MULTIPLE SCATTERING FROM RANDOM DISTRIBUTIONS OF INDIVIDUAL ROUGH SURFACE SCATTERERS

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ABSTRACT: Full wave approach is used to compute the single and the double scattered intensities from random rough surfaces. In this work the surface is assumed to consist of random distributions of individual rough surface scatterers with different mean square height, slope and mean depth. The results show that the double back scattered intensity is significant for normal incidence if the mean square slope is large and the scatterer is highly conducting.

I. FORMULATION OF THE PROBLEM

The model consists of random distributions of individual rough surface scatterers with different mean square height, slope, and mean depth. For simplicity it is assumed here that the surface of the individual scatterer is given by

$$h(x) = h_o \cos(\frac{2\pi x}{\Lambda}) + h_m \tag{1}$$

The mean depth of the individual scatterer is h_m, the mean square height is $<h^2>=h_o^2/2+h_m^2$, and the mean square slope is $(2\pi h_o/\Lambda)^2/2$. Thus h_o, Λ, h_m, and the location l_d of the individual surface scatterers are random variables in the expression for the double scattered fields (see fig.1). The average of the scattered intensity contains the factor [1]

$$<e^{jv_y(h_{mk} - h_{ml})}> = \sum_{k,l=1}^{N} e^{jv_y(h_{mk}-h_{ml})} p(h_{mk}, h_{ml})$$
$$= [\frac{N+N(N-1)\,|<e^{jv_y h_m}>|^2}{N^2}] \tag{2a}$$

where N is the number of individual scatterers.

$$v_y = k_o(\bar{n}^f - \bar{n}^i)\cdot \bar{a}_y \tag{2b}$$

In the above expression it is assumed that

$$p(h_{mk},h_{ml}) = p(h_{mk})\, p(h_{ml}) \qquad \text{for } k \neq l \tag{2c}$$

Furthermore it is assumed that $p(h_m)$ is Gaussian with zero mean and variance equal to $<h_m^2>$. Thus for $N \gg 1$ (2a) reduces to

$$<e^{jv_y(h_{mk}-h_{ml})}> = e^{-v_y^2<h_m^2>} \tag{3}$$

The above expression (3) is used as a factor to account for the fluctuations in the mean depth h_m. For the illustrative examples $4k_o^2<h_m^2>= 0.2$.

The surface height random variables h_o and Λ in (1) are assumed to be characterized by uniform or truncated Chi-squared joint probability density functions. Furthermore h_o and Λ are assumed to be uncorrelated random variables. The average of

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the scattered field magnitude square $|E(h_o,<h>,\Lambda,l_d)|^2$ over the surface height constants h_o and the width Λ is given by the double summation

$$<|E(h_o,\Lambda)|^2> = \sum_{h_o}\sum_{\Lambda} |E(h_o,\Lambda)|^2 \, P(h_o) \, P(\Lambda) \qquad (4a)$$

in which $E(h_o,\Lambda)$ are the full wave solutions for the single or the double scattered fields [2]-[5] while $P(h_o)$ and $P(\Lambda)$ are the probability density functions for h_o and Λ, respectively. The mean of the surface height is $<h_o>=h_{oo}$ and the mean of the surface width is $<\Lambda>=\Lambda_o$. Since the double scattered fields are evaluated for a finite range of h_o and Λ $((h_{oo}-\Delta h_o) \le h_o \le (h_{oo}+\Delta h_o)$, $(\Lambda_o-\Delta\Lambda) \le \Lambda \le (\Lambda_o+\Delta\Lambda))$ the truncated probability density functions are normalized such that

$$\int_{h_{oo}-\Delta h_o}^{h_{oo}+\Delta h_o} P(h_o)\, dh_o = 1, \quad \text{and} \quad \int_{\Lambda_o-\Delta\Lambda}^{\Lambda_o+\Delta\Lambda} P(\Lambda)\, d\Lambda = 1 \qquad (4b)$$

Let the random distance between two individual surface scatterers be l_d. The random variable l_d is assumed here to be characterized by triangular probability density function given by [1]

$$P(l_d) = \begin{cases} -l_d\left(\frac{1}{\Lambda_T^2}\right) + \left(\frac{1}{\Lambda_T}\right), & \text{for} \quad 0 < l_d < \Lambda_T \\ l_d\left(\frac{1}{\Lambda_T^2}\right) + \left(\frac{1}{\Lambda_T}\right), & \text{for} \quad -\Lambda_T < l_d < 0 \end{cases} \qquad (5)$$

in which Λ_T is assumed to be $(10\Lambda_o)$ and Λ_o is the median width of the individual surface scatterer. The statistical average over l_d of the scattered field magnitude square is obtained from the following expression

$$<e^{jv_x l_d}> = \text{sinc}^2\left(\frac{v_x\Lambda_T}{2}\right) \qquad (6a)$$

where

$$v_x = k_o(\bar{n}^f-\bar{n}^i)\cdot\bar{a}_x \qquad (6b)$$

Thus averaging over the location distance l_d is equivalent to multiplying the magnitude square scattered fields by (6a). Thus

