# SINGLE AND MULTIPLE SCATTER AND DEPOLARIZATION FROM TWO DIMENSIONAL ROUGH SURFACES

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**ABSTRACT:** Full wave approach has been used to obtain the like and cross polarized single and double scattered electromagnetic fields from deterministic two dimensional rough surfaces. The single scattered fields are expressed in terms of two dimensional integrals. However the double scattered fields are expressed in terms of four and six dimensional integrals. To evaluate the multiple scatter expressions a supercomputer is needed. The corresponding geometrical optics expressions are derived for the doubly scattered fields in terms of one and two dimensional integrals. The results show that for normal incidence double scatter is significant in the backward direction when the mean square slope of the highly conducting rough surface is large. The magnitude of the total field in the near backscatter direction fluctuates due to the interferences between the different multiple scatter waves and the single scattered wave.

## I. FORMULATION OF THE PROBLEM

The incident field is assumed to be a plane wave and far field expressions are obtained for the scattered electromagnetic waves. For (the suppressed) $\exp(j\omega t)$ time harmonic excitations the full wave solutions for the single scattered fields $G_s^f$ from the two dimensional rough surface $f(x,y,z)=y-h(x,z)=0$ is written in matrix form as [1],[2] :

$$G_s^f = \left(\frac{k_o}{2\pi j}\right)^2 \int\int\int\int D(\bar{n}',\bar{n}^i)\, e^{j\bar{v}'\cdot\bar{r}_s}\, e^{-jk_o\bar{n}'\cdot\bar{r}}\, U(\bar{r}_s)\, \frac{dn'_y}{n'_x}\, dn'_z\, dA_s\, G^i \qquad (1a)$$

where $\bar{n}^i$ and $\bar{n}'$ are unit vectors in the direction of the incident and scattered waves. The position vectors to the observation point and a point on the rough surface are $\bar{r}$ and $\bar{r}_s$, respectively. The rough surface element is $dA_s$. The unit vector normal to the rough surface $f(x,y,z)= y - h(x,z)$ is $\bar{n}$.

$$\bar{n}' = (n'_x\,\bar{a}_x + n'_y\,\bar{a}_y + n'_z\,\bar{a}_z), \quad \bar{n}^i = (n^i_x\,\bar{a}_x + n^i_y\,\bar{a}_y + n^i_z\,\bar{a}_z), \quad \bar{v}' = k_o(\bar{n}' - \bar{n}^i) \qquad (1b)$$

$$dA_s = \frac{dx_s\, dz_s}{\bar{n}\cdot\bar{a}_y}, \quad \bar{r}_s = x_s\,\bar{a}_x + h(x_s, z_s)\,\bar{a}_y + z_s\,\bar{a}_z, \quad \bar{r} = x\,\bar{a}_x + y\,\bar{a}_y + z\,\bar{a}_z \qquad (1c)$$

$$\bar{n} = \nabla f(x,y,z)/|\nabla f(x,y,z)| \qquad (1d)$$

The free space wave number is $k_o=\omega\sqrt{\mu_o\epsilon_o}$ . The elements of $2\times 1$ matrices $G^i$ and $G_s^f$ are the vertically and the horizontally polarized components of the incident and the scattered fields, respectively. The elements of the scattering matrix $D(\bar{n}',\bar{n}^i)$ depend upon the permitivity $\epsilon_r$ and the permeability $\mu_r$ of the medium $y<h(x,z)$, the direction of the normal to the surface $\bar{n}$, the polarization of the incident and scattered fields, and the direction of the incident and scattered fields. The shadow function $U(\bar{r}_s)$ is equal to one for points on the rough surface that are illuminated (by the incident waves) and visible (at the receiver). It is zero otherwise.

Assuming that $k_o r \gg 1$ ,the steepest descent method can be used to integrate (1) with respect to $n'_y$ and $n'_z$. Thus for an observation point in the direction $\bar{n}^f$

$$G_s^f = \left(\frac{k_o}{2\pi j}\right) \frac{e^{-jk_o r}}{r} \int\int \frac{D(\bar{n}^f,\bar{n}^i)}{\bar{n}\cdot\bar{a}_y}\, e^{jk_o(\bar{n}^f-\bar{n}^i)\cdot\bar{r}_s}\, U(\bar{r}_s)\, dx_s\, dz_s\, G^i \qquad (2)$$

