SINGLE AND DOUBLE LIKE ARE CROSS POLARIZED SCATTER FROM TWO DIMENSIONAL RANDOM ROUGH SURFACES– HIGH FREQUENCY APPROXIMATIONS

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ABSTRACT: Single and double scatter bistatic radar cross sections for two dimensional random rough surfaces are obtained. The full wave solutions are given as multi-dimensional integrals. The excitations are assumed to be plane waves. The high frequency approximations are applied to these expressions to reduce the dimensions of the multidimensional integrals. The large radius of curvature approximation is assumed in this work. The high frequency double scatter expressions are given by four dimensional integrals and the single scatter cross sections are given in closed form. This research is an extension to earlier work on double scatter from one dimensional random rough surfaces. The numerical results (using the high frequency approximations) show sharp enhancement in the backscatter direction at normal and oblique incident angles. This sharp enhancement is associated with the quasi anti-parallel double scatter path for both the like and cross polarized waves. The level and width of the peak in the backscatter direction depends on the mean square height and slope of the two dimensional random rough surfaces.

I. FORMULATION OF THE PROBLEM

The full wave solution for the double scatter far fields $G_d^f(\bar{r})$ from two dimensional rough surfaces ($y = h(x_s, z_s)$) is given in matrix notation by [1]

$$G_d^f(\bar{r}) = \left(\frac{k_o}{2\pi j}\right)^3 \frac{\exp(-jk_o r)}{r} \int D_{2'}(\bar{n}^f, \bar{n}')\exp(jk_o(\bar{n}^f \cdot \bar{r}_{s2'}))\exp(-jk_o\bar{n}' \cdot (\bar{r}_{s2'} - \bar{r}_{s1'}))$$
$$\times D_{1'}(\bar{n}', \bar{n}^i)\exp(-jk_o\bar{n}^i \cdot \bar{r}_{s1'})\frac{dn'_y dn'_z}{\sqrt{1-n'^2_y-n'^2_z}} U(\bar{r}_{s1'})U(\bar{r}_{s2'})dx_{s1'}dz_{s1'}dx_{s2'}dz_{s2'}G^i(0) \quad (1)$$

in which time harmonic excitations $\exp(j\omega t)$ are assumed and the free space wave number is $k_o = \omega\sqrt{\varepsilon_o\mu_o}$. The incident waves are in the direction $\bar{n}^i$ and the scattered waves are in the direction $\bar{n}^f$ to the receiver at $\bar{r}$, where,

$$\bar{n}^i = n_x^i\bar{e}_x + n_y^i\bar{e}_y + n_{zi}\bar{e}_z, \bar{n}^f = n_x^f\bar{e}_x + n_y^f\bar{e}_y + n_z^f\bar{e}_z, \bar{r} = x\bar{e}_x + y\bar{e}_y + z\bar{e}_z = r\bar{n}^f \quad (2)$$

The rough surface element scattering matrices at points $1'$ and $2'$ on the rough surface are $D_{1'}(\bar{n}', \bar{n}^i)$ and $D_{2'}(\bar{n}^f, \bar{n}')$. The elements of the scattering matrices depend on the slope of the rough surface [1]. Moreover, they depend on the polarization of the incident and scattered waves and the media on both sides of the rough interface. The incident fileds are assumed to be plane waves and the receiver is located in the far field. The wave vector associated with the scattered waves at the point on the surface $\bar{r}_{s1'}$ are in the direction $\bar{n}' = n'_x\bar{e}_x + n'_y\bar{e}_y + n'_z\bar{e}_z$. The position vectors to points $1'$ and $2'$ on the rough surfact (see Fig. 1) are respectively given, by

$$\bar{r}_{s1'} = x_{s1'}\bar{e}_x + h(x_{s1'}, z_{s1'})\bar{e}_y + z_{s1'}\bar{e}_z., \text{and} \bar{r}_{s2'} = x_{s2'}\bar{e}_x + h(x_{s2'}, z_{s2'})\bar{e}_y + z_{s2'}\bar{e}_z \quad (3)$$

At high frequencies, the shadow functions $U(\bar{r}_{s1'})$ and $U(\bar{r}_{s2'})$ are equal to one if the point at $\bar{r}_{s1'}$ is illuminated by the incident waves and visible at point $2'$ on the surface and if the point at $\bar{r}_{s2'}$ is illuminated by a point source at $1'$ and visible at the receiver [2]. The incoherent double scatter cross section is obtained on multiplying (1) by its complex conjugate and taking the statistical average of the product. Similar to the problem of scattering from one

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dimensional random rough surfaces [3], the major contributions to the double scatter cross sections are associated with the quasi parallel $\bar{n}' \approx \bar{n}''$ and the quasi anti-parallel $\bar{n}' \approx -\bar{n}''$ double scatter paths.

