Scattering from Multilayered Random Rough Surfaces Using the Steepest Descent Fast Multipole Method (SDFMM) and the Multiple Interaction Model

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Abstract

Scattering of electromagnetic waves from multilayered random rough surfaces is crucial for subsurface sensing applications. A multiple interaction method of moments (MoM) model is used in this work to analyze scattering from two-dimensional multilayered random rough ground (3-D scattering problem) especially when the underground layer is deeply buried under the air/ground interface. The presented model removes a barrier and enables the application of the Steepest Descent Fast Multipole Method (SDFMM) to certain 3-D non-quasi-planar structures. The conventional SDFMM has been used to analyze electromagnetic wave scattering from quasi-planar structures where the scatterer's height is a fraction of a free-space wavelength. The presented model is based on multiple interactions mechanism between the air/ground interface and the buried underground layer. The basic idea of the proposed multiple scattering model is to decompose the non-quasi-planar multilayered ground into two quasi-planar scatterers where the conventional SDFMM can be applied separately to each one. The interactions between the sub-quasi-planar scatterers are calculated using the electromagnetic vector potentials near-field expressions. This model is tested and validated with the MoM on a variety of geometries. The results show that the strongest signature of the buried scatterer is mainly due to the first multiple interaction mechanism (ground-object-ground) while the contributions from repeating this mechanism become insignificant even for lossless and/or slightly lossy underground.

I. INTRODUCTION

Electromagnetic subsurface sensing has recently become an attractive research area due to it wide civil and defense applications. There are numerous unfound buried objects; these objects could be, but are not limited to, the location of underground water, gas and/or water lines, identifying cracks in asphalt roads, cancerous tumors in the human breast, plastic anti-personnel or metallic anti-tank mines, or the location of hazardous environmental wastes, etc. In reality, these complicated scattering problems have three-dimensions (x, y and z) that cannot be solved in closed forms but rather can be solved using computational techniques, in particular using fast algorithms. Few fast computational techniques were developed in the literature, e.g., the Fast Multipole Method (FMM) [1]-[3], the Steepest Descent Fast Multipole Method (SDFMM) [4]-[6], the Sparse Matrix/Canonical Grid Method (SMCG) [7], [8], and the Spectral Algorithm combined with the Forward-Backward Method (FB/NSA) [9]. These fast techniques have shown superiority regarding the CPU time and memory requirements over the conventional methods (e.g. the method of moment (MoM), the finite-difference methods (FD) and the finite-element method (FEM). Recently, the SDFMM has been adopted to analyze the scattering from penetrable shallow objects buried under two-dimensional random rough ground [10]-[11]. The SDFMM has the great advantage of \( O(N) \) computational complexity for both the CPU time and computer memory, where \( N \) is the total number of electric and magnetic surface current unknowns. However, there is a barrier that prohibits using the SDFMM in some potential applications; the scatterer should have a quasi-planar structure with total height equal to a fraction of a free-space wavelength. On the other hand, there are several potential geometries that have non-quasi-planar structures such as (i) multilayered ground where the burial depth of
the underground layer is larger than the wavelength [12], (ii) a non-quasi-planar object buried under the quasi-planar rough ground, (iii) utility cylindrical pipes deeply buried under the rough ground, etc.

The basic idea of the multiple scattering model is to decompose a non-quasi-planar structure into two quasi-planar scatterers as shown in Fig. 1 where the conventional SDFMM can be applied separately to each one. The interactions between the sub-quasi-planar scatterers, e.g. between the ground and the buried object in Fig. 1, are calculated directly using the electromagnetic vector potential expressions for the electric and magnetic near-fields [13]. The proposed model sheds light on the physics involved in the subsurface scattering mechanism [12], [14]. In this work, we are emphasizing on (i) presenting and validating the multiple scattering model on a variety of geometries as shown in Fig. 2 and (ii) using the SDFMM in the proposed multiple scattering model for non-quasi-planar structures aiming to speed up the calculations by exploiting the superior $O(N)$ computational complexity of the SDFMM.

The proposed model algorithm will be described in Section II, numerical results are presented in Section III and concluding remarks are given in Section IV.

