Iterative Microwave Inversion Algorithm Based On The Adjoint-Field Method For Breast Cancer Application

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Summary. Our goal is to develop an inversion algorithm for reconstructing the shape of 3D breast tumors using electromagnetic data. The method of moments (MoM) forward solver is used to calculate the electric and magnetic equivalent surface currents at the tumor interface and consequently the scattered electromagnetic fields. Using a so-called 'adjoint scheme' for gradient calculation, the mismatch between calculated and measured fields at the receivers is used as new sources at all receiver locations and is back-propagated towards the tumor. The gradient is calculated then simultaneously for all nodes of the guessed tumor surface in order to obtain a correction displacement of each individual node of the surface which points into a descent direction of a least-squares cost functional. This process is repeated iteratively until the cost has decreased satisfactorily. Numerical results in 3D are presented based on the proposed technique using multiple transmitting sources/receivers at multiple microwave frequencies.

Key words: microwave imaging, shape reconstruction, level sets, adjoint scheme, method of moments

1 Introduction

Microwave tomographic imaging is showing significant promise as a new technique for the early detection of breast cancer. Its physical basis is the high contrast between the dielectric properties of the healthy breast tissue and the malignant tumors at microwave frequencies [Gabriel et al. (1996)]. As a consequence, microwave imaging systems which aim at detecting, localizing and characterizing tumors in the breast are being developed. Among them, we mention for example confocal imaging and near-field tomographic reconstructions (see [Fear et al. (2002)] and references therein).

Mathematically, microwave medical tomography amounts to solving a nonlinear inverse problem for some form of Maxwell's equations in which a given cost functional is minimized via an iterative algorithm. Traditional iterative algorithms, well suited for nonlinear inverse problems and based on pixel reconstruction techniques suffer from several drawbacks in this application, amongst them the need of strong regularization for stabilizing the algorithms which typically is done by adding a Tikhonov-Philips term to the cost functional. This, however, has the effect of severely smoothing out interfaces between tumors and surrounding tissue. Therefore, new approaches that avoid these difficulties need to be investigated. We will present here a shape-based approach for this application which allows to reconstruct quite general shapes by moving each individual surface node until a given cost functional is minimized. For more details on shape-based reconstruction schemes in various applications see for example the discussion led in [Dorn et al. (2006), El-Shenawee et al. (2006)].

2 Shape reconstruction in microwave imaging

Dropping out the time dependence $e^{i\omega_k t}$, we consider the system of Maxwell's equations

$$\nabla \times \mathbf{E}_{jk}(\mathbf{x}) - \alpha_k(\mathbf{x})\mathbf{H}_{jk}(\mathbf{x}) = 0 \tag{1}$$

$$\nabla \times \mathbf{H}_{jk}(\mathbf{x}) - \beta_k(\mathbf{x})\mathbf{E}_{jk}(\mathbf{x}) = 0 \tag{2}$$

in a domain $\Omega \subset \mathbf{R}^3$, where $\beta_k(\mathbf{x}) = \sigma(\mathbf{x}) + i\omega_k\epsilon(\mathbf{x})$ and $\alpha_k(\mathbf{x}) = -i\omega_k\mu(\mathbf{x})$ and where ω_k, $k = 1, \ldots, \underline{k}$, are the different (angular) frequencies of the applied fields. We will assume that $\alpha_k(\mathbf{x})$, $\beta_k(\mathbf{x})$ are constant outside some sufficiently large ball, with values denoted by $\alpha_{k,0}$ and $\beta_{k,0}$, respectively. With this assumption, we can apply the standard radiation condition outside this ball. The index j in (1), (2) indicates the different incoming radiation patterns (plane waves).

We will consider here the situation that the coefficient functions $\alpha_k(\mathbf{x})$ and $\beta_k(\mathbf{x})$ contain discontinuities along closed interfaces $\Gamma_m \subset \Omega$, $m = 1, \ldots, \underline{m}$, such that we add standard interface conditions to (1), (2). Given incoming plane waves corresponding to index jk, we can write the total field in the medium as

$$\mathbf{E}_{jk}^{tot} = \mathbf{E}_{jk}^{inc} + \mathbf{E}_{jk}^{scat}, \qquad \mathbf{H}_{jk}^{tot} = \mathbf{H}_{jk}^{inc} + \mathbf{H}_{jk}^{scat} \tag{3}$$

where $\mathbf{E}_{jk}^{inc}$ and $\mathbf{H}_{jk}^{inc}$ satisfy (1), (2) with $\alpha_k = \alpha_{k,0}$ and $\beta_k = \beta_{k,0}$. Let us assume that we have $\underline{l}$ receivers available at locations $\mathbf{d}_l$, $l = 1, \ldots, \underline{l}$. At these receiver positions, we can decompose the scattered electric fields as

$$\mathbf{E}^{scat}(\mathbf{d}_l) = \mathbf{E}^r(\mathbf{d}_l)\hat{r} + \mathbf{E}^\theta(\mathbf{d}_l)\hat{\theta} + \mathbf{E}^\phi(\mathbf{d}_l)\hat{\phi}. \tag{4}$$