The major contributions to the double and single scatter cross sections, in the high frequency limit, come from the neighborhood of the specular points of the rough surface [3]. The heights at any two neighboring points on the two dimensional rough surface are expressed as functions of the heights and slopes at the mean point between them. The surface element scattering coefficients are evaluated at the specular points after integrating with respect to the slopes. The joint probability density functions for the heights are assumed to be Gaussian. The high frequency approximation is used to reduce the expression for the double scatter cross section to four dimensional integrals. Thus the quasi parallel like $(P = Q)$ and cross $(P \neq Q)$ polarized double scatter cross sections are expressed as follows

$$\begin{aligned} \langle\sigma_{dp}^{PQ}\rangle &= \frac{2}{\pi}(k_o Lm)^2 P_2(\bar{n}^f) \\ &\times \sum_{R,S=V,H} \int \left[D_{2'}^{PS}(\bar{n}^f,\bar{n}') D_{1'}^{SQ}(\bar{n}',\bar{n}^i) D_{2''}^{*PR}(\bar{n}^f,\bar{n}'') D_{1''}^{*RQ}(\bar{n}'',\bar{n}^i)\right] \\ &\times \left.\frac{p(h_{x1},h_{x2},h_{z1},h_{z2})}{\left[n_y^f - (n_y' + n''_y)/2\right]^2 \left[-n_y^i + (n_y' + n''_y)/2\right]^2}\right|_{specular\ slope} \\ &\times sinc[kL_m(n_x' - n_x'')] sinc[kL_m(n_z' - n_z'')] \exp(-\langle h^2\rangle k_o^2 (n''_y - n_y')^2) \sin\vartheta' \sin\vartheta'' d\vartheta' d\vartheta'' d\varphi' \end{aligned}$$

in which sinc $\equiv \sin(x)/x$ and L_m is the mean width of a typical depression on the rough surface [1]. The probabilities that a point on the surface is not shadowed from the incident and scattered waves are given by $P_2(\bar{n}^i)$ and $P_2(\bar{n}^f)$, respectively [2]. The symbols V and H denote vertical and horizontal polarizations. The integration variables n_y and n_z in (1) are transformed to the spherical coordinate variables ϑ and φ in (4). The slopes at the specular points in (4) are given by

$$h_{x1s} = -\frac{\left[-n_x^i + \frac{n_x'+n_x''}{2}\right]}{\left[-n_y^i + \frac{n_y'+n_y''}{2}\right]}, \quad h_{x2s} = -\frac{\left[-n_x^f + \frac{n_x'+n_x''}{2}\right]}{\left[-n_y^f + \frac{n_y'+n_y''}{2}\right]} \tag{5}$$

$$h_{z1s} = -\frac{\left[-n_z^i + \frac{n_z'+n_z''}{2}\right]}{\left[-n_y^i + \frac{n_y'+n_y''}{2}\right]}, \quad h_{z2s} = -\frac{\left[-n_z^f + \frac{n_z'+n_z''}{2}\right]}{\left[-n_y^f + \frac{n_y'+n_y''}{2}\right]} \tag{6}$$

The high frequency quasi anti-parallel double scatter cross sections are given by

$$\begin{aligned} \langle\sigma_{dp}^{PQ}\rangle &= \frac{2}{\pi}(k_o Lm)^2 P_2(\bar{n}^i) P_2(\bar{n}^f) \\ &\times \sum_{R,S=V,H} \int \left[D_{2'}^{PS}(\bar{n}^f,\bar{n}') D_{1'}^{SQ}(\bar{n}',\bar{n}^i) D_{2''}^{*PR}(\bar{n}^f,\bar{n}'') D_{1''}^{*RQ}(\bar{n}'',\bar{n}^i)\right] \\ &\times \frac{p(h_{x1},h_{x2},h_{z1},h_{z2})}{\left[(n_y^f + n_y' - n_y'' - n_y^i)/2\right]^2 \left[n_y^f - n_y^i - n_y' + n_y'')/2\right]^2}\Big|_{specular\ slopes} \\ &sinc[k_o L_m(n_x' - n_x'' - n_x^f - n_x^i)] sinc[k_o L_m(n_z' - n_z'' - n_z^f - n_z^i)] \\ &\times \exp\left(-\langle h^2\rangle k_o^2 (n_y^f - n_y' - n_y'' + n_y^i)^2\right) \sin\vartheta' \sin\vartheta'' d\vartheta' d\vartheta'' d\varphi' d\varphi'' \end{aligned} \tag{7}$$

