Like and Cross Polarized Cross Sections for Two Dimensional Random Rough Surfaces: Bistatic Single and Double Scatter

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Abstract — In this work, full wave solutions for the single and double scatter radar cross sections from two dimensional random rough surfaces are given. The solutions are expressed as multidimensional integrals. The high frequency approximations are used to reduce the double scatter integrals from twelve to four dimensional. The single scatter cross section is given in closed form. The large radius of curvature approximation is used. The incident waves are assumed to be plane waves. The effects of changing the rough surface parameters, such as mean square height and mean square slope, on the double scatter cross sections are studied. The level and width of the peak in the backscatter direction depend on the mean square height and slope of the rough surface. The numerical results show sharp enhancement in the backscatter directions. This sharp backscatter enhancement, which is observed for all polarizations and for both normal and oblique incident angles, is associated with the quasi antiparallel double scatter paths.

FORMULATION OF THE PROBLEM

Noting that the major contributions to the double and single scatter cross sections, in the high frequency limit, come from the neighborhood of the specular points of the rough surface, the statistical average of the radar cross section (with respect to the random heights and large-scale slopes of the surface) for the quasi parallel, double scatter path ($\bar{n}' \approx \bar{n}''$) is given by

$$\left\langle \sigma^{PQ}_{dp} \right\rangle = \frac{(2k_0 Lm)^2}{\pi} P_2(\bar{n}^i) P_2(\bar{n}^f) \sum_{R,S=V,H} \left\{ \int\!\!\int \frac{\left[D^{PS}_{2'}(\bar{n}^f,\bar{n}') D^{SQ}_{1'}(\bar{n}',\bar{n}^i) D^{*PR}_{2''}(\bar{n}^f,\bar{n}'') D^{*RQ}_{1''}(\bar{n}'',\bar{n}^i)\right]}{\left(n^f_y - n'_y\right)\left(n^f_y - n''_y\right)\left(-n^i_y + n'_y\right)\left(-n^i_y + n''_y\right)} \right.$$
$$\cdot \frac{p(h_{xc1s}, h_{xc2s}, h_{zc1s}, h_{zc2s})}{\left(n^f_y - (n'_y + n''_y)/2\right)^2 \left(-n^i_y + (n'_y + n''_y)/2\right)^2}$$
$$\cdot \left[1 - P_2(|n'_y|)\right]\left[1 - P_2(|n''_y|)\right]$$
$$\cdot \operatorname{sinc}\left[k_0 L_m (n'_x - n''_x)\right] \operatorname{sinc}\left[k_0 L_m (n'_z - n''_z)\right]$$
$$\cdot \exp\left(-\langle h^2\rangle k_0^2 \left(n''_y - n'_y\right)^2\right)$$
$$\left. \cdot \sin\vartheta' \sin\vartheta'' d\vartheta' d\vartheta'' d\varphi' d\varphi'' \right\} \qquad (1)$$

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where $\bar{n}^i \bar{n}^f$ are unit vectors in the directions of the incident waves and scattered waves to the receiver, $\bar{n}'$ and $\bar{n}''$ are unit vectors in the directions of the spectrum of scattered waves between two points on the rough surface $1'$, $2'$, and $1''$, $2''$ respectively (see Fig. 1) and ϑ', ϕ' and ϑ'', ϕ'' are the spherical coordinate variables for $\bar{n}'$ and $\bar{n}''$. The surface element scattering coefficients D^{PQ} are evaluated at the specular point slopes h_{xc1s}, h_{xc2s}, h_{zc1s} and h_{zc2s}. The probability density functions of the (large scale) slopes $p(h_{xc1}, h_{xc2}, h_{zc1}, h_{zc2})$ and heights are assumed to be Gaussian. The free space wave number and the mean square height are k_0 and $\langle h^2 \rangle$.

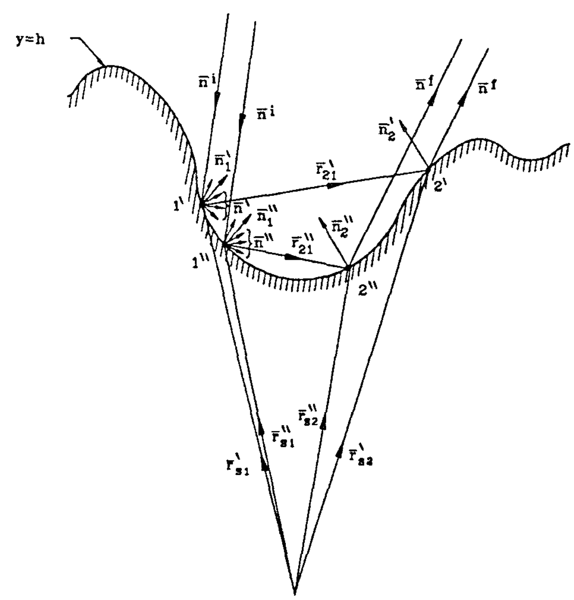

Fig. 1 Double scattered electromagnetic waves

Furthermore, L_m is the mean width of a typical depression on the rough surface [1], [2], [3]. The probabilities that the surface does not shadow the incident and scattered waves are given by $P_2(\bar{n}^i)$ and $P_2(\bar{n}^f)$, respectively [4], and $[1 - P_2]$ is associated with the probability of a double scatter event. The symbols V and H are for vertical and horizontal polarizations. For the quasi antiparallel double scatter paths, one gets the following expression for the high frequency double scatter cross section

