FULL WAVE MULTIPLE SCATTERING FROM ROUGH SURFACES

E. Bahar* and M. El-Shenawee
Electrical Engineering Department, University of Nebraska, Lincoln.

ABSTRACT: Using the full wave approach to rough surface scattering, the single and the double scattered electromagnetic fields from a perfect conducting one dimensionally rough surface are computed.The full wave expressions for the single and the double scattered far fields are given in terms of integrals. Double scatter in the backward direction from perfectly conducting surfaces is larger than single scatter only for near normal incidence when the rough surface mean square slopes are very large. The results could shed light on the observed fluctuations in the enhanced backscatter phenomenon as the angle of incidence varies from near normal to grazing angles.

I. FORMULATION OF THE PROBLEM

For suppressed $\exp(j\omega t)$ time excitation, the full wave solution for the single scattered fields from a two dimensional rough surface $f(x,y,z)= y-h(x,z)=0$ can be expressed as follows in the matrix form [1], [2] :

$$G^f = \left(\frac{k_o}{2\pi j}\right)^2 \int\!\!\int D(\bar{n}',\bar{n}^i)\, e^{j\bar{v}'\cdot\bar{r}_s'}\, e^{-jk_o\bar{n}'\cdot\bar{r}'}\, \frac{dn_y'}{n_x'}\, dn_z'\, \frac{dx_s\, dz_s}{\bar{n}\cdot\bar{a}_y}\, G^i \tag{1a}$$

where,

$$k_o\bar{n}' = k_o\,(n_x'\bar{a}_x + n_y'\bar{a}_y + n_z'\bar{a}_z), \qquad k_o\bar{n}^i = k_o(n_x^i\bar{a}_x + n_y^i\bar{a}_y), \tag{1b}$$

$$\bar{v}' = k_o(\bar{n}' - \bar{n}^i), \tag{1c}$$

$$\bar{r}_s' = x_s\bar{a}_x + h(x_s,z_s)\bar{a}_y + z_s\bar{a}_z, \qquad \bar{r}' = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z, \tag{1d}$$

$$\bar{n} = \nabla f/|\nabla f| = (-h_x\bar{a}_x + \bar{a}_y - h_z\bar{a}_z)/(1+h_x^2+h_z^2)^{1/2} \tag{1e}$$

in which $k_o=\omega\sqrt{\mu_o\epsilon_o}$ is the free space wavenumber. The radius vectors from the origin to the rough surface and to the observation point are $\bar{r}_s'$ and $\bar{r}'$, respectively , and $\bar{n}'$ and $\bar{n}^i$ are the unit vectors in the directions of the scattered and incident waves. The elements of the 2×1 matrices G^i and G^f are the vertically and the horizontally polarized field components of the incident and scattered waves, respectively, and the elements of the 2×2 scattering matrix $D(\bar{n}',\bar{n}^i)$ depend on the polarizations and the directions of the incident and scattered waves, the media on both sides of the rough interface, and the unit vector $\bar{n}$ normal to the rough surface [1],[2]. The full wave solutions have also been applied to random rough surfaces [2].

For a one dimensional rough surface $h(x)$, $D(\bar{n}',\bar{n}^i)$ is a diagonal matrix (no depolarization) provided that $\bar{n}^i\cdot\bar{a}_z=0$. In this case the integral in (1a) with respect to z_s reduces to a Dirac Delta function $\delta(n_z')$. Thus on performing the integration with respect to n_z' (1) reduces to:

$$G^f = \frac{-k_o}{2\pi} \int\!\!\int \frac{D(\bar{n}',\bar{n}^i)}{\bar{n}\cdot\bar{a}_y}\, e^{j(\bar{v}'\cdot\bar{r}_s)}\, e^{-jk_o(\bar{n}'\cdot\bar{r})}\, \frac{dn_y'}{n_x'}\, dx_s\, G^i \tag{2a}$$

where,

$$\bar{r}_s = x_s\bar{a}_x + h(x_s)\,\bar{a}_y, \text{ and } \qquad \bar{r} = x\bar{a}_x + y\bar{a}_y \tag{2b}$$

Assuming that $k_o r \gg 1$, we can use the steepest descent method to integrate (2a) with respect to n_y' to obtain the single scattered far fields,

$$G^f = G_o\, k_o \int \frac{D(\bar{n}^f,\bar{n}^i)}{(\bar{n}\cdot\bar{a}_y)}\, e^{j\bar{v}\cdot\bar{r}_s}\, dx_s\, G^i \tag{3a}$$

where $\bar{n}^f$ is the unit vector in the direction of the vector $\bar{r}$ to the observation point and,

