The Linear Sampling Method for the Acceleration of the Level Set Algorithm

Ahmed M. Hassan, Tyler Bowman, and Magda El-Shenawee
Department of Electrical Engineering
University of Arkansas
Fayetteville, AR 72701, USA
amhassan@uark.edu, tcbowman@uark.edu, magda@uark.edu

***Abstract*—The robust shape reconstruction level set algorithm has proven its capability for reconstructing the exact shape and location of unknown 2D and 3D targets. However, the computational time of the level set algorithm can be extensive especially for complex multi-scale targets. In order to eliminate this burden, the qualitative Linear Sampling Method (LSM) is investigated to generate the initial guess for the level set algorithm. In this case, the obtained results show a reduction of almost 90% of the CPU time required by the level set algorithm.**

I. Introduction

This paper explores using the qualitative Linear Sampling Method (LSM) [1] to estimate the initial guess of the level set algorithm [2]-[3]. The LSM technique reconstructs a rough estimate of the shape of the unknown target consuming a considerably low computational time [1]. This rough estimate of the target, calculated using the LSM, is used as the initial guess of the level set algorithm which will converge within less number of iterations since the initial guess is close to the unknown target. Therefore, the role of the LSM is to reconstruct an estimate of the general layout of the target whereas the level set algorithm is evoked to refine that estimate and reconstruct the exact shape. The hybridized level set and LSM algorithm will combine the accuracy of the level set algorithm with the speed of the LSM.

As discussed in detail in our previous work, the level set technique is capable of accurately reconstructing the shape and the location of multiple unknown objects [2]-[3]. The accuracy of the algorithm has been proven in the reconstruction of both two dimensional (2D) [2] and three dimensional (3D) targets [3]. The main idea behind the level set algorithm is to model the evolving object during the reconstruction as the zero level of a higher order function [2]-[3]. This is termed implicit modeling and has the advantage of topological flexibility in modeling multiple unknown objects. The level set algorithm is initialized using an arbitrary guess of the unknown target. This initial guess evolves iteratively to the true object directed by forces, termed *deformation velocity*, which are calculated using the fields scattered from the unknown target [2]-[3].

However, according to the complexity of the target, the level set algorithm could require thousands of iterations to converge which can lead to extensive computational time. In particular, multi-scale targets, which are targets that are large with respect to the probing wavelength but also have fine features, require a large number of iterations to be reconstructed accurately. If the initial guess could be chosen close to the unknown target, using the LSM qualitative reconstruction, the level set could require significantly fewer iterations and the computational time is reduced considerably.

In Section II, the LSM algorithm will be described whereas numerical results and conclusions are presented in Section III and Section IV, respectively.

II. Linear Sampling Method

The Linear Sampling Method involves discretizing the imaging domain into $L_x \times L_y$ pixels. At each pixel, the following far field Fredholm equation is solved [1]:

$$\int_{\Omega} u_{\infty}(\hat{x}, d)\, g(\theta, z)\, d\theta = \Phi_{\infty}(\hat{x}, z) \tag{1}$$

where $u_{\infty}(\hat{x}, \theta)$ is the far field pattern scattered from unknown target(s) measured at an angle $\hat{x}$ due to a plane wave excitation at angle θ and the function $\Phi_{\infty}(\hat{x}, z)$ represents the far field pattern measured at an angle $\hat{x}$ due to a point source at z [1]. The unknown in (1) is $\|g(:,z)\|$ which is calculated at all pixels z. Contours of $\|g(:,z)\|$ are then plotted which provide a general idea of the number, location and the shape of the objects in the unknown target. The contours of $\|g(:,z)\|$ with small values reside inside the unknown target whereas the contours with large values reside outside the unknown target. The contour of $\|g(:,z)\|$ whose value is equal to a certain threshold is used as the approximate reconstruction of the target. The determination of the optimum value for the threshold is still an open challenge [1]. Equation (1) is highly ill-conditioned and, therefore, requires regularization before it can be solved efficiently. One possible choice for regularization is the Tikhonov regularization described in [1].

III. Numerical Results

In this section, two LSM reconstruction examples for 2D perfect electric conductor (PEC) targets are presented: (*i*) a star-shaped target as shown in Fig. 1a and (*ii*) a target consisting of five objects of various shapes as shown in Fig.

978-1-4673-0462-7/12/$31.00 ©2012 IEEE

1b. The imaging domain ranges from -0.4 m to +0.4 m when the star-shaped target is considered whereas the imaging domain ranges from -1m to +1m in both *x*- and *y*-directions when the five objects target is considered. A frequency of 1GHz is employed in the reconstruction of the star-shaped target whereas a slightly higher frequency of 1.4GHz is employed in the reconstruction of the five objects target since they are relatively smaller. In the LSM technique, the number of incident and measurement directions is proportional to the frequency employed and the anticipated size of the unknown target which can be estimated from the size of the imaging domain [1]. Since both frequency and imaging domain employed in the target in Fig. 1b are larger than those used in Fig. 1a, 20 incident and 20 measurement angles are employed in reconstructing Fig. 1b compared with 100 incident and 100 measurement angles employed in Fig. 1b. The angles are uniformly distributed from 0° to 360°. The number of pixels employed in reconstructing Fig. 1a is 250×250 and is 125×125 pixels in Fig. 1b, to keep the computational time low since more angles are employed in the this case.

