Multiple Scattering from Random Rough Surfaces Based on a Deterministic Full Wave Model

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Abstract

Full wave solutions for the double scattered fields from deterministic (sinusoidal) rough surfaces are obtained. It is found that rough surfaces of maximum slope less than 45° do not exhibit backscatter enhancement. The double scattered fields from rough surfaces of maximum slope larger than 45° are significant compared with the single scattered fields. At very high frequencies, the major contribution to the double scattered fields come from those points on the surface at which the phase is stationary. Double scatter radar cross sections for random rough surfaces are obtained based on the deterministic (sinusoidal) model.

FORMULATION OF THE PROBLEM

For the suppressed $\exp(j\omega t)$ time excitation, the full wave solutions for the double scattered far fields from one dimensional rough surfaces are obtained (Bahar and El-Shenawee, 1990). For the one dimensional rough surface y = h(x), the total double scattered far field $G_d^{fP}(\bar{r})$ is given as:

$$G_d^{fP}(\bar{r}) = \frac{k_0^3}{2\pi} e^{j\pi/2} e^{-jk_0 r} \int_{x_{s1}=0}^{\Lambda} \int_{x_{s2}=0}^{\Lambda} \times \frac{D_1^P(\bar{n}_{21},\bar{n}^i) D_2^P(\bar{n}^f,\bar{n}_{21})}{\left[k_0\left(-n_y^i + n_{21y}\right)\right]\left[k_0\left(n_y^f - n_{21y}\right)\right]} \times \frac{\exp\left\{jk_0\left(\bar{n}^f \cdot \bar{r}_{s2} - \bar{n}^i \cdot \bar{r}_{s1} - r_{21}\right)\right\}}{\sqrt{r_{21} r}} \times U(\bar{r}_{s1}) U(\bar{r}_{s2}) dx_{s1} dx_{s2} G^{iP}(0) \quad \text{(1a)}$$

in which $k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$ is the free space wave number and the length of the rough surface is Λ. The surface element slope dependent scattering coefficients are $D_1^P(\bar{n}_{21},\bar{n}^i)$ and $D_2^P(\bar{n}^f,\bar{n}_{21})$. The superscript P in (1a) denotes the polarization of the incident and scatter waves, P = H (horizontal) and P = V (vertical). The incident plane wave amplitude at the origin is $G^{iP}(0)$. The unit vectors in the directions of the incident and scattered waves are $\bar{n}^i$ and $\bar{n}^f$, respectively, see Fig. 1

$$\bar{n}^i = n_x^i \bar{a}_x + n_y^i \bar{a}_y, \text{ and } \bar{n}^f = n_x^f \bar{a}_x + n_y^f \bar{a}_y \quad \text{(1b)}$$

The position vector to the observer is $\bar{r} = x\bar{a}_x + y\bar{a}_y$, while the position vectors to the points at $\bar{r}_{s1}$ and at $\bar{r}_{s2}$ on the rough surface are

$$\bar{r}_{s1} = x_{s1}\bar{a}_x + h(x_{s1})\bar{a}_y, \text{ and } \bar{r}_{s2} = x_{s2}\bar{a}_x + h(x_{s2})\bar{a}_y \quad \text{(2)}$$

The vector from point 1 to point 2 on the rough surface is $\bar{r}_{21} = \bar{r}_{s2} - \bar{r}_{s1}$, and its unit vector is $\bar{n}_{21}$. The local unit vectors normal to the rough surface at points 1 and 2 are given by $\bar{n}_1$ and $\bar{n}_2$, respectively, see Fig 1. For high frequencies, the shadow function $U(\bar{r}_{s1})$ is equal to one if point 1 on the rough surface is illuminated by the incident waves and visible at point 2 on the rough surface (see Fig.1). It is equal to zero otherwise. Similarly, the shadow function $U(\bar{r}_{s2})$ is equal to one if point 2 on the surface is illuminated by a source at point 1 on the surface and observed at the receiver, and is equal to zero otherwise.