$$
\begin{aligned}
\left\langle \sigma^{PQ}_{dp} \right\rangle = {} & \frac{(2k_0 Lm)^2}{\pi} P_2(\bar{n}^i) P_2(\bar{n}^f) \sum_{R,S=V,H} \\
& \left\{ \int\!\!\int \frac{\left[D^{PS}_{2'}(\bar{n}^f,\bar{n}') D^{SQ}_{1'}(\bar{n}',\bar{n}^i) D^{*PR}_{2''}(\bar{n}^f,\bar{n}'') D^{*RQ}_{1''}(\bar{n}'',\bar{n}^i)\right]}{\left(n^f_y - n'_y\right)\left(n^f_y - n''_y\right)\left(-n^i_y + n'_y\right)\left(-n^i_y + n''_y\right)} \right. \\
& \cdot \frac{p(h_{xc1s}, h_{xc2s}, h_{zc1s}, h_{zc2s})}{\left((n^f_y + n'_y - n''_y - n^i_y)/2\right)^2 \left((n^f_y - n'_y + n''_y - n^i_y)/2\right)^2} \\
& \cdot \left[1 - P_2(|n'_y|)\right]\left[1 - P_2(|n''_y|)\right] \\
& \cdot \operatorname{sinc}\left[k_0 L_m (n^f_x + n^i_x - n'_x - n''_x)\right] \\
& \cdot \operatorname{sinc}\left[k_0 L_m (n^f_z + n^i_z - n'_z - n''_z)\right] \\
& \cdot \exp\left(-\langle h^2 \rangle k_0^2 (n^f_y - n'_y - n''_y + n^i_y)^2\right) \\
& \left. \cdot \sin\vartheta' \sin\vartheta'' d\vartheta' d\vartheta'' d\varphi' d\varphi'' \right\}
\end{aligned} \tag{2}
$$

The slopes at the specular points for the quasi antiparallel case are given by h_{xc1s}, h_{xc2s}, h_{zc1s} and h_{zc2s}. Note that a stationary phase (quasi optics) integration over the wave vector variables $\bar{n}'$ and $\bar{n}''$ is not performed since the points 1 and 2 on the rough surface are not necessarily far apart ($k_0 r'_{21}$ and $k_0 r''_{21}$ are not large compared to one; see Fig. 1). The sharp enhancement in the backscatter direction $\left(-\bar{n}^i = \bar{n}^f\right)$ is associated with the quasi antiparallel ($\bar{n}' \approx -\bar{n}''$) double scatter cross section (2). The expressions for the slopes at the specular points for the quasi parallel and the quasi antiparallel cases are not the same. For backscatter at normal incidence the major contributions to the double scatter cross sections come from quasi horizontal paths between points 1 and 2 (see Fig. 1) and the slopes at the stationary points are approximately $\pm 45^\circ$.

NUMERICAL EXAMPLES

The incoherent double scatter cross sections (quasi parallel + quasi antiparallel) for two dimensional rough surfaces are plotted in Figs. 2, 3 and 4 as functions of the scatter angle $\vartheta^f \cos\varphi^f$ (where $\varphi^i = 0$, $\varphi^f = 0, \pi$). The two dimensional rough surface is assumed to be coated with gold of permittivity $\varepsilon_r = -9.888312 - j1.051766$ at $\lambda = 0.633\mu m$. The incident angle is equal to 10^{o}. The Rayleigh roughness parameter $\beta = 4k_0^2\langle h^2 \rangle$ is assumed to equal 394.105 in Figs. 2 and 4. In Figs. 2 and 3, the VV polarized double scatter cross sections are shown. The effect of changing the mean square slope (m.s.s.) is shown in Fig. 2 where it is assumed to be 0.25, 0.4, 0.5, 0.65, 0.75, 0.85 and 1.0. The level of the peak enhanced backscatter cross section is highest for mean

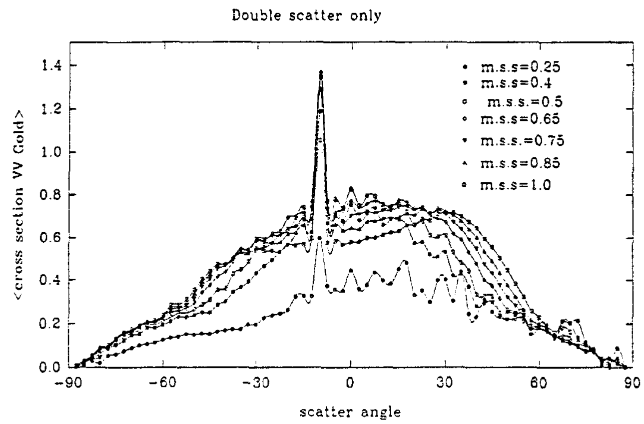

Fig. 2 $\vartheta^i = 10^\circ, \varepsilon_r = 9.888312 - j1.051766,$
$\lambda = 0.633\mu m, \beta = 394.105$

