The Steepest Descent Fast Multipole Method (SDFMM) for Solving Combined Field Integral Equation Pertinent to Rough Surface Scattering

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Introduction

The steepest descent fast multipole method (SDFMM) is an integral equation based technique aimed to accelerate the iterative solution of the method of moments (MOM) equations for large scale quasi-planar structures. The method constitutes a hybridization of the multilevel fast multipole algorithm (MLFMA) [1] and the steepest descent path (SDP) method. The SDFMM has been successfully used to analyze scattering from three-dimensional quasi-planar structures [2]. Both the memory requirements and the computational cost of the SDFMM are of $O(N)$ as opposed to $O(N^2)$ for the conventional MOM technique using an iterative solver.

Recently, several enhancements have been introduced to the SDFMM to efficiently and accurately simulate scattering from large scale rough surfaces. These enhancements include error estimates that are essential to *a priori* selection of the SDFMM parameters, such as the number and location of the SDP integration points, the interpolation orders, and the number of harmonics needed for the MLFMA [3].

The objective of this work is to solve the combined field integral equation (CFIE) using the enhanced SDFMM to analyze electromagnetic scattering from practical random rough surfaces. For scattering problems involving low grazing angle (LGA) incident waves, we found that the CFIE converges very fast compared with the electric field integral equation (EFIE), which converges extremely slowly for the same problem. Scattering at low grazing angles (LGA) requires modeling of an infinite rough surface by a finite surface large enough such that the incident antenna beam excites no edge currents. In order to thoroughly investigate such a special scattering problem, a Monte Carlo simulation using many random surface realizations should be conducted which necessitates a fast converging and accurate technique such as the enhanced SDFMM.

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Formulation

Assuming a time dependence $\exp(-i\omega t)$, the electric and magnetic field integral equations that enforce the appropriate boundary conditions on a perfectly conducting rough surface are

$$-\hat{t}\cdot\bar{E}^i(\bar{r})=\hat{t}\cdot\int_S ds'\left(i\omega\mu_o\bar{J}(\bar{r}')-\frac{1}{i\omega\varepsilon_o}\nabla\nabla'\cdot\bar{J}(\bar{r}')\right)g(\bar{r},\bar{r}') \tag{1}$$

$$-\hat{n}\times\bar{H}^i(\bar{r})=\hat{n}\times\nabla\times\int_S ds' g(\bar{r},\bar{r}')\bar{J}(\bar{r}')-\bar{J}(\bar{r}) \tag{2}$$

in which the Green's function is $g(\bar{r},\bar{r}')=\exp(ik_o|\bar{r}-\bar{r}'|)/(4\pi|\bar{r}-\bar{r}'|)$, and $\hat{t}$ and $\hat{n}$ are unit vectors tangential and normal to the surface, respectively. To solve Eqns. (1-2), the unknown current density $J(\bar{r}')$ is approximated as

$$\bar{J}(\bar{r}')=\sum_{n=1}^{N}I_n\bar{j}_n(\bar{r}') \tag{3}$$

where $\bar{j}_n$ is a set of the RWG basis functions. Upon substituting Eqn. (3) into Eqns. (1-2) and testing with a set of RWG functions $\bar{j}_m$, a system of $N\times N$ equations $\bar{\bar{Z}}\cdot\bar{I}=\bar{V}$ is obtained. The details of the SDFMM are given in [2] and they will not be repeated here. The geometry is assumed to be a perfectly conducting open surface for which the EFIE (1) is suitable, but to speedup the solution, the CFIE, viz. a combination of Eqns. (1) and (2) as α EFIE + $(1\text{-}\alpha)$ MFIE, is used in this work. The incident wave is assumed to be a Gaussian beam that has been carefully tapered towards the edges of the surface to allow for the use of the MFIE (2) - and hence the CFIE - for this open structure.

