FULL WAVE MULTIPLE SCATTERING FROM ONE DIMENSIONAL RANDOM ROUGH SURFACES AND HIGH FREQUENCY STATIONARY PHASE APPROXIMATIONS

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ABSTRACT: Using the full wave approach, integral expressions for the double scattered radar cross sections are given. The rough surface is assumed to be characterized by a Gaussian joint probability density function for the surface heights and slopes at two points. The surface height autocorrelation function and its Fourier transform (the rough surface spectral density function) are also assumed to be Gaussian. The expressions for the double scattered cross section are expressed as six dimensional integrals which account for the correlations between the heights and the slopes of the random rough surface. In this paper we also use the stationary phase approximation to reduce the expressions for the double scattered cross section from six to two dimensional integrals. For the stationary phase approximations to the full wave solutions it is not necessary to assume Gaussian joint probability density functions or Gaussian surface height autocorrelation functions. It is shown that enhanced backscatter is due to double scatter when the rough surfaces mean square slopes and heights are large.

I. FORMULATION OF THE PROBLEM

The full wave solutions for the double scattered far fields $G_d^f(\bar{r})$ from one dimensional rough surfaces ($y=h(x)$) is given by [1]

$$G_d^f(\bar{r})=\frac{k_o^2}{4\pi^2}\sqrt{\frac{2\pi}{k_o r}}\, e^{j\pi/4}\, e^{-jk_o r}\int D_2(\bar{n}^f,\bar{n}')\, D_1(\bar{n}',\bar{n}^i)\ \exp\{jk_o(\bar{n}^f\cdot\bar{r}'_{s2}-\bar{n}^i\cdot\bar{r}'_{s1}-\bar{n}'\cdot\bar{r}'_{21})\}$$
$$\cdot\, U(\bar{r}'_{s1})\, U(\bar{r}'_{s2})\,\frac{dn'_y}{n'_x}\, dx'_{s1}\, dx'_{s2}\, G^i(0) \qquad (1a)$$

in which $\bar{n}^i$ and $\bar{n}^f$ the unit vectors in the directions of the incident and scattered fields, respectively, are given by

$$\bar{n}^i=n_x^i\,\bar{a}_x + n_y^i\,\bar{a}_y, \qquad \bar{n}^f=n_x^f\,\bar{a}_x + n_y^f\,\bar{a}_y \qquad (1b)$$

and $\exp(j\omega t)$ time excitations are assumed. The spectrum of the scattered fields from the surface at $\bar{r}'_{s1}$ and to the surface at $\bar{r}'_{s2}$ are in the direction of the unit vector $\bar{n}'$ (see fig.1). The free space wave number is $k_o=\omega\sqrt{\mu_o\epsilon_o}$. The scattering matrices at points 1′ and 2′ on the surface are $D_1(\bar{n}',\bar{n}^i)$ and $D_2(\bar{n}^f,\bar{n}')$, respectively. The elements of these scattering matrices depend on the polarizations of the incident and scattered waves, the media on both sides of the rough interface, and the local normals $\bar{n}'_1$ and $\bar{n}'_2$ at points 1′ and 2′, respectively. The elements of the 2×1 matrices $G^i(0)$ and $G_d^f(\bar{r})$ are the vertically and the horizontally polarized field components of the incident and scattered fields, respectively. For high frequencies the shadow function $U(\bar{r}'_{s1})$ is equal to one if the point at $\bar{r}'_{s1}$ is illuminated by the incident waves and visible to the point at $\bar{r}'_{s2}$. The shadow function $U(\bar{r}'_{s2})$ is equal to one if the point at $\bar{r}'_{s2}$ is illuminated by a source at $\bar{r}'_{s1}$ and visible to the receiver [2]. For one dimensionally rough surfaces

$$\bar{r}'_{s1}=x'_{s1}\bar{a}_x+h(x'_{s1})\bar{a}_y,\ \ \bar{r}'_{s2}=x'_{s2}\bar{a}_x+h(x'_{s2})\bar{a}_y,\ \ \bar{r}'_{21}=\bar{r}'_{s2}-\bar{r}'_{s1},\ \ \bar{n}'=n'_x\bar{a}_x+n'_y\bar{a}_y \qquad (1c)$$

