DOUBLE SCATTER RADAR CROSS SECTIONS FOR TWO DIMENSIONAL RANDOM ROUGH SURFACES-HIGH FREQUENCY APPROXIMATION

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ABSTRACT: Full wave solutions for the single and double scatter radar cross sections from two dimensional random rough surfaces are obtained. The solutions are given as multi-dimensional integrals. The high frequency approximation is applied to these expressions to reduce the dimensions of the integrals. The incident waves are assumed to be plane waves. The large radius of curvature approximation is assumed in this work. The high frequency double scatter expressions are given as four dimensional integrals and the single scatter cross sections is given in closed form. This work is an extension of the problem of double scatter from one dimensional random rough surfaces. The numerical results, using the high frequency approximations, show sharp enhancement in the backscatter direction at normal and oblique incident angles. This sharp enhancement is associated with the quasi anti-parallel double scatter path. The level and width of the peak in the backscatter direction depends on the mean square height and slope of the two dimensional random rough surfaces.

I. FORMULATION OF THE PROBLEM

The full wave solution for the double scatter far fields $G_d^f(\bar{r})$ from two dimensional rough surfaces ($y = h(x_s, z_s)$) is given in matrix notation by [1]

$$G_d^f(\bar{r}) = \left(\frac{k_o}{2\pi j}\right)^3 \frac{\exp(-jk_o r)}{r} \int D_{2'}(\bar{n}^f, \bar{n}')\exp(jk_o(\bar{n}^f \cdot \bar{r}_{s2'}))\exp(-jk_o\bar{n}' \cdot (\bar{r}_{s2'} - \bar{r}_{s1'}))$$
$$\times D_{1'}(\bar{n}', \bar{n}^i)\exp(-jk_o\bar{n}^i \cdot \bar{r}_{s1'})\frac{dn'_y dn'_z}{\sqrt{1-n'^2_y-n'^2_z}}U(\bar{r}_{s1'})U(\bar{r}_{s2'})dx_{s1'}dz_{s1'}dx_{s2'}dz_{s2'}G^i(0) \quad (1)$$

in which time harmonic excitations exp(jωt) are assumed and the free space wave number is $k_o = \omega\sqrt{\varepsilon_o\mu_o}$. The incident waves are in the direction $\bar{n}^i$ and the scattered waves are in the direction $\bar{n}^f$ to the receiver at $\bar{r}$, where,

$$\bar{n}^i = n_x^i\bar{e}_x + n_y^i\bar{e}_y + n_{zi}\bar{e}_z, \bar{n}^f = n_x^f\bar{e}_x + n_y^f\bar{e}_y + n_z^f\bar{e}_z; \bar{r} = x\bar{e}_x + y\bar{e}_y + z\bar{e}_z \quad (2)$$

The rough surface element scattering matrices at points $1'$ and $2'$ on the rough surface are $D_{1'}(\bar{n}', \bar{n}^i)$ and $D_{2'}(\bar{n}^f, \bar{n}')$. The elements of the scattering matrices depend on the slope of the rough surface [1]. Moreover, they depend on the polarization of the incident and scattered waves and the media on both sides of the rough interface. The incident fileds are assumed to be plane waves and the receiver is located in the far field. The wave vector associated with the scattered waves at the point on the surface $r_{s1'}$ are in the direction $\bar{n}' = n'_x\bar{e}_x + n'_y\bar{e}_y + n'_z\bar{e}_z$ (see Fig.1). The position vectors to points $1'$ and $2'$ on the rough surfact (see Fig. 1) are respectively given, by

$$\bar{r}_{s1'} = x_{s1'}\bar{e}_x + h(x_{s1'}, z_{s1'})\bar{e}_y + z_{s1'}\bar{e}_z., \text{and} \bar{r}_{s2'} = x_{s2'}\bar{e}_x + h(x_{s2'}, z_{s2'})\bar{e}_y + z_{s2'}\bar{e}_z \quad (3)$$

At high frequencies, the shadow functions $U(\bar{r}_{s1'})$ and $U(\bar{r}_{s2'})$ are equal to one if the point at $\bar{r}_{s1'}$ is illuminated by the incident waves and visible at point $2'$ on the surface and if the point at $\bar{r}_{s2'}$ is illuminated by a point source at $1'$ and visible at the receiver [2]. The incoherent double scatter cross section is obtained on multiplying (1) by its complex conjugate and taking the statistical average of the product. Similar to the problem of scattering from the one dimensional random rough surfaces [3], the major contributions to the double scatter cross sections are associated with the quasi parallel $\bar{n}' \approx \bar{n}''$ and the quasi anti-parallel $\bar{n}' \approx \bar{n}''$ double scatter paths.