II. FORMULATION

The four integral equations describing the unknown equivalent electric and magnetic surface currents for the problem of a single object buried beneath two-dimensional rough ground were derived in [10]-[11]. Upon applying Galerkin’s method for testing and using the RWG vector basis functions for approximating the surface currents [15], these integral equations are transformed into a set of linear system of equations $\overline{Z}I = \overline{V}$ given by [10]:

$$
\begin{pmatrix}
\overline{Z}_{g.g} & \overline{Z}_{g.obj} \\
\overline{Z}_{obj.g} & \overline{Z}_{obj.obj}
\end{pmatrix}
\begin{pmatrix}
\overline{I}_g \\
\overline{I}_{obj}
\end{pmatrix} =
\begin{pmatrix}
\overline{V}_g \\
0
\end{pmatrix}
$$

(1)

where $\overline{Z}_{g.g}$ is a submatrix representing interactions between elements only on the ground surface; $\overline{Z}_{g.obj}$ is a submatrix representing interactions between elements on the ground surface and elements on the object surface; $\overline{Z}_{obj.g}$ is a submatrix representing interactions between elements on the object surface and elements on the ground surface; and $\overline{Z}_{obj.obj}$ is a submatrix representing interactions between elements only on the object surface. It was shown in [10] that the total matrix $\overline{Z}$ has order of $2(N + P) \times 2(N + P)$, where $N$ is the number of vector basis functions on the ground and $P$ is the number of vector basis functions on the object. The factor of two is to account for both the electric and magnetic surface currents. Moreover, $\overline{Z}_{g.g}$ (and $\overline{Z}_{obj.obj}$) is exactly the impedance matrix obtained in the PMCHW integral equations [16]. The vector $\overline{V}_g$ is composed of the tested tangential incident electric field $\overline{E}_g^{inc}$ and the tested normalized magnetic field $\eta_1 \overline{H}_g^{inc}$ on the ground surface. The unknown current coefficients $\overline{I}_g$ and $\overline{I}_{obj}$ were solved for in [10] and [11] by applying the SDFMM directly to Eq. 1. Conversely, in this work, the multiple scattering model will be used to iteratively solve for the unknown current coefficients in

$$
\begin{align}
\overline{Z}_{g.g} \overline{I}_g^{(n)} &= \overline{V}_g^{(n)} \\
\overline{Z}_{obj.g} \overline{I}_{obj}^{(n)} &= \overline{V}_{obj}^{(n)}
\end{align}
$$

(2a, 2b)

where $n = 1, 2, 3, \ldots$ is the number of iterations or the number of multiple scattering mechanisms between the ground and the buried object as shown in Fig. 2b. The mechanism of the multiple scattering model begins by calculating the incident waves from the source (transmitting antenna) on the rough ground $\overline{V}_g^{(0)}$ with the assumption that there are no buried objects under the ground. Then the SDFMM is used to calculate the induced electric and magnetic surface currents on the ground as $\overline{I}_g^{(0)}$ and $\overline{M}_g^{(0)}$, respectively. In return, these currents are used to induce incident electric and magnetic fields on the surface of the
buried scatterer $\overline{V}_{\text{obj}}^{(1)}$, which are calculated directly using the near-field expressions [13]. Depending on the geometry of the buried object, the SDFMM or the MoM is used to calculate the induced electric and magnetic surface currents on the buried object, $\overline{J}_{\text{obj}}^{(1)}$ and $\overline{M}_{\text{obj}}^{(1)}$. Finally, these currents are used to induce electric and magnetic fields on the ground surface as $\overline{V}_{g}^{(1)}$ using [13], and the SDFMM can be used to solve for the induced electric and magnetic currents on the ground, $\overline{J}_{g}^{(1)}$ and $\overline{M}_{g}^{(1)}$, respectively. This completes one multiple scattering mechanism (e.g. ground-object-ground). The process is to be repeated till the solution of currents on the ground and on the buried object converges.

As will be shown in the following section, a significant reduction in the CPU and memory requirements is anticipated upon using the multiple scattering mode where only $\overline{Z}_{g,g}$ and $\overline{Z}_{\text{obj, obj}}$ are used in the iterative calculations while in the conventional SDFMM and/or the conventional MoM, the four matrices $\overline{Z}_{g,g}$, $\overline{Z}_{\text{obj, obj}}$, $\overline{Z}_{g,\text{obj}}$, and $\overline{Z}_{\text{obj, g}}$ are to be used. In many applications $\overline{Z}_{g,\text{obj}}$ and $\overline{Z}_{\text{obj, g}}$ have the same order as $\overline{Z}_{g,g}$ and $\overline{Z}_{\text{obj, obj}}$ as will be demonstrated in Section III.

III. NUMERICAL RESULTS

A variety of geometries represented by five examples are used in this section to test and validate the proposed multiple scattering model with the conventional MoM. The numerical results will present (i) the electric and magnetic surface currents on the ground, (ii) the electric and magnetic surface currents on the buried scatterer, and (iii) the scattered electric field above the ground due to just the buried scatterer (i.e., scatterer's signature). In all results presented in this section, the incident wave is assumed a Gaussian beam tapered towards the edges of the ground [17] with horizontally polarized incident electric field (i.e., in the y-direction) and due to just the air/ground interface as well. The half-beam width of the beam is $L/5$ where the ground has dimensions $L \times L$.