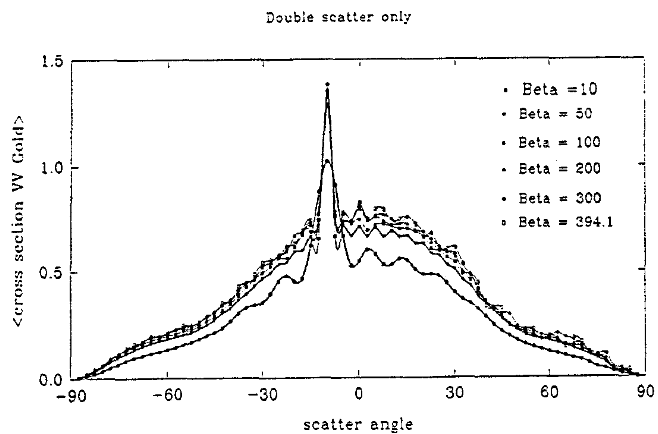

Fig. 3 $\vartheta^i = 10^\circ, \varepsilon_r = 9.888312 - j1.051766,$
$\lambda = 0.633\mu m$, mean square slope=0.5

square slope equal to 1.0 and it decreases as the mean square slope decreases to 0.25. In Fig. 3, the mean square slope is assumed to be 0.5 and the Rayleigh roughness parameter is assumed to be 10, 50, 100, 200, 300, and 394.105. The level of the peak enhanced backscatter increases as the Rayleigh roughness parameter increases. The width of the enhanced backscatter peak increases as both the Rayleigh roughness parameter and the mean square slope decrease. The cross sections for different polarizations (VV, HH, VH, HV) are shown in Fig. 4 in which the mean square slope is assumed to be 0.5 and β is assumed to be 394.105. Enhanced backscatter is observed for the four polarizations considered. The levels of the peak double scatter cross sections (in the backward direction) are approximately the same for both the like and cross polarized cases. The sharp backscatter enhancement observed for all polarizations, is associated with the quasi antiparallel double scatter paths.

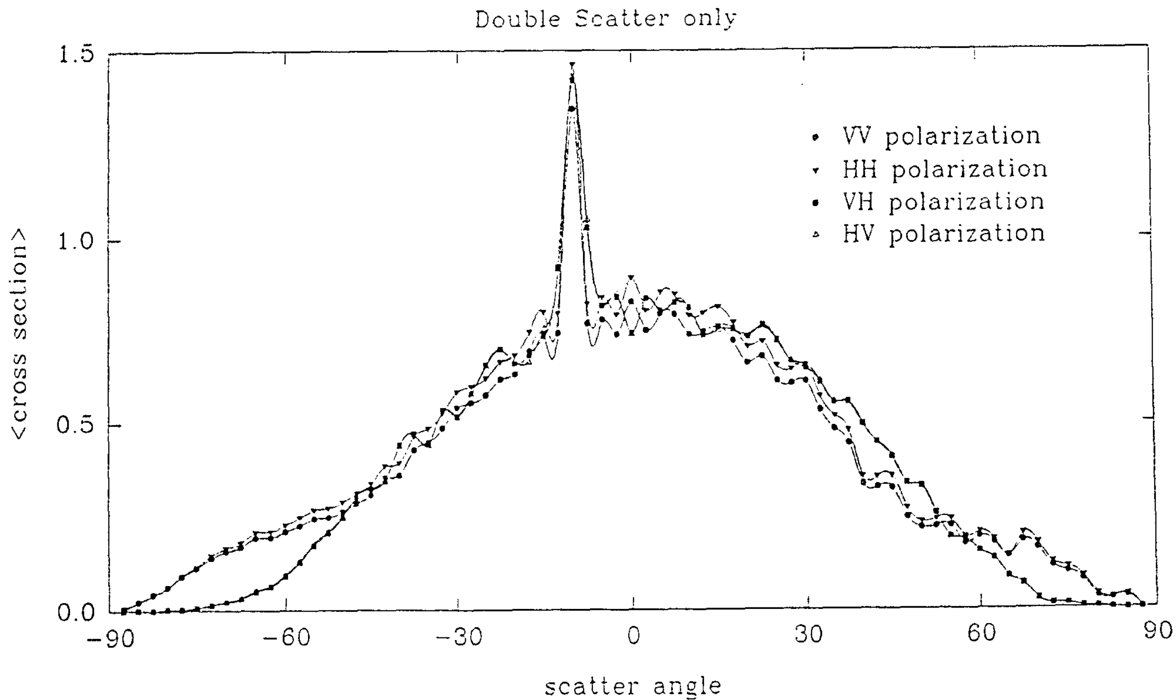

Fig. 4 $\vartheta^i = 10°, \epsilon_r = 9.888312 - j1.051766$, $\lambda = 0.633\mu m$, mean square slope=0.5, β=394.105

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