Details of the derivation of the full wave solutions are given elsewhere [1]. The double scatter intensity is obtained by multiplying the expression for the field (1a) by its complex conjugate $G_d^f(\bar{r})^*$. Thus,

$$<\sigma_{double}>=\frac{k_o^3}{4\pi^2}\frac{1}{2L}\int <D_{2'}(\bar{n}^f,\bar{n}')\ D_{1'}(\bar{n}',\bar{n}^i)\ D_{2''}^*(\bar{n}^f,\bar{n}'')\ D_{1''}^*(\bar{n}'',\bar{n}^i)$$

$$\cdot \exp\{jk_o(n_x^f(x_{s2}'-x_{s2}'')-n_x^i(x_{s1}'-x_{s1}'')-n_x'(x_{s2}'-x_{s1}')+n_x''(x_{s2}''-x_{s1}''))\}$$

$$\cdot \exp\{jk_o(n_y^f(h_2'-h_2'')-n_y^i(h_1'-h_1'')-n_y'(h_2'-h_1')+n_y''(h_2''-h_1''))\}$$

$$\cdot U(\bar{r}_{s1}')\ U(\bar{r}_{s1}'')U(\bar{r}_{s2}')\ U(\bar{r}_{s2}'')>\ dx_{s1}'\ dx_{s1}''\ dx_{s2}'\ dx_{s2}''\ \frac{dn_y'}{\sqrt{1-(n_y')^2}}\ \frac{dn_y''}{\sqrt{1-(n_y'')^2}} \tag{2a}$$

in which $\bar{n}''$ is a unit vector in the direction of the spectrum of the conjugate scattered fields from point 1'' to point 2'' on the surface. The heights at the points 1', 1'', 2', and 2'' on the surface are given by

$$h_1'=h(x_{s1}'),\quad h_1''=h(x_{s1}''),\quad h_2'=h(x_{s2}'),\quad h_2''=h(x_{s2}'') \tag{2b}$$

and the quantities associated with the conjugate field are

$$\bar{r}_{s1}''=x_{s1}''\bar{a}_x+h(x_{s1}'')\bar{a}_y,\quad \bar{r}_{s2}''=x_{s2}''\bar{a}_x+h(x_{s2}'')\bar{a}_y,\quad \bar{n}''=n_x''\bar{a}_x+n_y''\bar{a}_y \tag{2c}$$

In the high frequency limit, it is assumed that the major contributions to the double scattered cross sections come from the neighborhoods of the specular points of the surface. It is assumed that $k_o\rho \gg 1$, where ρ is the radius of curvature of the rough surface, and that $4k_o^2<h^2> \gg 1$, where $<h^2>$ is the mean square height. Following Barrick [3] the heights and the slopes of any two neighbor points on the rough surface are expanded about the mean point between them. Thus the stationary phase approximations are used to reduce the expressions for the double scatter cross sections from six to two dimensional integrals.

The significant contributions to the double scattered intensities come from two different double scatter paths (see fig.2-a,b). In this manuscript, they are referred to as quasi parallel ($\bar{n}' \approx \bar{n}''$) regular path (see fig.2-a) and the quasi anti-parallel ($\bar{n}' \approx -\bar{n}''$) cross path (see fig.2-b). Applying Barrick's approach [3] to (2) we get for the double scatter regular cross section (see fig.2a)

$$<\sigma_{regular}>=\int_{\theta'=-\pi}^{\pi}\int_{\theta''=-\pi}^{\pi}\Big[\frac{D_2(\bar{n}^f,\bar{n}')\ D_1(\bar{n}',\bar{n}^i)\ D_2^*(\bar{n}^f,\bar{n}'')\ D_1^*(\bar{n}'',\bar{n}^i)}{[-n_y^i+(n_y'+n_y'')/2]\ [n_y^f-(n_y'+n_y'')/2]}$$

$$\cdot p(h_{x1})\ p(h_{x2})\ \exp\{-k_o^2<h^2>(n_y''-n_y')^2\}(2k_oL_m)\ \mathrm{Sinc}\{k_oL_m(n_x''-n_x')\}\Big]_{h_{x1}=h_{x1s},\ h_{x2}=h_{x2s}}$$

