EXAMINATION OF LARGE RADIUS OF CURVATURE APPROXIMATION FOR ROUGH SURFACE SCATTER CROSS SECTIONS

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ABSTRACT: The objective of this paper is to examine the impact of the large radius of curvature approximation on the random rough surface scatter cross section. The full wave expressions for the single scatter cross sections for one dimensional random rough surfaces are given. The height and slope correlations are accounted for in this work. The single scatter cross section is expressed in terms of three dimensional integrals over the slopes at two distinct points on the surface and the spatial distance between them. Using the large radius of curvature approximation, the solution reduces to a two dimensional integral. This is because the slope variables at two neighboring points (within few correlation lengths) are assumed to be the same. The joint probability density function of the heights and slopes and the autocorrelation function for the heights are assumed to be Gaussian. When the large radius of curvature approximation is not valid, double humps appear in the plots for the cross section around the quasi specular direction.

I. FORMULATION OF THE PROBLEM

For the exp(jωt) time excitations, the full wave solutions for the diffuse single scatter far fields $E_s^{fp}(\bar{r})$ from one dimensional rough surfaces (y = h(x)) are given by [1]

$$E_s^{fp}(\bar{r}) = \frac{-k_0}{\sqrt{2\pi k_0 r}} e^{j(\pi/4 - k_0 r)} \int_{x_s=-2L}^{2L} \frac{D_{\bar{n}}^{p}(\bar{n}^f,\bar{n}^i)}{(n_y^f - n_y^i)} \exp\left(jk_0 x_s (n_x^f - n_x^i)\right) \cdot\left[\exp\left(jk_0 h(x_s)(n_y^f - n_y^i)\right) - 1\right] U(\bar{r}_s) dx_s \quad (1)$$

in which $\bar{n}^i = n_x^i \bar{e}_x + n_y^i \bar{e}_y$ and $\bar{n}^f = n_x^f \bar{e}_x + n_y^f \bar{e}_y$ are the unit vectors in the directions of the incident and scattered fields, respectively. The surface element scattering coefficient $D_{\bar{n}}^{p}(\bar{n}^f,\bar{n}^i)$ is slope dependent and $\bar{n}$ is the local normal to the rough surface [2]. It depends on the polarizations of the incident and scattered waves and the media on both sides of the rough interface. The incident fields are assumed to be plane waves and the receiver is located in the far field. The position vector to any point on the rough surface is $\bar{r}_s = x_s\bar{e}_x + h(x_s)\bar{e}_y$. The free space wave number is $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$. At high frequencies the shadow function $U(\bar{r}_s)$ is equal to one if the point at $\bar{r}_s$ is illuminated by the incident waves and visible at the receiver [3]. The single scatter cross section is obtained by multiplying (1) by its complex conjugate. The statistical average of the incoherent diffuse single scatter cross section is given by [1]:

$$\langle\sigma_I^p\rangle = 2k_0\, P_2(\bar{n}^f)P_2(\bar{n}^i) \int_{x_d=0}^{\infty} \int_{h_{x1}=-\infty}^{\infty} \int_{h_{x2}=-\infty}^{\infty} D_{\bar{n}_1}^{p} D_{\bar{n}_2}^{p*}\left[p(h_{x1},h_{x2})\right\{\chi_2(v_y,-v_y)\exp\left(\frac{v_y^2 B^2}{\langle h_x^2\rangle(1+A)}\right)$$

$$\cdot\cos\left(\frac{v_y B(h_{x1}+h_{x2})}{\langle h_x^2\rangle(1+A)}+v_x x_d\right)-\chi(v_y)\exp\left(\frac{v_y^2 B^2}{2\langle h_x^2\rangle(1-A^2)}\right)\cos\left(\frac{v_y B(h_{x2}-Ah_{x1})}{\langle h_x^2\rangle(1-A^2)}+v_x x_d\right)$$

$$-\chi(v_y)\exp\left(\frac{v_y^2 B^2}{2\langle h_x^2\rangle(1-A^2)}\right)\cos\left(\frac{v_y B(h_{x1}-Ah_{x2})}{\langle h_x^2\rangle(1-A^2)}+v_x x_d\right)\Bigg\}$$

$$-p(h_{x1})p(h_{x2})\left\{\chi^2(v_y)-2\chi(v_y)\right\}\cos(v_x x_d)\Big]dh_{x2}dh_{x1}dx_d \qquad (2)$$

in which $D^p_{\bar{n}_1}$ and $D^p_{\bar{n}_2}$ are the surface element slope dependent coefficients at two distinct points on the surface. In (2) the correlation between the slopes at the two points on the surface is given by A [1], and the correlation between the height at one point and the slope at the other point is given by B [1]. The surface height autocorrelation function is assumed to be Gaussian. The surface height joint characteristic function is $\chi_2(v_y,-v_y)$ [1], the characteristic function is $\chi(v_y)$, and $\bar{v}=k_0(\bar{n}^f-\bar{n}^i)$. The joint probability density function of the slopes $p(h_{x1},h_{x2})$ is assumed to be Gaussian and the mean square slope is $\langle h_x^2\rangle$. The shadow function for the incident and the scattered waves are given by $P_2(\bar{n}^i)$ and $P_2(\bar{n}^f)$, respectively [4]. Upon using the standard large radius of curvature approximation, the slopes at the two neighboring points on the surface, are assumed to be the same [3]. This reduces the three dimensional integral (2) to the following two dimensional integral:

$$\langle\sigma_I^p\rangle=2k_0P_2(\bar{n}^i)P_2(\bar{n}^f)\int_{x_d=0}^{\infty}\int_{h_x=-\infty}^{\infty}|D^p_{\bar{n}}|^2 p(h_x)\left[\exp\left\{-0.5v_y^2\left(2\langle h^2\rangle(1-R)-\frac{B^2}{\langle h_x^2\rangle}\right)\right\}\right.$$

$$\cdot\cos\left\{v_y\frac{h_x B}{\langle h_x^2\rangle}-v_x x_d\right\}-\exp\left\{-0.5v_y^2\left(\langle h^2\rangle-\frac{B^2}{\langle h_x^2\rangle}\right)\right\}\cos\left\{\frac{v_y}{\langle h_x^2\rangle}Bh_x-v_x x_d\right\}$$

$$\left.-\left\{\exp\left\{-v_y^2\langle h^2\rangle\right\}-\exp\left\{-0.5v_y^2\langle h^2\rangle\right\}\right\}\cos\{v_x x_d\}\right]dh_x dx_d \qquad (3)$$

in which R represents the autocorrelation function for the heights. Notice that in (2) the integral contains two slope variables versus one in (3) where the large radius of curvature approximation is used.

III. NUMERICAL EXAMPLES

The diffuse incoherent single scatter cross sections of one dimensional rough surface are plotted in Figs.1 and 2 as functions of the scatter angle $\vartheta^f\cos\varphi^f$ (where $\varphi^i=0,\varphi^f=0,\pi$). The probability density function of the slopes are assumed to be Gaussian. The mean square slope of the surface is $\langle h_x^2\rangle=0.508$, the root mean square height is $\sqrt{\langle h^2\rangle}=1.73\mu m$, and the correlation length is $l_c=3.43\mu m$. The relative permittivity of the good conducting surface is $\varepsilon_r=-62.787-j4.948$ at wave length λ_0=1.152μm. In Figs.1 and 2, the cross sections are plotted without using the large radius of curvature approximation ($h_{x1}\neq h_{x2}$) and with using the large radius of curvature approximation ($h_{x1}\approx h_{x2}$). The incident angle is zero in Fig.1 and is 10° in Fig.2. The results show double humps in the quasi specular direction when the large radius of curvature approximation is not used (2). These double humps in the quasi specular direction are observed in recent experiments [5].

CONCLUSIONS

Use of the large radius of curvature approximation simplifies the computations of the cross sections of rough surfaces. However when the radii of curvature are not very large double humps appear in the plots for the scatter cross sections about the quasi specular direction.

ACKNOWLEDGMENT

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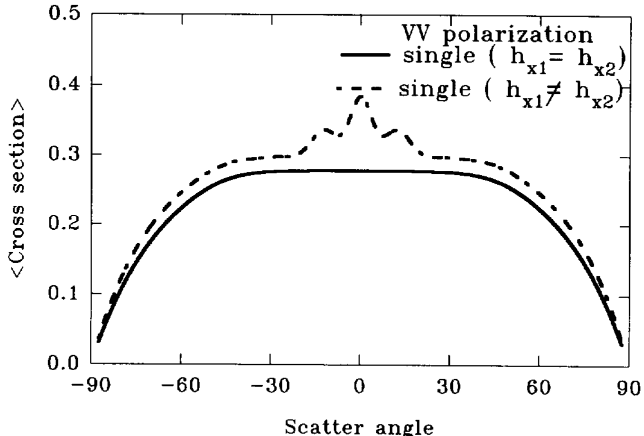

VV polarization
single ($h_{x1} = h_{x2}$)
single ($h_{x1} \neq h_{x2}$)
0.5
0.4
0.3
0.2
0.1
0.0
<Cross section>
−90
−60
−30
0
30
60
90
Scatter angle

Fig. 1

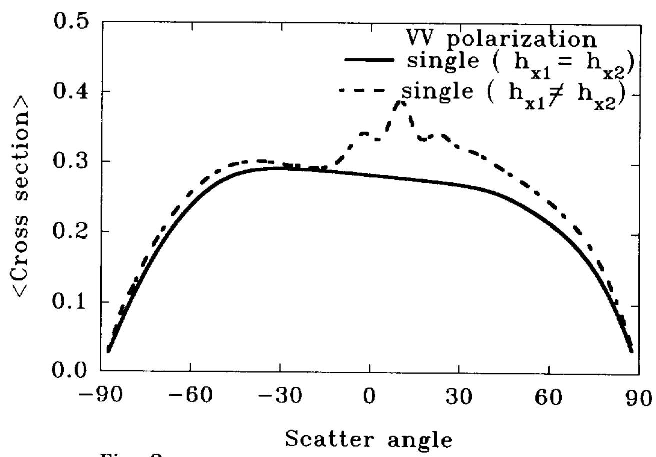

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Fig. 2