$$<|E(h_o,h_m,\Lambda,l_d)|^2> = \text{sinc}^2\left(\frac{v_x\Lambda_T}{2}\right) e^{-v_y^2<h_m^2>} \cdot\left\{\sum_{h_o}\sum_{\Lambda}|E(h_o,\Lambda)|^2 \, P(h_o)\, P(\Lambda)\right\} \qquad (7)$$

in which the limits of the first summation (for h_o) are $h_{omin.}$ and $h_{omax.}$ while the limits of the second summation (for Λ) are $\Lambda_{min.}$ and $\Lambda_{max.}$. The different realizations of h_o and Λ for the one dimensional rough surfaces $h(x)$ are arranged in an $m \times n$ matrix. The number of rows (m) of the matrix represents the different values of the height constant h_o. The number of columns (n) of the matrix represents the different values of the surface width Λ.

II. ILLUSTRATIVE EXAMPLES

The elements of the $m \times n=11 \times 33$ matrix represent 363 realizations of one dimensionally rough surface scatterers of different heights h_o and widths Λ (1). For all the examples presented here, $4k_o^2<h_m^2> = 0.2$ and $\Lambda_T = 10\Lambda_o$.

For the random surface realizations the median height is $h_{oo}/\lambda_o = 2.0$ and the median width is $\Lambda_o/\lambda_o = 14.0$ corresponding to a mean square slope $<h_x^2>$=0.402 and maximum slope 41.9°. The relative permittivity of the surfaces is assumed to be ϵ_r = - 11.43 - j1.24 (gold coated at $\lambda_o = 0.633\mu m$). In fig.2 the magnitude of the horizontally polarized scattered fields (single and double) are plotted in the plane of incidence ($\phi^i - \phi^f = 0, \pi$). The results in fig.2 show that for this realization the single scattered fields are larger than the double scattered fields (even for backscatter) at normal incidence. This is because for a maximum slope of 41.9°, pairs of stationary phase points do not exist for backscatter at normal incidence [2]-[5]. The single and the double scattered incoherent intensities $<|E^{HH}|^2>$ averaged over 363 realizations are shown in fig.3 in which truncated χ^2 distributions of h_o and Λ are assumed. The effects of including the random variables l_d and h_m are presented in fig.4. The ensemble considered in fig.4 contains realizations with maximum slope greater than 45°. Thus for backscatter at normal incidence, pairs of stationary phase points do exist and double scatter is larger than single scatter. These preliminary results can be used to interpret recent experimental data [6].

ACKNOWLEDGEMENT

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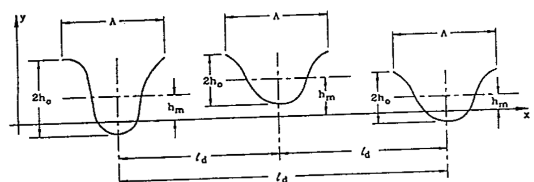

Fig.1 Rough surface model consisting of random distributions of individual surface scatterers.

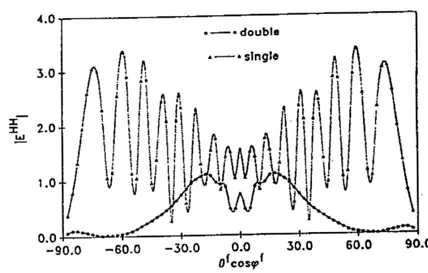

Fig. 2 $|E^{HH}|$ for $\theta^i = 0$, $\phi^i = 0$, $\Lambda = 14.0$, $h_o = 2.0$, $< h_x^2 > = 0.4028$, max. slope = 41.911°, gold ($\epsilon_r = -11.43 - j1.24$, $\lambda_o = 0.633\ \mu m$).

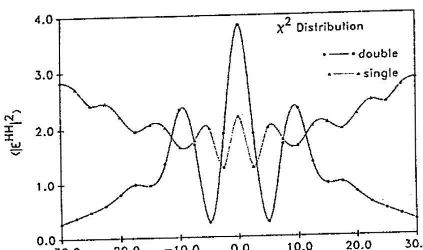

Fig. 3 $< |E^{HH}|^2 >$ for $\theta^i = 0$, $\phi^i = 0$, $\Lambda_o = 14.0$, $h_{oo} = 2.0$, $< h_x^2 > = 0.4028$, max. slope = 41.911°, gold ($\epsilon_r = -11.43 - j1.24$, $\lambda_o = 0.633\ \mu m$).

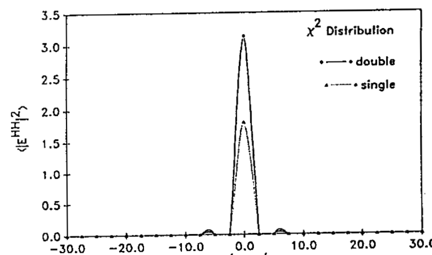

Fig. 4 $< |E^{HH}|^2 >$ for $\theta^i = 0$, $\phi^i = 0$, $\Lambda_o = 14.0$, $h_{oo} = 2.0$, $< h_x^2 > = 0.4028$, max. slope = 41.911°, gold ($\epsilon_r = -11.43 - j1.24$, $\lambda_o = 0.633\ \mu m$).