90CH2776-3/0000-1548 S1.00 ©1990 IEEE

$$G_o = \frac{-1}{\sqrt{2\pi k_o r}} e^{j\pi/4} e^{-jk_o r}, \quad r = \sqrt{x^2 + y^2}, \quad \bar{v} = k_o(\bar{n}^f - \bar{n}^i) \tag{3b}$$

The full wave expression (2) can also be used to determine the single scattered field G_2 incident upon the rough surface at $\bar{r}_{s2}$(see Fig. 1). Thus using (2a) it follows that

$$G_2 = \frac{-k_o}{2\pi} \int\int \frac{D(\bar{n}' \cdot \bar{n}^i)}{\bar{n}_1 \cdot \bar{a}_y} e^{-jk_o \bar{n}^i \cdot \bar{r}_{s1}} e^{-jk_o \bar{n}' \cdot \bar{r}_{21}} \frac{dn_y'}{n_x'} dx_{s1} G^i \tag{4}$$

where $\bar{r}_{21} = \bar{r}_{s2} - \bar{r}_{s1} = (x_{s2} - x_{s1})\, \bar{a}_x + (h(x_{s2}) - h(x_{s1}))\, \bar{a}_y$, is the radius vector from point 1 to point 2 on the surface in the x-y plane, and $\bar{n}_1$ is the normal to the rough surface at $\bar{r} = \bar{r}_{s1}$.

Assuming that $k_o r_{21} \gg 1$, we can apply the steepest descent method to integrate (4) with respect to n_y'. Thus the integrand in (4) reduces to

$$dG_2 = \frac{-1}{\sqrt{2\pi k_o r_{21}}} e^{j\pi/4} e^{-jk_o r_{21}} \frac{D(\bar{n}_{21}, \bar{n}^i)}{\bar{n}_1 \cdot \bar{a}_y} e^{-jk_o \bar{n}^i \cdot \bar{r}_{s1}} k_o\, dx_{s1} G^i \tag{5}$$

in which $r_{21} = |\bar{r}_{21}|$. The above approximation is obviously not valid for regions around the point $\bar{r}_{s1} \approx \bar{r}_{s2}$, where $k_o r_{21}$ is not sufficiently large.

The full wave solution for the double scattered fields G_d^f, are obtained by replacing $G^i \exp(-j\bar{n}^i \cdot \bar{r}_s)$ in (2a) by G_2. Thus it follows that

$$G_d^f = \frac{-k_o}{2\pi} \int\int \frac{D(\bar{n}'', \bar{n}^i)}{(\bar{n}_2 \cdot \bar{a}_y)} e^{jk_o \bar{n}'' \cdot \bar{r}_{s2}} e^{-jk_o \bar{n}'' \cdot \bar{r}} G_2 \frac{dn_y''}{n_x''} dx_{s2} \tag{6}$$

Assuming that $k_o r \gg 1$, we can apply the steepest descent method to integrate (6) with respect to n_y'',

$$G_d^f = \frac{k_o^2}{4\pi^2} \int\int\int \frac{D(\bar{n}^f, \bar{n}')\, D(\bar{n}', \bar{n}^i)}{(\bar{n}_2 \cdot \bar{a}_y)(\bar{n}_1 \cdot \bar{a}_y)}$$
$$\cdot\, e^{jk_o(\bar{n}^f \cdot \bar{r}_{s2} - \bar{n}^i \cdot \bar{r}_{s1} - \bar{n}' \cdot \bar{r}_{21})} \sqrt{\frac{2\pi}{k_o r}}\, e^{j\pi/4} e^{-jk_o r} \frac{dn_y'}{n_x'} dx_{s1}\, dx_{s2}\, G^i \tag{7}$$

Assuming that $k_o r_{21} \gg 1$, we can apply steepest descent approximation to integrate (7) with respect to n_y'. Thus the integrand of (7) reduces to

$$d^2 G_d^f = \left(\frac{k_o}{2\pi}\right) e^{j\pi/2} \frac{D(\bar{n}^f, \bar{n}_{21})\, D(\bar{n}_{21}, \bar{n}^i)}{(\bar{n}_2 \cdot \bar{a}_y)(\bar{n}_1 \cdot \bar{a}_y)\sqrt{r_{21} r}}$$
$$\cdot\, e^{jk_o(\bar{n}^f \cdot \bar{r}_{s2} - \bar{n}^i \cdot \bar{r}_{s1})} e^{-jk_o r_{21}} e^{-jk_o r} dx_{s1}\, dx_{s2}\, G^i \tag{8}$$