Here, $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$ are the polar unit vectors at the points $\mathbf{d}_l$. With this, we can define the linear measurement operators M_{jkl} by

$$M_{jkl}\mathbf{E}_{jk} \;=\; \mathbf{E}^{\theta}_{jk}(\mathbf{d}_l)\hat{\theta} + \mathbf{E}^{\phi}_{jk}(\mathbf{d}_l)\hat{\phi} \tag{5}$$

which measure the 'plane wave components' of the scattered fields at the given receiver location. We will assume in the following that the coefficient α_k is fixed and known to be $\alpha_k = \alpha_{k,0}$. We will write then $M_{jk}\mathbf{E}_{jk}(\beta)$ for the vector of all measured fields which correspond to the parameter β, the frequency ω_k and the incoming plane wave with index j. Furthermore, g_{jk} will denote the corresponding physically measured ('true') data. With this notation, we can define the least squares cost functional

$$\mathbf{J}_{jk}(\beta) \;=\; \frac{1}{2}\|R_{jk}(\beta)\|^2 \tag{6}$$

where $R_{jk}(\beta) = M_{jk}\mathbf{E}_{jk}(\beta) - g_{jk}$ is the residual operator for indices (jk). In the shape inverse problem, we assume that

$$\beta(\mathbf{x}) \;=\; \begin{cases} \beta_i \text{ for } & \mathbf{x} \in D, \\ \beta_e \text{ for } & \mathbf{x} \in \Omega \backslash D. \end{cases} \tag{7}$$

When deforming in a given step of the iterative inversion scheme the current shape D by a vector field $\mathbf{v}$ (i.e., each point $\mathbf{x} \in D$ is displaced according to $\mathbf{x} \;\rightarrow\; \mathbf{x} + \mathbf{v}(\mathbf{x})$) then the fields and therefore also the least squares cost will change. We want to find a vector field such that $\mathbf{J}$ is reduced by the corresponding deformation. It has been shown in [Dorn et al. (2006)] that the deformation of the boundary $\Gamma = \partial D$ by a sufficiently small vector field $\mathbf{v}$ gives rise to a change in the cost

$$\delta\mathbf{J}_{jk} \;=\; \mathrm{Re}\int_{\partial D} [R'_{jk}(\beta)^* R_{jk}(\beta)]\overline{\beta_i - \beta_e}\mathbf{v}(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x})\, ds(\mathbf{x}) \tag{8}$$

where $R'_{jk}(\beta)^*$ denotes the formal adjoint operator of the linearized residual operator $R'_{jk}(\beta)$ and $\mathbf{n}(\mathbf{x})$ is the normal direction to the boundary Γ in the point $\mathbf{x}$. Therefore, it is sufficient to find a vector field in the normal direction to the boundary $\mathbf{v}_d(\mathbf{x}) = F_d(\mathbf{x})\mathbf{n}(\mathbf{x})$ which points into a descent direction of the cost $\mathbf{J}$. Obviously, we can choose

$$F_d(\mathbf{x}) \;=\; -\gamma \mathrm{Re}\left\{R'_{jk}(\beta)^* R_{jk}(\beta)\overline{\beta_i - \beta_e}\right\} \tag{9}$$

for a sufficiently small positive step size $\gamma > 0$. Plugging $\mathbf{v} = \mathbf{v}_d$ into (8) shows us that then the cost is reduced. The expressions $R'_{jk}(\beta)^* R_{jk}(\beta)$ are calculated by an efficient 'adjoint scheme' as explained for example in [Dorn et al. (1999)]. This scheme requires us to run just one forward and one adjoint simulation for a given frequency and incoming wave in order to evalute the gradient expressions (9) at all nodes simultaneously.

3 Numerical experiments

In our numerical experiment shown here the true object is a sphere of radius 1 cm located at the center of the computational domain of $20 \times 20 \times 20\mathrm{cm}^3$.