At very high frequencies, the major contributions to the double scattered fields come only from the neighborhood of the points at $\bar{r}_{s1}$ and $\bar{r}_{s2}$ on the surface at which the phase $k_0\Phi(x_{s1},x_{s2})$ in the integrand of (1a) is stationary. The phase function is given by

$$\Phi(x_{s1},x_{s2}) = \left(\bar{n}^f \cdot \bar{r}_{s2} - \bar{n}^i \cdot \bar{r}_{s1} - r_{21}\right) \quad \text{(3)}$$

On differentiating $\Phi(x_{s1},x_{s2})$ with respect to x_{s1} and x_{s2} respectively and equating the two equations to zero, the solution is obtained for the pth pair of stationary phase (specular) points x_{s1p} and x_{s2p} (see Fig.2). The incident and the scattered waves are assumed to be at normal in Fig. 2. It is shown, for the typical (sinusoidal) surface depression of maximum slope larger than 45°, that there are four distinct ray paths (p = 1,2,3,4) that contribute to the double scattered fields. One pair is above the level where the surface slope is stationary and the curvature of the surface changes sign. The second pair is below that level.

ILLUSTRATIVE EXAMPLES

The rough surface is assumed to be the one dimensional sinusoidal surface y = h(x) and is given by

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$$h(x) = h_0 \cos(2\pi x / \Lambda) \tag{4}$$

The parameters h_0 and Λ are related to the mean square slope. It is convenient to normalize all distances with respect to the free space wavelength λ_0. In table 1, the stationary phase points (x_{s1p}, x_{s2p}) at $\theta^i = \theta^f = 0$ are given for the rough surfaces which have $\Lambda / \lambda_0 = 13.2793$. When the rough surface height (h_0 / λ_0) is 2.1932 corresponding to mean square slope $\langle h_x^2 \rangle = 0.5384$ and maximum slope 46.06°, the two pairs of stationary phase paths are interchangeable as shown in table 1. This indicates that there are a total of four almost equal stationary phase paths for backscatter normal incidence, see Fig. 2. As the maximum slope of the rough surface decreases from 46.06° to 45°, the two pairs of almost equal stationary phase paths reduce to one pair. On decreasing the maximum slope of the rough surface to 42.15°, no double scatter stationary phase points are found on the surface. In Figs.3 and 4, the magnitudes of the scattered electric field (single, double, single+double) are plotted as functions of the scatter angle $(\theta^f \cos\phi^f)$ at $\theta^i = \theta^f = 0$ and $\phi^i - \phi^f = 0, \pi$. In Fig.3, the rough surface height (h_0 / λ_0) is 2, the corresponding mean square slope is $\langle h_x^2 \rangle = 0.5384$, and the maximum slope is 46.06°. In Fig.4, (h_0 / λ_0) is 2.3721, the corresponding mean square slope is $\langle h_x^2 \rangle = 0.2776$, and the maximum slope is 36.69°. In Fig.3, (maximum slope of the rough surface is 46.06°) there are four distinct double scatter stationary phase paths. While in Fig.4 (maximum slope of the rough surface is 36.69°), there are no stationary phase points on the rough surface. This explains the reduction in the magnitude of the double scattered fields shown in Fig.4.

Full wave solutions for the double scatter radar cross sections of one dimensional random rough surfaces are obtained based on this deterministic (sinusoidal) model. There are two quasi parallel and two quasi anti-parallel pairs of paths associated with each typical depression in the expression for the double scatter radar cross sections (Bahar and El-Shenawee, 1994). Thus there are two quasi parallel and two quasi anti-parallel pairs of paths associated with each typical depression in the expression for the double scatter cross sections, see Figs.5a and 5b. In general there could be more than two quasi parallel and quasi anti-parallel pairs of paths on the average, depending on the mean radii of curvature of the rough surfaces and therefore on the small scale roughness of the rough surfaces. This explains why the observed backscatter enhancement increases significantly as the small scale content of the rough surface increases.

ACKNOWLEDGMENT

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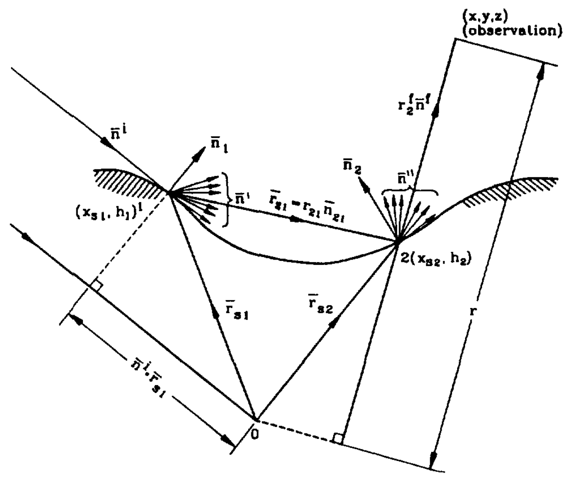

Figure 1.
The double scattered electromagnetic waves.