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The single scattered fields $G_2(\bar{r}_{s2})$ that impinge upon the rough surface at $\bar{r}_{s2}$ can be obtained from (1a) by replacing $\bar{r}_s$, $\bar{r}$, and $\bar{n}$ by $\bar{r}_{s1}$, $\bar{r}_{s2}$, and $\bar{n}_1$, respectively, where $\bar{n}_1$ is the normal to the rough surface at $\bar{r}_{s1}$ (see fig.1a), thus

$$G_2(\bar{r}_{s2})=(\frac{k_o}{2\pi j})^2 \int \frac{D(\bar{n}',\bar{n}^i)}{\bar{n}_1 \cdot \bar{a}_y} e^{-jk_o\bar{n}' \cdot \bar{r}_{21}} e^{-jk_o\bar{n}^i \cdot \bar{r}_{s1}} U(\bar{r}_{s1}) \frac{dn'_y}{n'_x} dn'_z \, dx_{s1} dz_{s1} \, G^i \quad (3a)$$

$$\bar{r}_{21}=\bar{r}_{s2}-\bar{r}_{s1} \quad (3b)$$

On substituting $G^i \exp(-\bar{n}^i \cdot \bar{r}_s)$ in (1) by $G_2(\bar{r}_{s2})$ in (3a) the double scattered electromagnetic fields $G_d^f$ from the two dimensional rough surface is obtained

$$G_d^f=(\frac{k_o}{2\pi j})^4 \int \frac{D(\bar{n}'',\bar{n}')}{\bar{n}_2 \cdot \bar{a}_y} \frac{D(\bar{n}',\bar{n}^i)}{\bar{n}_1 \cdot \bar{a}_y} e^{-jk_o\bar{n}' \cdot \bar{r}_{21}} e^{-jk_o\bar{n}^i \cdot \bar{r}_{s1}} e^{jk_o\bar{n}'' \cdot \bar{r}_{s2}} e^{-jk_o\bar{n}'' \cdot \bar{r}}$$

$$\cdot U(\bar{r}_{s1}) U(\bar{r}_{s2}) \frac{dn'_y \, dn'_z}{n'_x} \frac{dn''_y dn''_z}{n''_x} dx_{s1}dz_{s1} \, dx_{s2}dz_{s2} \, G^i \quad (5a)$$

in which $\bar{n}''$ is the direction of the scattered electromagnetic fields from the rough surface at point $\bar{r}_{s2}$. Thus the double scattered fields $G_d^f$ from two dimensional rough surfaces (5a) are given in terms of eight dimensional integrals. Assuming that the receiver is in the far field ($k_o r \gg 1$) the steepest descent method can be used to integrate (5a) with respect to both $n''_y$ and $n''_z$, thus

$$G_d^f= (\frac{k_o}{2\pi j})^3 \frac{e^{-jk_o r}}{r} \int [ G^i e^{-jk_o\bar{n}^i \cdot \bar{r}_{s1}} D(\bar{n}', \bar{n}^i) \frac{dx_{s1} \, dz_{s1}}{(\bar{n}_1 \cdot \bar{a}_y)} U(\bar{r}_{s1})] [\frac{dn'_y \, dn'_z}{n'_x} e^{-jk_o\bar{n}' \cdot \bar{r}_{21}}]$$

$$\cdot [ D(\bar{n}^f, \bar{n}') \frac{dx_{s2} \, dz_{s2}}{(\bar{n}_2 \cdot \bar{a}_y)} e^{jk_o\bar{n}^f \cdot \bar{r}_{s2}} U(\bar{r}_{s2})] \quad (6)$$

Assuming that $k_o r_{21} \gg 1$, the steepest descent method can be applied to integrate (6) with respect to $n'_y$ and $n'_z$ over the spectrum of waves from $\bar{r}_{s1}$ to $\bar{r}_{s2}$. Thus the six dimensional integral in (6) is reduced to the following four dimensional integral,

$$G_d^f = (\frac{k_o}{2\pi j})^2 \frac{e^{-jk_o r}}{r} \int [ G^i e^{-jk_o\bar{n}^i \cdot \bar{r}_{s1}} D(\bar{n}_{21}, \bar{n}^i) \frac{dx_{s1} \, dz_{s1}}{(\bar{n}_1 \cdot \bar{a}_y)} U(\bar{r}_{s1})] [ \frac{e^{-jk_o r_{21}}}{r_{21}} ]$$

$$\cdot [ D(\bar{n}^f, \bar{n}_{21}) \frac{dx_{s2} \, dz_{s2}}{(\bar{n}_2 \cdot \bar{a}_y)} e^{jk_o\bar{n}^f \cdot \bar{r}_{s2}} U(\bar{r}_{s2}) ] \quad (7a)$$

$$r_{21}=((x_{s2}-x_{s1})^2+(h_2(x_{s2},z_{s2})-h_1(x_{s1},z_{s1}))^2+(z_{s2}-z_{s1})^2)^{1/2}, \qquad \bar{n}_{21}=\frac{\bar{r}_{21}}{r_{21}} \quad (7b)$$