The major contributions to the double and single scatter cross sections, in the high frequency limit, come from the neighborhood of the specular points of the rough surface. The heights at any two neighboring points on the two dimensional rough surface are expanded and written as functions of the heights and slopes of the mean point between them. The surface element scattering coefficients are evaluated at the specular points after integrating with respect to the slopes. The probability density functions of the heights and slopes are assumed to be Gaussian. The high frequency approximation is used to reduce the double scatter cross section expressions to four dimensional integrals. The quasi parallel double scatter cross section is expressed as follows

$$\begin{aligned}\langle \sigma_{dp}^{PQ} \rangle &= 2k_o R_m P_2(\bar{n}^i) P_2(\bar{n}^f) \\ &\times \sum_{R,S=V,H} \int \left[D_{2'}^{PS}(\bar{n}^f, \bar{n}') D_{1'}^{SQ}(\bar{n}', \bar{n}^i) D_{2''}^{*PR}(\bar{n}^f, \bar{n}'') D_{1''}^{*RQ}(\bar{n}'', \bar{n}^i) \right] \\ &\times \left. \frac{p(h_{x1}, h_{x2}, h_{z1}, h_{z2})}{\left[n_y^f - (n_y' + n''_y)/2 \right]^2 \left[-n_y^i + (n_y' + n''_y)/2 \right]^2} \right|_{specular\ slope} \\ &\times \frac{J_1(k_o R_m \alpha_p)}{\alpha_p} \exp(-\langle h^2 \rangle k_o^2 (n''_y - n_y')^2) \sin\vartheta' \sin\vartheta'' d\vartheta' d\vartheta'' d\varphi' d\varphi'' \\ P, Q &= V, H \end{aligned} \tag{4}$$

in which J_1 is the Bessel funciton of order one, $\alpha_p = \sqrt{(n_x' - n''_x)^2 + (n_z' - n''_z)^2}$, and $2R_m$ is the mean width of a typical depression on the rough surface [3]. The probability that the surface does not shadow the incident and scattered waves are given by $P_2(\bar{n}^i)$ and $P_2(\bar{n}^f)$, respectively [2]. The symbols V and H denote vertical and horizontal polarizations. The integration variables n_y and n_z in (1) have been changed to the new spherical coordinate variables ϑ and φ in (4). The slopes at the specular points in (4) are given by

$$h_{x1s} = -\frac{\left[-n_x^i + \frac{n_x' + n_x''}{2} \right]}{\left[-n_y^i + \frac{n_y' + n_y''}{2} \right]}, \quad h_{x2s} = -\frac{\left[-n_x^f + \frac{n_x' + n_x''}{2} \right]}{\left[-n_y^f + \frac{n_y' + n_y''}{2} \right]} \tag{5}$$

$$h_{z1s} = -\frac{\left[-n_z^i + \frac{n_z' + n_z''}{2} \right]}{\left[-n_y^i + \frac{n_y' + n_y''}{2} \right]}, \quad h_{z2s} = -\frac{\left[-n_z^f + \frac{n_z' + n_z''}{2} \right]}{\left[-n_y^f + \frac{n_y' + n_y''}{2} \right]} \tag{6}$$

The high frequency quasi anti-parallel double scatter cross section is given by

$$\begin{aligned}\langle \sigma_{dp}^{PQ} \rangle &= 2k_o R_m P_2(\bar{n}^i) P_2(\bar{n}^f) \\ &\times \sum_{R,S=V,H} \int \left[D_{2'}^{PS}(\bar{n}^f, \bar{n}') D_{1'}^{SQ}(\bar{n}', \bar{n}^i) D_{2''}^{*PR}(\bar{n}^f, \bar{n}'') D_{1''}^{*RQ}(\bar{n}'', \bar{n}^i) \right] \\ &\times \frac{p(h_{x1}, h_{x2}, h_{z1}, h_{z2})}{\left[(n_y^f + n_y' - n_y'' - n_y^i)/2 \right]^2 \left[n_y^f - n_y^i - n_y' + n_y'')/2 \right]^2} \Big|_{specular\ slopes} \\ &\times \frac{J_1(k_o R_m \alpha_a)}{\alpha_a} \\ &\times \exp\left(-\langle h^2 \rangle k_o^2 (n_y^f - n_y' - n_y'' + n_y^i)^2 \right) \sin\vartheta' \sin\vartheta'' d\vartheta' d\vartheta'' d\varphi' d\varphi'' \end{aligned} \tag{7}$$

in which $\alpha_p = \sqrt{(-n_x^f + n_x' + n_x'' - n_x^i)^2 + (-n_z^f + n_z' + n_z'' - n_z^i)^2}$ and the slopes at the specular points are given by

$$h_{x1s} = -\frac{\left[-n_x^i + n_x^f + n_x' - n_x''\right]}{\left[-n_y^i + n_y^f + n_y' - n_y''\right]}, \quad h_{x2s} = -\frac{\left[n_x^f - n_x^i - n_x' + n_x''\right]}{\left[n_y^f - n_y^i - n_y' + n_y''\right]} \tag{8}$$

$$h_{z1s} = -\frac{\left[-n_z^i + n_z^f + n_z' - n_z''\right]}{\left[-n_y^i + n_y^f + n_y' - n_y''\right]}, \quad h_{z2s} = -\frac{\left[n_z^f - n_z^i - n_z' + n_z''\right]}{\left[n_y^f - n_y^i - n_y' + n_y''\right]} \tag{9}$$

The sharp enhancement in the backscatter direction ($-\bar{n}^i = \bar{n}^f$) is associated with the quasi anti-parallel ($\bar{n}' \approx -\bar{n}''$) double scatter path (3). Notice that the slopes in (3) are different from (2).