Results and Conclusions

In Figs. 1 and 2, the random rough surface is modeled by a finite plate of dimensions equal to $50.5\lambda\times12.5\lambda$. Using eight unknowns per wave length, the number of MOM current unknowns is equal to 120,696. A Gaussian beam with incident angle equal to 80° from the normal direction is used for excitation in this work. The surface is assumed to be perfect conductor with Gaussian statistics. The rms height and the correlation length of the rough surface are assumed to be 0.3λ and 1.0λ, respectively. The tolerance of the iterative solver is chosen to be 0.1%. The vertically co- and cross polarized bistatic RCS of only one realization are plotted versus the scatter angle $\theta^s\cos\phi^s$, where $\phi^s=0,\pi$ and $-\pi/2\leq\theta^s\cos\phi^s\leq\pi/2$. In Fig. 3, a comparison of the convergence of the solutions obtained by solving the EFIE using the TFQMR iterative solver, the EFIE using the GMRES iterative solver, and the CFIE using the TFQMR iterative solver is shown for 1% tolerance. The dimensions of the rough surface used in this comparison is equal to $80\lambda \times 17\lambda$, the number of MOM current unknowns equal to 260,344, the rms height is equal to 0.5λ, and the correlation length is equal to 1.0λ. The results show the dramatic decrease in the number of iterations upon solving the CFIE compared with solving the EFIE even with using an optimum and expensive iterative solver such as the GMRES. The ultimate goal of this work is to conduct a Monte Carlo simulation to investigate the LGA scattering phenomena. The preliminary results show that the SDFMM is a prime candidate for analyzing the electromagnetic scattering from rough surfaces at low grazing incident angles.

References

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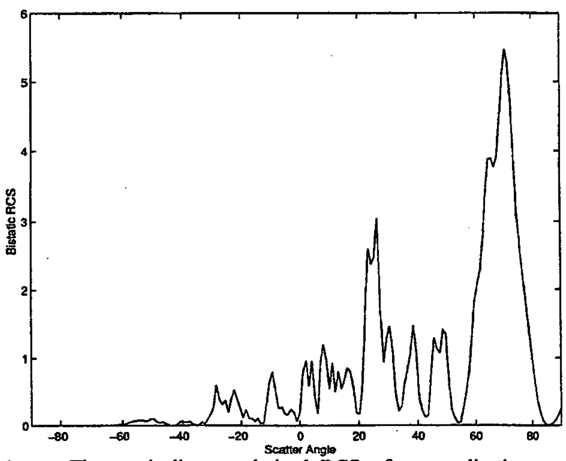

Figure 1. The vertically co-polarized RCS of one realization versus the scatter angle $\theta^s \cos\phi^s$.

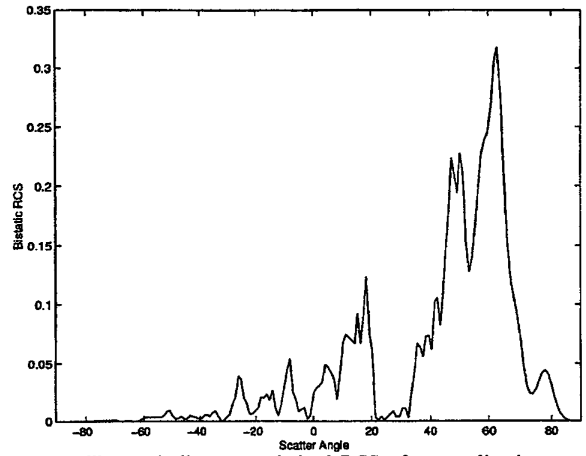

Figure 2. The vertically cross-polarized RCS of one realization versus the scatter angle $\theta^s \cos\phi^s$.

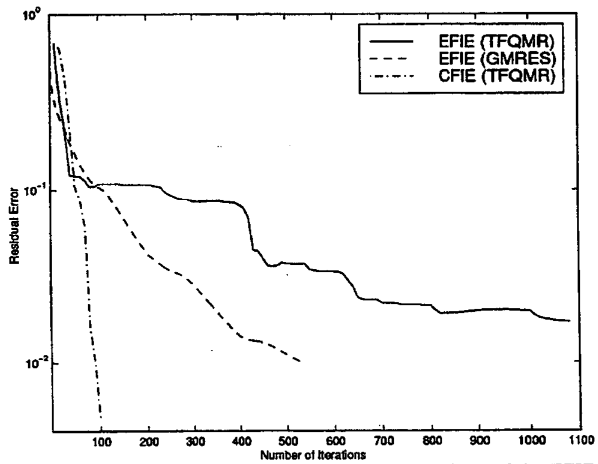

Figure 3. Comparison of the convergence of the solutions of the EFIE using TFQMR (solid line), EFIE using GMRES (dotted line), and CFIE with $\alpha = 0.5$ using TFQMR (dashed line) for a perfectly conducting rough surface.