In general if the assumption $k_o r_{21} \gg 1$ is not satisfied the steepest descent approximation cannot be used and the six dimensional integral (6) should be evaluated instead. However the expression (7) can be used [1],[2] even if $k_o r_{21}$ is not large because the integrand itself vanishes as $\bar{r}_{s1}$ approaches $\bar{r}_{s2}$.

## II. STATIONARY PHASE GEOMETRICAL OPTICS APPROXIMATIONS

At very high frequencies the major contributions to the doubly scattered fields come from the neighborhood of points on the rough surface where the phase term $k_o\Phi(x_{s1}, z_{s1}, x_{s2}, z_{s2})$ is stationary. For backscatter at normal incidence $\Phi$ is given as

$$\Phi(\rho_1, \theta_1, \rho_2, \theta_2) = - (\rho_1{}^2 + \rho_2{}^2+ h_{21}{}^2- 2 \rho_1 \rho_2 \cos(\theta_2-\theta_1))^{1/2} + h_1+h_2 \quad (8a)$$

where

$$h(x_s, z_s) = - h_o \cos[ \frac{2\pi}{\Lambda} (x_s{}^2+ z_s{}^2)^{1/2} ] = - h_o \cos (\frac{2\pi}{\Lambda} \rho) , \quad \frac{-\Lambda}{2} \le x_s, z_s \le \frac{\Lambda}{2} \quad (8b)$$

$$h_{21} = h_2 - h_1 , \qquad h_i = h(x_{si}, z_{si}) , \quad i = 1, 2 \quad (8c)$$

On setting the partial derivatives of $\Phi$ with respect to $\rho_1$, $\rho_2$, $\theta_1$, and $\theta_2$ equal to zero one obtains the locus of the stationary phase points. For backscatter at normal incidence the stationary phase points lie on circles of radius $\rho_{op}(\theta_1, \theta_2)$ given by

$$\rho_{op}(\theta_1, \theta_2)= \frac{\Lambda}{2\pi} \sin^{-1}\left( \frac{\Lambda}{2\pi h_o} \sin\left(\frac{\theta_2-\theta_1}{2}\right) \right) \tag{9}$$

Thus integrating (7a) with respect to $\rho_1$ and $\rho_2$ using the stationary phase approximation [3] one gets

$$G_d^f = \left(\frac{k_o}{2\pi j}\right)^2 \frac{e^{-jk_o r}}{r} \sum_{p=1}^{N} \left\{ \int_0^{2\pi} \int_0^{2\pi} \left[ \frac{D(\bar{n}^f,\bar{n}_{21})\, D(\bar{n}_{21},\bar{n}^i)}{(\bar{n}_2 \cdot \bar{a}_y)\,(\bar{n}_1 \cdot \bar{a}_y)} \rho_1 \rho_2 \frac{2\pi j\sigma}{k_o\sqrt{|\alpha\beta-\gamma^2|}} \right.\right.$$

$$\left.\left. \cdot \frac{e^{jk_o((\rho_1^2+\rho_2^2-2\rho_1\rho_2\cos(\theta_2-\theta_1))^{1/2} -2h)}}{r_{21}} \right]\Bigg|_{\rho_1=\rho_2=\rho_{op}(\theta_1,\theta_2)} d\theta_1\, d\theta_2 \right\} \tag{10a}$$

The stationary phase paths from point $\bar{r}_{s1}$ to point $\bar{r}_{s2}$ on the circles $\rho_{o1}$ and $\rho_{o2}$ are along the diameter, since $\theta_2 - \theta_1 = \pi$ (see fig.1b), thus on integrating (10a) with respect to $\theta_2$ we get

$$G_d^f = \left(\frac{k_o}{2\pi j}\right)^2 \frac{e^{-jk_o r}}{r} \sum_{p=1}^{N} \left\{ \int_0^{2\pi} \left[ \frac{D(\bar{n}^f,\bar{n}_{21})}{(\bar{n}_2 \cdot \bar{a}_y)} \frac{D(\bar{n}_{21},\bar{n}^i)}{(\bar{n}_1 \cdot \bar{a}_y)} \frac{e^{jk_o\Phi(\rho_1,\rho_2,\theta_1,\theta_1+\pi)}}{r_{21}} \frac{2\pi j\sigma}{k_o\sqrt{|\alpha\beta-\gamma^2|}} \right.\right.$$