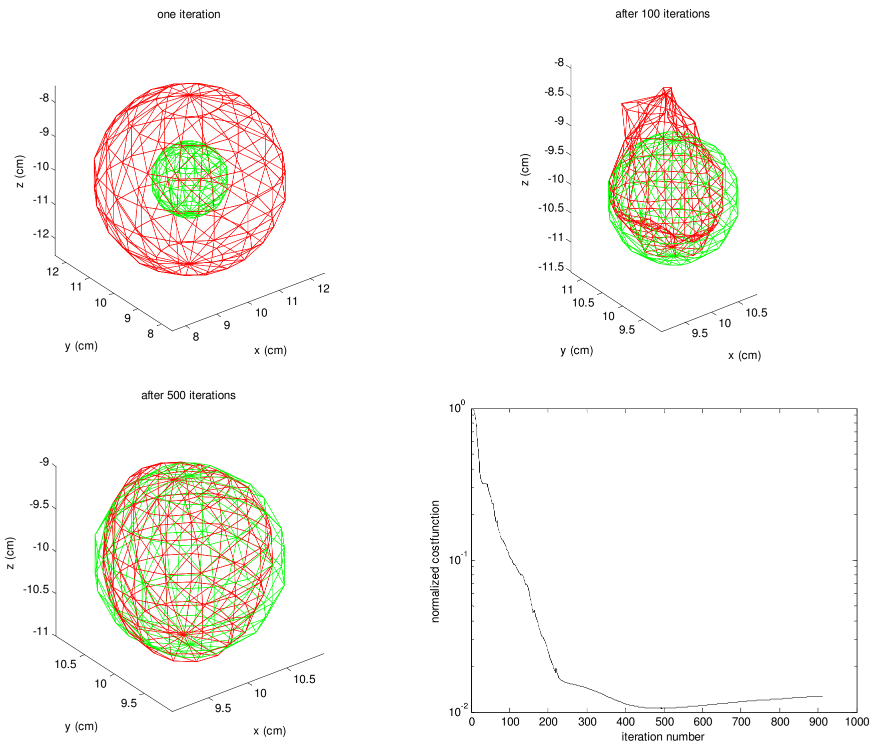

Fig. 1. Reconstruction of a small sphere. The true object is displayed in gray colour, and the reconstructed object by black colour in each iteration. Top left: after one iteration; top right: after 100 iterations; bottom left: after 500 iterations. The bottom right shows the evolution of the cost.

Inside the object, the relative dielectric constant is $50 - j12$, and in the background it is $9 - j1.2$. The total number of transmitters (receivers) is 30 with 5 of them being located at each plane of constant azimuth angle (starting at $\theta = 0.1\pi$ and ending at $\theta = 0.9\pi$) with φ between 0 and 2π . Two frequencies are used here, namely $f = 3$ GHz and $f = 5$ GHz ($\omega = 2\pi f$). Plane waves are used to excite the object with incident polarization in the θ-direction. The results shown here are for the co-polarization case, where both the incident and scattered plane waves are in the θ-direction. Synthetic data is generated using the method of moments, where the surface of the object is discretized into surface nodes and triangular patches similar to the work of [El-Shenawee et al. (2006)]. The number of discretization points in the θ- and

φ-directions are 8 and 16, respectively, for the true object, while the object generated at each inversion iteration is discretized into 10 and 16 points, respectively. The gradient-based algorithm using adjoint fields is implemented such that the location of each surface node will be corrected into the normal direction of the current boundary by an amount given in equation (9) for each node using a fixed small step-size factor γ (being 10^{-5} at 3 GHz and 10^{-3} at 5 GHz). A regularization step is applied after each update which amounts to filtering neighbouring nodes by an averaging filter in order to obtain a smooth surface. This smoothing operation will be discussed in more detailes in a forthcoming publication.

In this work, the main focus is on reconstructing the shape of the object assuming the knowledge of its position and electrical properties. The initial guess in this case is a sphere of radius 2 cm located at the same position as the true object. Figure 1 shows the true object (gray) and the guessed object (black) at iteration numbers 1 (top left), 100 (top right) and 500 (bottom left), where the latter one corresonds approximately to the lowest cost value which we could achieve during our reconstruction. The evolution of the total cost (summed over all indices) is displayed in the bottom right image of the figure. We mention that the reconstruction at the final iteration number 900 looks quite similar to that one at number 500. We conclude that our algorithm has converged in a stable way to the correct sphere.

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