$$\cdot [1-P_2(|n_y'|)]\ [1-P_2(|n_y''|)]\ d\theta'\ d\theta''\ P_2(\bar{n}^i)\ P_2(\bar{n}^f) \tag{3a}$$

where

$$n_y'=\cos\theta',\qquad n_x'=\sin\theta',\qquad n_y''=\cos\theta'',\qquad n_x''=\sin\theta'' \tag{3b}$$

and

$$h_{x1s}=\frac{-[-n_x^i+(n_x'+n_x'')/2]}{[-n_y^i+(n_y'+n_y'')/2]},\qquad h_{x2s}=\frac{-[n_x^f-(n_x'+n_x'')/2]}{[n_y^f-(n_y'+n_y'')/2]} \tag{3c}$$

in which $P_2(|n_y'|)$ and $P_2(|n_y''|)$ are the probabilities that no part of the surface will intersect the rays in the directions of the wave spectra $\bar{n}'$ and $\bar{n}''$ upon scattering by the surface at points 1' and 1'', respectively [2]. Thus $([1-P_2(|n_y'|)]\ [1-P_2(|n_y''|)])$ is the probability that the single scattered field will intersect the surface in the direction of the wave spectra $\bar{n}'$ and $\bar{n}''$. Thus $([1-P_2(|n_y'|)]\ [1-P_2(|n_y''|)])$ represents the probability for multiple scatter to occur. Moreover, the probabilities that the surface will not intersect the incident and the scattered waves are given by $P_2(\bar{n}^i)$ and $P_2(\bar{n}^f)$, respectively [2]. The distance L_m is the mean path for the double scattered wave. The probability density functions for the slopes at the two points of the double scatter path are given by $p(h_{x1})$ and $p(h_{x2})$, and h_{x1s} and h_{x2s} are the slopes at the specular (stationary phase) points. The angles between the unit vectors $\bar{n}'$ and $\bar{n}''$ and the vertical y-direction are θ' and θ'', respectively. Similarly the double scatter cross section for the cross (anti-parallel) path (see fig.2b) is given by

$$<\sigma_{cross}> = 4 \int_{\theta'=-\pi}^{\pi} \int_{\theta''=-\pi}^{\pi} \Big[\frac{D_2(\bar{n}^f,\bar{n}')\, D_1(\bar{n}',\bar{n}^i)\, D_2^*(\bar{n}'',\bar{n}^i)\, D_1^*(\bar{n}^f,\bar{n}'')}{[n_y^f-n_y^i-n_y'+n_y''']\,[n_y^f-n_y^i+n_y'-n_y'']}$$

$$\cdot\, p(h_{x1})\, p(h_{x2}) \exp\{-k_o^2<h^2>(n_y^f+n_y^i-n_y''-n_y')^2\}$$

$$\cdot\, (2k_oL_m)\, \mathrm{Sinc}\{k_oL_m(n_x^f+n_x^i-n_x''n_x')\}\Big]_{h_{x1}=h_{x1s},h_{x2}=h_{x2s}}$$

$$\cdot\, [1-P_2(|n_y'|)]\,[1-P_2(|n_y''|)]\, d\theta'\, d\theta''\, P_2(\bar{n}^i)\, P_2(\bar{n}^f) \qquad (4a)$$

where

$$h_{x1s}=\frac{-[n_x^f-n_x^i+n_x'-n_x'']}{[n_y^f-n_y^i+n_y'-n_y'']}, \qquad h_{x2s}=\frac{-[n_x^f-n_x^i-n_x'+n_x'']}{[n_y^f-n_y^i-n_y'+n_y'']} \qquad (4b)$$

Thus adding (3a) and (4a) we get

$$<\sigma_{double}> = <\sigma_{regular}> + <\sigma_{cross}> \qquad (5)$$

Note that the denominators of (3a) and (4a) vanish only if the slopes at the specular points are infinity. In these cases $p(h_{x1})\to 0$ and $p(h_{x2})\to 0$. It is found that the sharp enhanced backscatter is attributed to the cross term (4) of the double scatter cross section when the rough surface mean square slopes are $<h_x^2> \geq 0.5$ and $4k_o^2<h^2> \geq 40$. Thus the correlation length is of the order of the rms height.