$$\left.\left. \cdot \left(\frac{2\pi\rho_1\, \rho_2(\rho_1{}^2+\rho_2{}^2+2\rho_1\, \rho_2)^{1/2}}{k_o}\right)^{1/2} \right]\Bigg|_{\rho_1=\rho_2=\rho_{op}} e^{j\pi/4}\, d\theta_1 \right\} \tag{10b}$$

Note that in (10a) $\rho_{op}(\theta_1,\theta_2)$ is a function of $(\theta_2-\theta_1)$ and in (10b) it is a constant. The number of stationary phase circles is $N=2$ when the maximum slope of the rough surface is greater than 45° and $N=1$ when it is exactly 45°. There are no stationary phase circles if the maximum slope of the rough surface is less than 45°. The parameters $\alpha$, $\beta$, $\gamma$, and $\sigma$ are associated with the second derivatives of $\Phi$ [3].

## III. ILLUSTRATIVE EXAMPLES

The magnitudes of the single and the double scatter fields and of the phasor sum are plotted as functions of $\theta^f\cos\phi^f$ (in the incident plane $\phi^i - \phi^f=0, \pi$) (figs.2,3). The height of the surface is given by (8b) with $\Lambda/\lambda_o=9.27932$ and $h_o/\lambda_o=1.5932$. Thus the mean square slope is $<h_\rho>= 0.5819$ and the maximum slope is 47.17°. The medium $y<h(x, z)$ is assumed to be gold ($\epsilon_r= -11.43$ -j1.24 at $\lambda_o=0.633\mu m$, $\mu_r=1$).


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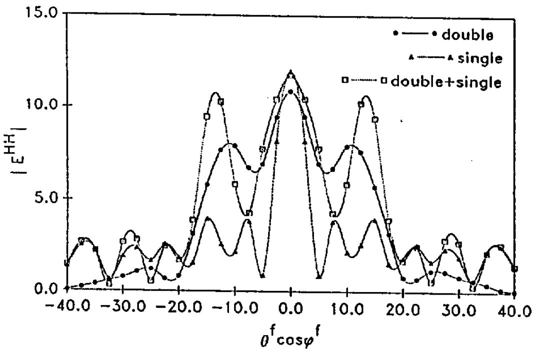


Fig. 2

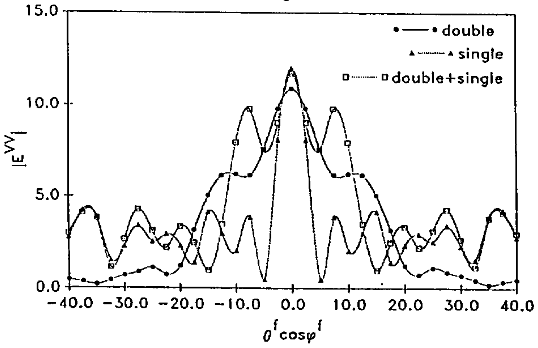


Fig. 3

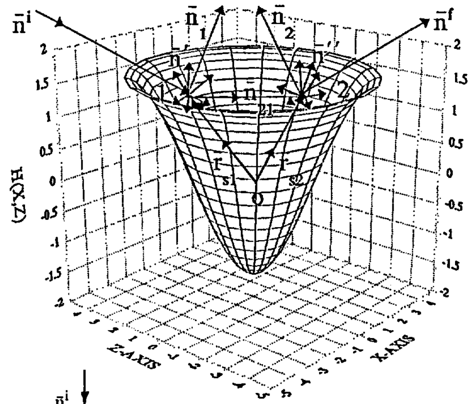


Fig. 1a

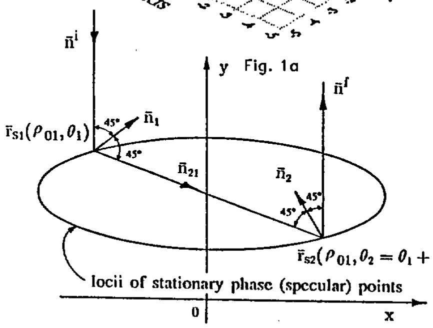


Fig. 1b