The contours of all values of $\log\left\| g_z^{\alpha} \right\|$ are plotted for the star-shaped target and for the five objects target in Fig. 2a and Fig. 2b, respectively. The contour plots in Fig. 2 give a general idea of the target shape and location. The reconstructions in Fig. 2a and Fig. 2b are achieved within 4 minutes and 9.6 minutes, respectively. The best contour in both Fig. 2a and Fig. 2b is selected and plotted versus the true target in Fig. 3a and Fig. 3b, respectively. It is clear that the LSM reconstructions are relatively close to the shape of the true target. Hence, they are anticipated to provide efficient initial guesses to the level set algorithm especially since the LSM reconstructions are achieved in CPU minutes versus the CPU hours required when using the level set algorithm [2]-[3]. The comparison of the CPU time and accuracy of the level set algorithm versus the LSM will be presented in the conference. Results using the hybridized level set / LSM algorithm will also be presented in the conference.

IV. CONCLUSIONS

The LSM technique was proven capable of reconstructing a general estimate of the target in few CPU minutes. This estimate will be used as an initial guess to the level set algorithm in order to accelerate the reconstruction. The effect of several of the LSM parameters, such as the number of pixels in the computational domain, the number of incident and measurement angles, the threshold for $\log\left\| g_z^{\alpha} \right\|$, on accelerating the level set algorithm will be investigated.

ACKNOWLEDGEMENT

This work was sponsored by the Army Research Laboratory and was accomplished under Cooperative Agreement Number **W911NF-10-2-0093**. The views and conclusions contained in this document are those of the authors and should not be interpreted as representing the official policies, either expressed or implied, of the Army Research Laboratory or the U.S. Government. The U.S. Government is authorized to reproduce and distribute reprints for Government purposes notwithstanding any copyright notation herein.

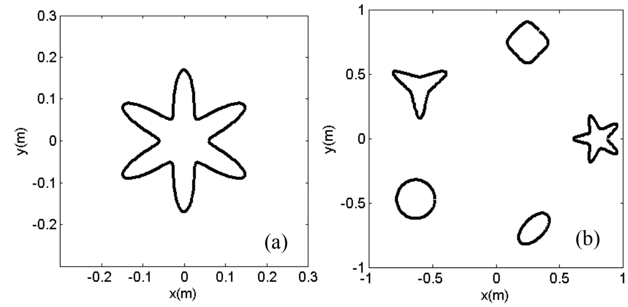

Figure 1. (a) star shaped and (b) five objects 2D PEC targets.

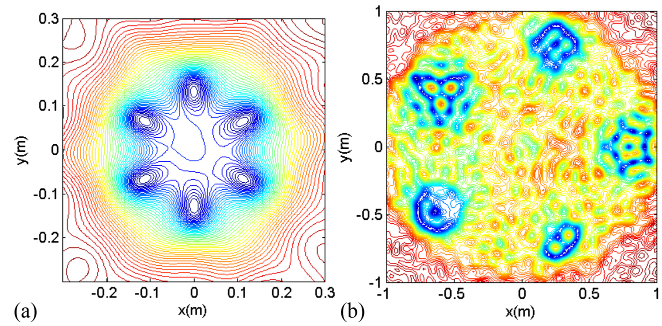

Figure 2. The LSM contours corresponding to different values of $\log\left\| g_z^{\alpha} \right\|$ for (a) the star shaped target and (b) the five objects target.

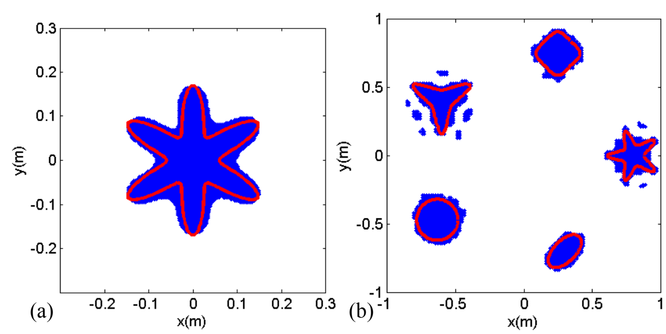

Figure 3. The final LSM final reconstruction obtained by selecting the contour closest to the true (a) star-shaped and (b) the 5 objects target.

REFERENCES

[1] F. Cakoni, D. Colton, P. Monk, *The Linear Sampling Method in Inverse Electromagnetic Scattering*, Society for Ind. and Applied Math., 2011

[2] M. R. Hajihashemi and M. El-Shenawee, “TE versus TM for the Shape Reconstruction of 2-D PEC Targets using the Level-Set Algorithm,” *IEEE Trans. Geosci. & Rem. Sens.*, vol. 48, no. 3, pp. 1159-1168, 2010.

[3] M. R. Hajihashemi and M. El-Shenawee, "Level Set Algorithm for Shape Reconstruction of Non-Overlapping Three-Dimensional Penetrable Targets," *IEEE Trans. Geosci. & Rem. Sens.*, vol.50, no.1, pp.75-86, 2012.