In Example 1 shown in Fig. 2a-b, the flat ground has dimensions $3.04\lambda_o \times 3.04\lambda_o$ with an x-directed horizontal cylinder of length $b = 3.04\lambda_o$, radius $a = 0.15\lambda_o$, and burial depth $z = -0.65\lambda_o$ measured from its center, where $\lambda_o$ is the free-space wavelength. The relative dielectric constant of the ground and the buried cylinder are assumed $\varepsilon_{xg} = 2.5 - j0.18$ and $\varepsilon_{y} = 7.9 - j0.0029$, respectively. The incident angle in this example is $\theta^i = 0$. Excellent agreement between the multiple scattering model and the MoM for the electric and magnetic surface currents on the flat ground and on the buried cylinder is shown in Figs. 3a-d. For qualitative comparisons, the magnitudes of the surface currents are plotted versus the y-direction at $x = 1.52\lambda_o$ in these figures. Four multiple scattering mechanisms, as described in Fig. 2b, are used to obtain these results. Similar agreement is observed for results plotted versus the x-direction at $y = 1.52\lambda_o$. An insignificant error is observed in Figs. 3c and 3d that could be due to the error introduced in approximating the cylindrical surface with triangular patches. As shown in these figures, the presence of the buried cylinder caused a significant change in the initial ground currents $\overline{J}_{g}^{(0)}$ and $\overline{M}_{g}^{(0)}$ only after the first multiple scattering mechanism. Similarly, Fig. 4a shows the convergence of the magnetic surface current on the upper half of the buried cylinder, plotted versus the x-direction at $y = 1.52\lambda_o$, while Fig. 4b shows the convergence of the magnetic surface current on both the upper and lower halves of the buried cylinder, plotted versus the y-direction at $x = 1.52\lambda_o$. The results confirm that the surface currents converge after only one ground-object-ground scattering mechanism, as described in Fig. 2a.

In Example 2, the geometry is a multilayered flat ground as shown in Fig. 2c, where the ground has dimensions $3.04\lambda_o \times 3.04\lambda_o$ and the underground layer has burial depth of $z = -0.45\lambda_o$. The relative dielectric constant of the ground and the underground layers are assumed $\varepsilon_{xg} = 2.5 - j0.18$ and
\( \varepsilon_{3r} = 4.2 - j0.29 \), respectively. The incident angle in this example is \( \theta' = 10^\circ \). All results show excellent agreement with the MoM (not presented here).

In Example 3, the geometry is a multilayered rough ground with flat underground layer as shown in Fig. 2d. The ground dimensions, the burial depth of underground layer, the relative dielectric constants, and the incident angle are the same as in example 2. The roughness parameters of the upper rough air/ground interface are given by the rms height and the correlation length of the random rough surface as \( \sigma = 0.08\lambda_o \) and \( l_c = 0.5\lambda_o \), respectively. All comparisons with the MoM strongly validate the multiple scattering model used in this example similar to previous examples (not presented here).

In Example 4, the geometry is flat ground with a buried sphere as shown in Fig. 2e where the dimensions of the ground are \( 4.08\lambda_o \times 4.08\lambda_o \). The sphere has radius of \( a = 0.5\lambda_o \) and is buried at \( z = -0.75\lambda_o \) measured from its center. The relative dielectric constants of the ground and the buried sphere are assumed \( \varepsilon_{2r} = 2.5 - j0.18 \) and \( \varepsilon_{3r} = 4.5 - j0.029 \), respectively. The incident angle in this example is \( \theta' = 0 \). In Figs. 5a and 5b, the magnitude of magnetic surface current on the air/ground interface is plotted versus the \( x \)-direction at \( y = 2.04\lambda_o \). Three solutions are presented in Figs. 5a; (i) solution obtained using the conventional MoM for the whole scatterer (hollow circles), (ii) solution obtained by employing the MoM in the multiple scattering model (solid circle), (iii) solution obtained by employing the SDFMM in the multiple scattering model (+ symbol). The results show excellent agreement between these three methods. Moreover, validations based on the scattered electric fields observed above the ground at \( z = 0.5\lambda_o \) and due just to the buried sphere are shown in Fig 5b where results are plotted versus the \( x \)-direction at \( y = 2.04\lambda_o \). Excellent agreement between the three methods is also observed in these figures. However a slight difference between the multiple scattering model and the conventional MoM results is observed in Fig. 5b. This slight difference can be attributed due to the definition of the scattered fields due to just the buried object in both methods. In the conventional MoM method we calculated the total scattered electric fields twice; once with the buried sphere and once without the buried sphere, then the results are subtracted from each other using complex vectors [10]-[11]. On the other hand, for the multiple scattering model we use the obtained surface electric and magnetic currents on the ground due to only the presence of the sphere, i.e., \( \vec{\mathcal{J}}^{(1)}_g + \vec{\mathcal{J}}^{(2)}_g + \vec{\mathcal{J}}^{(3)}_g \) and \( \vec{M}^{(1)}_g + \vec{M}^{(2)}_g + \vec{M}^{(3)}_g \), and use the near-field expression in [13] to compute the scattered electric fields.

