z increases, the forecast error decreases. But at CV_{NZ}^2=0.5 and 0.25, we see that as the π_Z increases the forecast error increases for a while and then starts decreasing. It can be inferred that when lumpiness is small the forecast error increases with π_Z and then starts decreasing. In addition to this, we observe that the forecast error associated with the demand scenarios having CV_{NZ}^2=0.5 and 0.25 are very low i.e. with low intermittency. Meanwhile the demand scenarios with CV_{NZ}^2=0.95 are highly intermittent followed by the demand scenarios with CV_{NZ}^2=0.75 with relatively medium.

The above inferences were made based on the demand scenarios with $\phi_{1,NZ} = -0.8$ and the comparison based upon the MAD associated with Syntetos (0.2). When we compared the demand scenarios using the MAD as well as MSE and weighted error of the other forecasting techniques, we came up with the same inferences. The MCB was also made between the demand-scenarios within the other levels of $\phi_{1,NZ}$. This also yielded the same inferences. Besides, another MCB was performed across all the 80 demand scenarios (i.e. comparing all the 80 scenarios together), and the trends observed in the error. This also yielded the same inferences. The categorization scheme can be summarized as: demand scenarios with CV_{NZ}^2=0.95 are highly intermittent followed by the demand scenarios with

CV_{NZ}^2=0.75 with relatively medium intermittency and finally the demand scenarios with CV_{NZ}^2=0.5 or 0.25 with low intermittency.

2.2 MCB Technique across Forecasting Techniques

If the categorization scheme can be used to indicate the best forecasting technique, then it may have important practical applications. In each of the 80 levels of demand, we compared the forecasting techniques based on the weighted error. The multiple comparisons were done by using Hsu's approach, and the best forecasting techniques were selected for each of the 80 demand scenarios. The forecasting technique that was most repeated within a category was considered as the best forecasting technique within that demand category. Subsequently, we developed a classification table that covers extensive ranges of demand characteristics and the best forecasting technique associated with the range. This can be implemented in an inventory system to make recommendations on forecasting techniques most appropriate to a particular item. An example classification table is given in Figure 3.

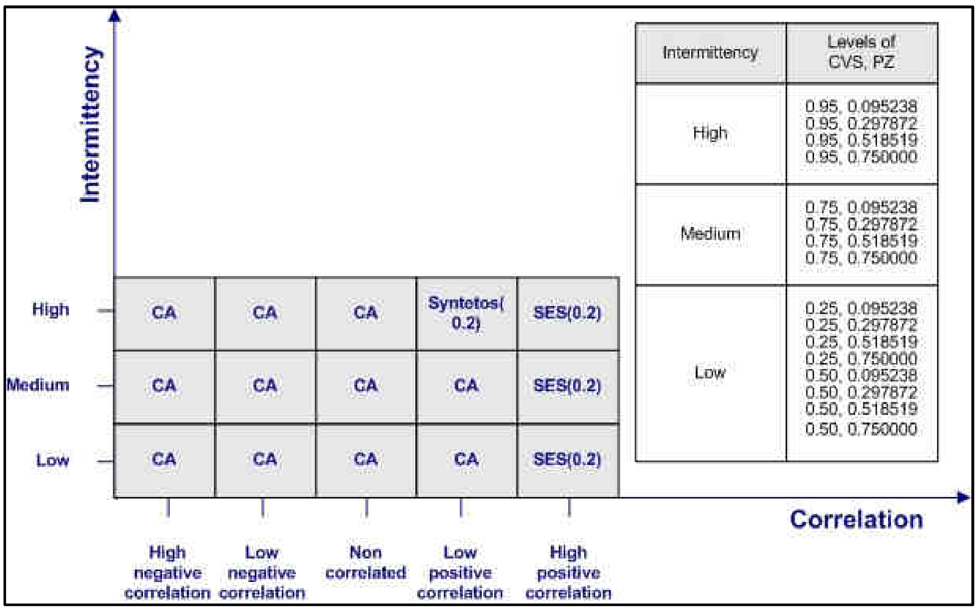

Figure 3 Recommended Forecasting Technique by Weighted Error

Figures 3 recommends a forecasting technique based on equally weighing MAD, MSE, and MAPE. The X-axis consists of the level of lag 1 correlation of the demand series under study. The Y-axis plots the high, medium and low intermittence. The intermittence categories and their corresponding $\pi_Z$and CV_{NZ}^2 levels are listed in the table at the right side of the chart. In order to use the table, the correlation level and the intermittency level of the demand series must first be identified. For example, if π_Z=0.75 andCV_{NZ}^2=0.95, the table suggests that the demand is characterized as highly intermittent. Then, if this demand is non-correlated, the forecasting technique suitable for non-correlated high intermittent demand from the chart: CA (in this case) can be selected.

Twenty stock items were randomly selected from a US Navy inventory system. The corresponding demand series were analyzed. After calculating the demand characteristics, the values were identified in the chart and the corresponding recommended forecasting techniques were selected. For the randomly selected demand series the current technique was the cumulative average method. The percentage difference as well as the difference between the MAD associated with CA and the MAD associated with the forecasting technique recommended by the chart was calculated. Both indicated that significant benefit can be achieved by using the chart to select the forecasting technique. For example, we observed that following the forecasting technique suggested by the chart for weighted error will cause an average percentage reduction of 7.03% on MAD. The analysis suggested significant benefits in using the chart-recommended forecasting technique in order to improve overall forecast accuracy.

3. Conclusions and Future Research

The results from this preliminary study show that a classification procedure can be developed and used to choose an appropriate forecasting technique for demand series. First, the estimates of the three demand attributes (demand attribute vector) of the stock keeping unit are computed. Second, the corresponding demand category is identified and then the forecasting technique to which the demand category is mapped is chosen. This meta-forecasting technique was applied on a real data and was found to reduce the forecast error, when compared with the existing technique. Future work will consider better classification approaches based on the best forecasting technique itself instead of the forecast error. Using Multinomial Logistic Regression, Discriminant Analysis, Nearest Neighbor Clustering or Artificial Neural Networks, we can train the classifier to predict the best forecasting technique based on demand attributes computed from a demand series. Such a classifier would be of great benefit when used within a large enterprise resource planning system to set the most appropriate forecast technique for a given stock item automatically.

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