II. ILLUSTRATIVE EXAMPLES

The incoherent double scatter cross sections of two dimensional rough surface are plotted in Figs. 2 and 3 and functions of the scatter angle $\vartheta^f \cos\varphi^f$ (where $\varphi^i = 0, \varphi^f = 0, \pi$). The probability density functions for the heights and the slopes are assumed to be Gaussian. The mean square slope of the surface is $\langle h_x^2 \rangle = 0.508$ and the Raleigh parameter is β=41.65. The two dimensional random rough surface is assumed to be perfectly conducting. The incident angle is zero in Fig.2 and it is 10^O in Fig.3. The results exhibit the sharp enhancement in the backscatter direction.

III. CONCLUSIONS

The results for the double scatter radar cross sections exhibit a sharp enhancement in the backscatter direction for normal and oblique incident angles. This sharp enhancement is associated with the quasi anti-parallel double scatter path. The height and the width of the peak in the backscatter direction depends on the mean square height and slope of the two dimensional random rough surface. The high frequency approximations make the computations more tractable.

IV. ACKNOWLEDGMENT

This work is supported partially by the ARO under grant number DAAL03-87-K-0085, the Center for Electro-Optics at the University of Nebraska-Lincoln and the Supercomputer facility at Cornell University.

V. REFERENCES

1. M. El-Shenawee, "Full Wave Multiple Scattering and Depolarization of Electromagnetic Fields from Rough Surfaces," in partial fulfillment of requirements for the degree of Doctor of Philosophy in Electrical Engineering, University of Nebraska-Lincoln, August 17, 1991.

2. M.I. Sancer, "Shadow-Corrected Electromagnetics Scattering from a Randomly Rough Surface," IEEE Trans. Antennas Propagation, vol. AP-17, No.5, pp.577-585, September 1969.

3. E. Bahar and M. El-Shenawee, "Vertically and Horizontally Polarized Diffuse Double Scatter Cross Sections of One Dimensional Random Rough Surfaces That Exhibit Enhanced Backscatter-Full Wave Solutions," Journal of Optical Society of America A, pp.2271-2285, August 1994.

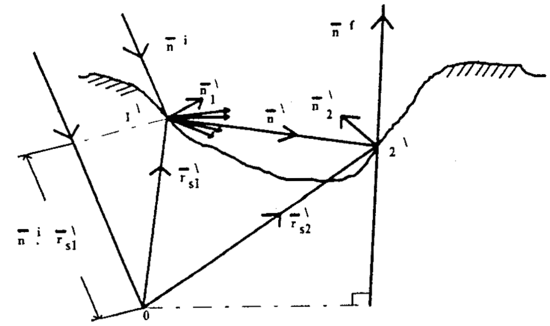

Fig. 1 Double Scatter from Rough Surfaces

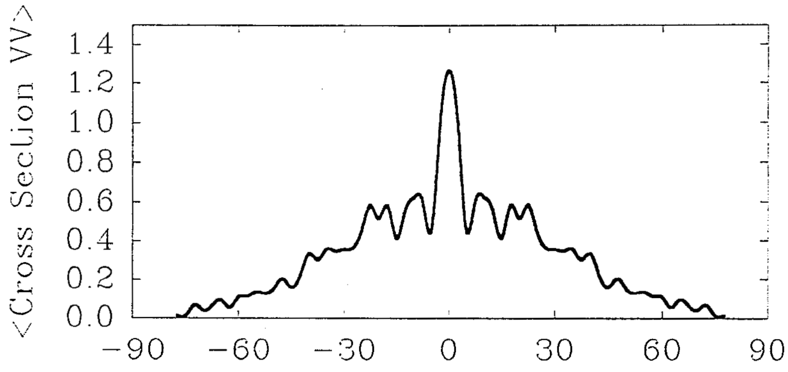

Fig. 2

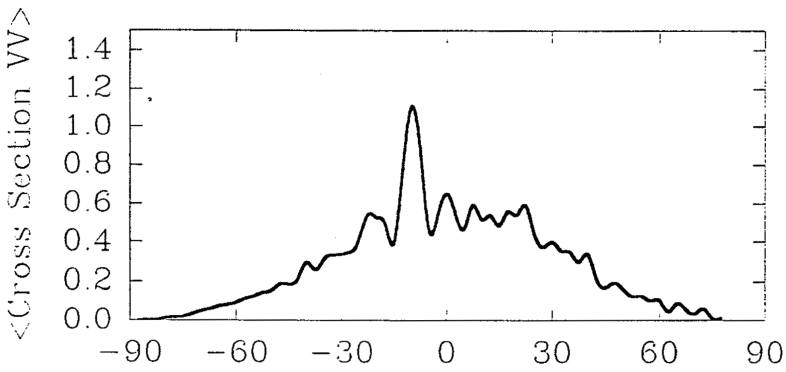

Fig. 3