Even though the above approximation for the integrand is only valid when $k_o r_{21} \gg 1$, it can be used to evaluate G_d^f instead of (7) since the scattering coefficients (D) vanish as $\bar{r}_{s2} \to \bar{r}_{s1}$. Note that the point at $\bar{r}_{s1}$ on the rough surface should be illuminated by the incident plane wave and visible at the point at $\bar{r}_{s2}$ on the surface, and the point at $\bar{r}_{s2}$ should be illuminated by a point source at $\bar{r}_{s1}$ on the surface and visible by the observation at $\bar{r}$.

II. GEOMETRIC OPTICS APPROXIMATION

At very high frequencies the major contributions to the double scattered fields come only from the points 1 and 2 on the surface at which the phase $k_o \phi(x_{s1}, x_{s2})$ in the integrand of (8) is stationary. The function $\phi(x_{s1}, x_{s2})$ is given by

$$\phi(x_{s1}, x_{s2}) = (\bar{n}^f \cdot \bar{r}_{s2} - \bar{n}^i \cdot \bar{r}_{s1} - r_{21}) \tag{9}$$

On differentiating $\phi(x_{s1}, x_{s2})$ with respect to x_{s1} and x_{s2} respectively, and equating the two equations simultaneously to zero, we get the solution for the pth pair of stationary phase (specular) points x_{1p}, x_{2p}(see Fig.1). Thus in the high frequency limit, the stationary phase method can be used to evaluate the double integral in (8).

$$G^f_{d.Go.} \approx K \sum_{p=1}^{N} \left\{ \left[e^{-jk_o(\bar{n}^i \cdot \bar{r}_{s1p})} \frac{D(\bar{n}_{21p}, \bar{n}^i)}{(\bar{n}_{1p} \cdot \bar{a}_y)} \right] \cdot \left[\frac{e^{-jk_o r_{21p}}}{\sqrt{k_o r_{21p}}} \right] \right.$$

$$\left. \cdot \left[\frac{D(\bar{n}^f, \bar{n}_{21p})}{(\bar{n}_{2p} \cdot \bar{a}_y)} e^{-jk_o r_2^f} \right] \right\} \qquad , \quad p=1,2,\ldots N \tag{10}$$

where $G^f_{d.Go.}$ is the geometrical optics solution for the double scattered electromagnetic field. The subscript p defines the pth stationary phase path associated with the pair of specular points at $\bar{r}_{s1p}$ and $\bar{r}_{s2p}$. The normals at these points are $\bar{n}_{1p}$ and $\bar{n}_{2p}$. The distance vector between the pth pair of specular points is $\bar{r}_{21p}=\bar{r}_{s2p}-\bar{r}_{s1p}= r_{21p}\bar{n}_{21p}$, where $\bar{n}_{21p}$ is a unit vector. Furthermore, $r_2^f= r-\bar{r}_{s2p}\cdot\bar{n}^f$ is the distance from point 2 on the surface to the observation point and N is the number of the stationary phase paths on the rough surface (see Fig.1). The constant K is proportional to $G^i/(k_o r)^{1/2}$[3]. The diagonal terms of the scattering matrices reduce to the Fresnel reflection coefficients at the stationary phase points x_{1p}, x_{2p}.

III. ILLUSTRATIVE EXAMPLES

The rough surface h(x) is assumed to be a deterministic perfectly conducting one dimensional surface with different heights and mean square slopes to simulate different realizations of random surfaces.

$$h(x) = h_o \cos(2\pi x/\Lambda)\ , \tag{11}$$

where Λ is the period. All distances are normalized with respect to the free space wave length λ_o. In Figs. 2-4, the horizontally polarized single and double scattered fields are plotted as functions of $-\pi/2 \leq \theta^f \cos\phi^f \leq \pi/2$ (in the plane of incidence) for different incident angles and different mean square heights and slopes. The interference between the different doubly scattered contributions along the distinct stationary phase paths (p=1,2,...,N) in (10) could explain the observed fluctuations in the scattered fields near backscatter for normally incident excitations[4].