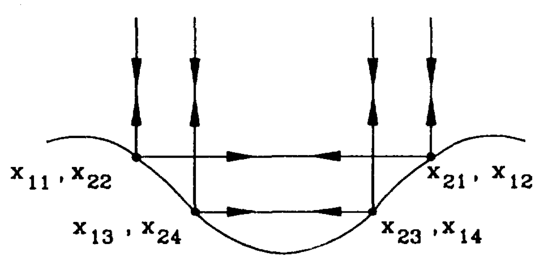

Figure 2.
Stationary phase double scatter paths (p = 1,2,3,4).

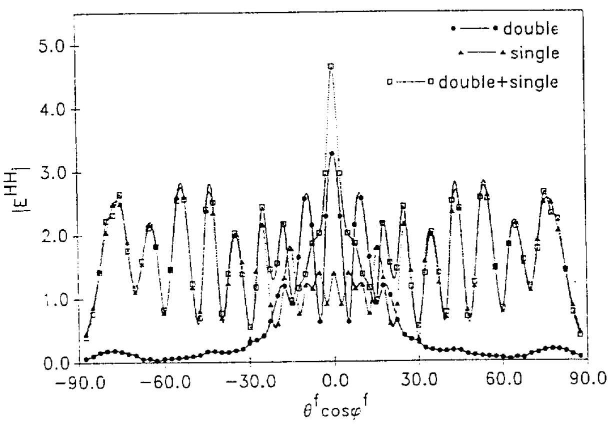

Figure 3.

$\left|E^{HH}\right|$ for $\phi^i = 0, \theta^i = 0, \Lambda = 12.1095, h_0 = 2, \left\langle h_x^2 \right\rangle = 0.5384$

max. $slope = 46.06^o, |\varepsilon_r| >> 1(perfect\ \ conductor), \mu_r = 1$

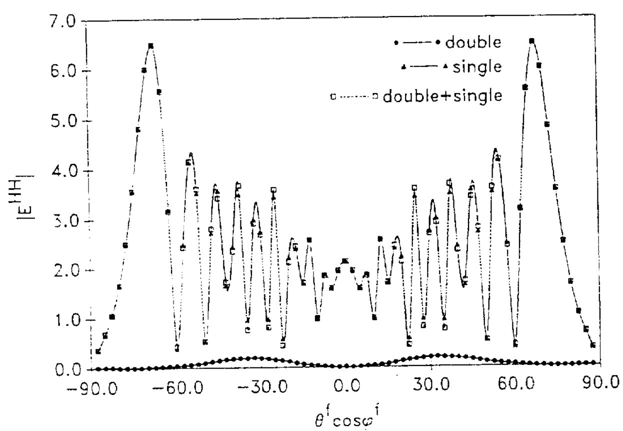

Figure 4.

$\left|E^{HH}\right|$ for $\phi^i = 0, \theta^i = 0, \Lambda = 20, h_0 = 2.3721, \left\langle h_x^2 \right\rangle = 0.2776$

max. $slope = 36.69^o, |\varepsilon_r| >> 1(perfect\ \ conductor), \mu_r = 1$

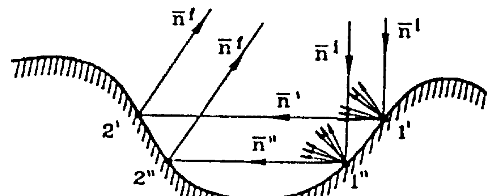

Figure 5a.
Quasi parallel path.

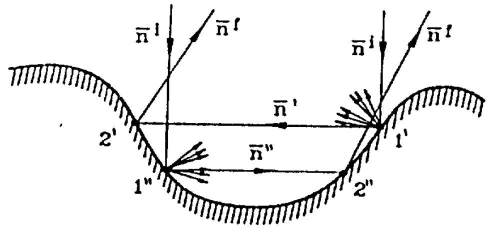

Figure 5b.
Quasi anti-parallel path.

TABLE 1. *The stationary phase points* (x_{s1p}, x_{s2p})

Λ	h_o	$\langle h_x^2 \rangle$	max. slope	x_{1p}	x_{2p}
13.2793	2.1932	0.5384	46.06°	2.7479 10.5313 3.8916 9.3876	10.5313 2.7479 9.3876 3.8916
13.2793	2.11347	0.5	45°	3.3198 9.9595	9.959 3.3197
13.279	1.91347	0.4098	42.15°	–	–