and the slopes at the specular points are given by

$$h_{x1s} = -\frac{\left[-n_x^i + n_x^f + n_x' - n_x''\right]}{\left[-n_y^i + n_y^f + n_y' - n_y''\right]}, \quad h_{x2s} = -\frac{\left[n_x^f - n_x^i - n_x' + n_x''\right]}{\left[n_y^f - n_y^i - n_y' + n_y''\right]} \tag{8}$$

$$h_{z1s} = -\frac{\left[-n_z^i + n_z^f + n_z' - n_z''\right]}{\left[-n_y^i + n_y^f + n_y' - n_y''\right]}, \quad h_{z2s} = -\frac{\left[n_z^f - n_z^i - n_z' + n_z''\right]}{\left[n_y^f - n_y^i - n_y' + n_y''\right]} \tag{9}$$

The sharp enhancement in the backscatter direction ($-\bar{n}^i = \bar{n}^f$) is associated with the quasi anti-parallel ($\bar{n}' \approx -\bar{n}''$) double scatter path (3). Note that the slopes in (3) and (2) are different.

II. ILLUSTRATIVE EXAMPLES

The incoherent double scatter cross sections for two dimensional rough surfaces are plotted in Figs. 1, 2 and 3 as functions of the scatter angle $\vartheta^f \cos\varphi^f$ in the incident plane (where $\varphi^i = 0, \varphi^f = 0, \pi$). The probability density functions for the slopes and the surface height auto-correlation function are assumed to be Gaussian. The mean square slope of the surface is $m.m.s. = 0.5$ and the Raleigh parameter is $\beta = 4k_o^2 < h^2 >$ (see captions). The two dimensional random rough surface is assumed to be gold coated (ε_r = -9.89 - j1.05). The incident angle is 10^O. The results exhibit the sharp enhancement in the backscatter direction for both the like and cross-polarized waves.

III. CONCLUSIONS

The results for the double scatter radar cross sections exhibit a sharp enhancement in the backscatter direction for normal and oblique incident angles. This sharp enhancement is associated with the quasi anti-parallel double scatter path for both the like and cross polarized waves. The height and the width of the peak in the backscatter direction depends on the mean square height and slope of the two dimensional random rough surface (Fig. 1). While the single scatter cross polarized cross sections (in the plane of incidence) are negligible compared to the like polarized cross section (Fig. 2), the double scatter like and cross polarized cross sections are of the same order of magnitude (Fig. 3).

IV. ACKNOWLEDGMENT

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V. REFERENCES

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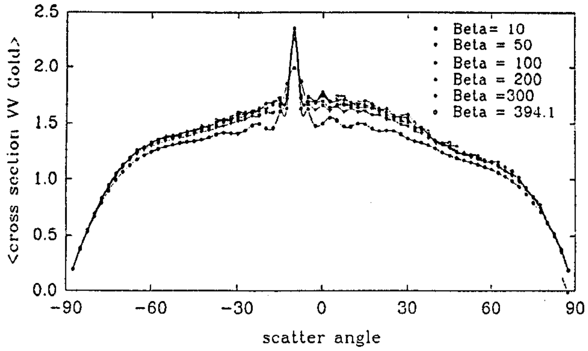
Total (double+single)



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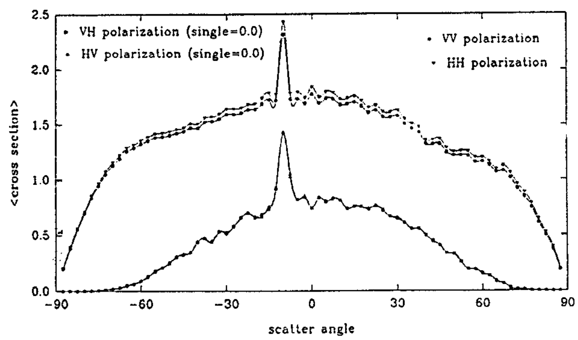

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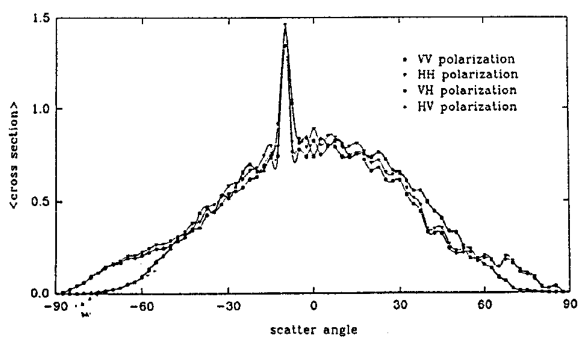
Double Scatter only



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