ACKNOWLEDGMENT

This work was conducted using the Cornell National Supercomputer Facility, and sponsored by the U.S. Army Research Office Contract DAAL0387-K-0085 and ONR Contract A00014-87-K-0177. The authors wish to thank W. Flood and D. Johnsen for their comments and encouragement.

REFERENCES

[1] E. Bahar, Full-Wave Solutions for the Scattered Radiation, Fields from Rough Surfaces with Arbitrary Slope and Frequency, IEEE Trans. Antennas Propagat., vol. AP-28, No. 1, January 1980.

[2] _____, Full-Wave Solutions for the Depolarization of the Scattered Radiation Fields by Rough Surfaces of Arbitrary Slope, IEEE Trans. Antennas Propagat., vol. AP-29, No. 3, May 1981.

[3] Born and Wolf, Principles of Optics, Pergamon Press 1965.

[4] K.A. O'Donnell and E.R. Mendez, An Experimental Study of Scattering from Characterized Random Surfaces, Optics Section, Blacket Laboratory Imperial College, London SW7 2BZ, 1987.

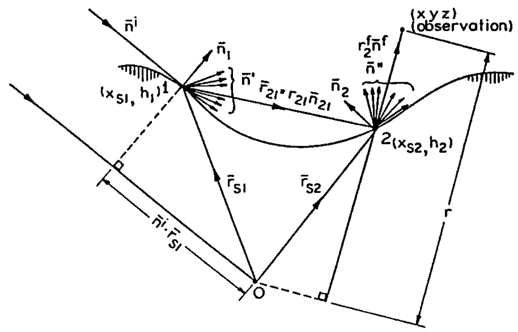

Fig. 1 The double scattered electromagnetic waves.

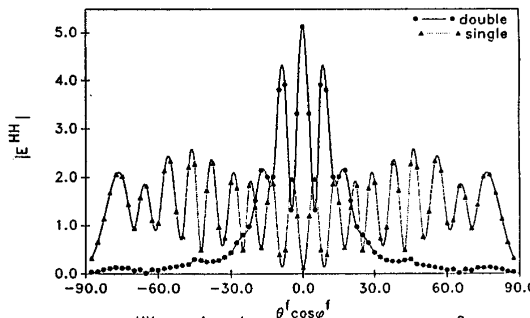

Fig.2 $|E^{HH}|$ for $\Phi^i=0, \theta^i=0, \Lambda=13.2793, h_o=2.1932, \langle h_x^2\rangle=.5384$, max. slope$=46.06^\circ, |\epsilon_r|\gg 1, \mu_r=1$.

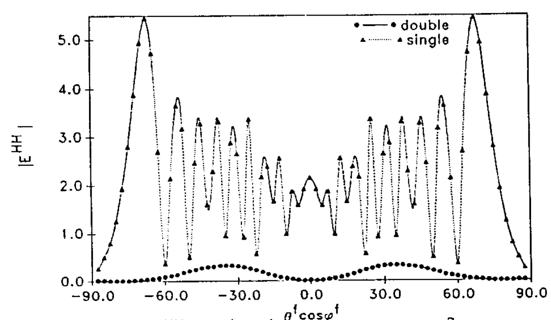

Fig.3 $|E^{HH}|$ for $\Phi^i=0, \theta^i=0, \Lambda=20, h_o=2.3721, \langle h_x^2\rangle=.2776$, max. slope$=36.69^\circ, |\epsilon_r|\gg 1, \mu_r=1$.

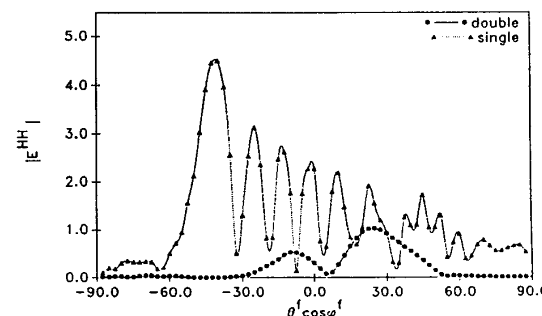

Fig.4 $|E^{HH}|$ for $\Phi^i=0, \theta^i=45^\circ, \Lambda=13.2793, h_o=2.1932, \langle h_x^2\rangle=.5384$, max. slope$=46.06^\circ, |\epsilon_r|\gg 1, \mu_r=1$.