II. ILLUSTRATIVE EXAMPLES

Both the single and the double (regular, quasi parallel and cross, quasi anti-parallel) scatter cross sections are computed for one dimensional random rough surfaces $h(x)$. In figs.3-4, the cross sections $<\sigma>$ are plotted as functions of the scatter angle $\theta^f\cos\phi^f$ where $(\phi^f-\phi^i)=0,\pi$. The mean square slope of the rough surface is $<h_x^2>=0.508$. The Rayleigh parameter is $\beta=41.077$ ($\beta=4k_o^2<h^2>$), the relative permittivity is $\epsilon_r=-424.64-j81.144$, the wave length is $\lambda=3.392\mu$.m., and the angles of incidence are $\theta^i=0$, 10, respectively. These results show that there is no significant difference between the vertically and the horizontally polarized waves when the high frequency approximation is used. Moreover, the sharp enhanced backscatter is due to the cross term of the double scatter cross section. The intensity fluctuations about the backscatter directions are due to interactions between the scattered intensities at the two points on the rough surface. These also appear in the experiments. Preliminary computations using the full wave six dimensional integrals indicate that the double scattered fields are distinctly polarization dependent. These results are also more sensitive to the fluctuations in the mean square height than the corresponding high frequency results. The high frequency full wave results are nevertheless indicative of the backscatter enhancement phenomenon that has been observed in numerous experiments.

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References

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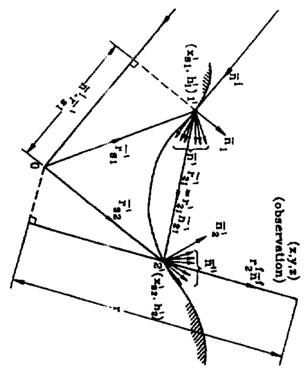

Fig.1 The double scattered electromagnetic waves.

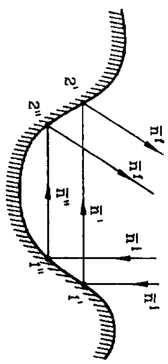

Fig.2-a Quasi-parallel, regular path $\bar{n}' \approx \bar{n}''$

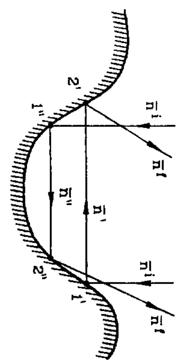

Fig.2-b Quasi-antiparallel, cross path $\bar{n}' \approx -\bar{n}''$

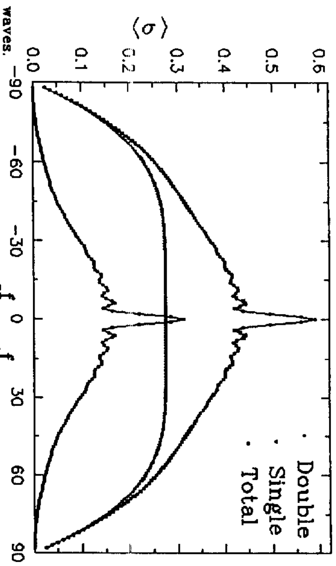

Fig. 3

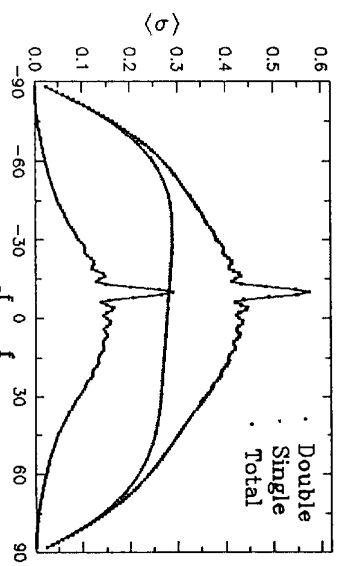

Fig. 4