Notice that the quantities \( \vec{\mathcal{J}}^{(0)}_g \) and \( \vec{M}^{(0)}_g \) represent the surface currents on the ground with the assumption that there is no buried objects, i.e., only the ground is present and are not used to calculate the signature of the buried sphere.

In Example 5, the geometry is a multilayered rough ground as shown in Fig. 2f where the dimensions of the ground and the incident angle are the same as in example 4. The underground rough layer has a burial depth of \( z = -0.95\lambda_o \) measured from its mean plane to the rough air/ground interface mean plane. The relative dielectric constants of the ground and the underground layer are assumed \( \varepsilon_{2r} = 2.5 - j0.18 \) and \( \varepsilon_{3r} = 3.7 - j0.2 \), respectively. The roughness parameters of the air/ground interface are \( \sigma_1 = 0.06\lambda_o \) and \( l_{c1} = 0.5\lambda_o \) while they are \( \sigma_2 = 0.05\lambda_o \) and \( l_{c2} = 0.4\lambda_o \) for the underground layer. In Fig. 5a, the magnitudes of the magnetic surface currents on both the rough air/ground interface and the underground rough layer are plotted versus the \( x \)-direction at \( x = 2.04\lambda_o \). The results show an excellent validation between the model using the MoM and the model using the SDFMM. In Fig. 5b, the scattered electric fields observed above the ground at \( z = 0.5\lambda_o \) are plotted versus the \( y \)-direction at \( x = 2.04\lambda_o \). The scattered fields due to just the rough air/ground interface are calculated using only the surface currents \( \vec{\mathcal{J}}^{(0)}_g \) and \( \vec{M}^{(0)}_g \) while the scattered fields due to just the underground layer are calculated using only the
surface currents $\mathbf{J}^{(1)}_g + \mathbf{J}^{(2)}_g + \mathbf{J}^{(3)}_g$ and $\mathbf{M}^{(1)}_g + \mathbf{M}^{(2)}_g + \mathbf{M}^{(3)}_g$. As expected, the results in Fig. 5b show that the signature of the air/ground interface is significantly larger than that of the underground layer. Moreover, the results of this example clearly confirm the previous observations that the solution converges after the first multiple scattering mechanism (i.e., air/ground interface-underground layer-air/ground interface).

It should be mentioned that the relative residual error used in the TFQMR solver is $10^{-5}$ in all results of this section. For efficient results, the multiple scattering model should not be used when the buried objects are very shallow due to the inaccuracy of the near-field expressions in [13], but instead, the complete SDFMM, which has been successfully used in analyzing these structures [10]-[11], should be used.

IV. CONCLUSIONS

A new multiple scattering model to compute the signature of non-shallow objects buried under the rough ground (3-D scattering problem) is presented and validated in this work. The advantages of this model are (i) superior computational requirements than using the conventional MoM especially when the SDFMM is employed in the model and (ii) removing a barrier in the conventional SDFMM since it should be only applied to quasi-planar structures. It has been shown in this work that upon using the proposed multiple scattering model, certain potential non-quasi-planar structures, e.g., three multilayered ground, are decomposed into two quasi-planar layers where the SDFMM is separately applied. Interestingly, all obtained results show that both the electric and magnetic surface current solutions converge after only a single multiple scattering mechanism (i.e., from air/ground interface to buried object and then back to air/ground interface). This assures that no extra CPU time is needed to achieve the correct solutions.

ACKNOWLEDGMENTS

This research was sponsored by the Northeastern University’s Demining MURI grant # DAA 0-55-97-0013 and in part by the College of Engineering at the University of Arkansas. The SDFMM was originally developed by V. Jandhyala, E. Michielssen and W. Chew at the UIUC.

References


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![Diagram](image_url)

**Figure 1.** Decomposition of non-quasi-planar structure into two quasi-planar structures showing their interactions.
Figure 2. Multiple scattering mechanism between (a) the air/ground interface and a buried horizontal cylinder showing one ground-object-ground mechanism, i.e., \( n = 1 \) (b) the same as in (a) but showing four ground-object-ground mechanisms, i.e., \( n = 4 \), (c) the air/ground flat interface and an underground flat layer, (d) the air/ground rough interface and an underground flat layer, (e) the flat air/ground interface and a buried sphere, (f) the air/ground rough interface and an underground rough layer.
Figure 3. Magnitude of surface current on the flat air/ground interface shown at $x=1.52\lambda_0$ for (a) magnetic current $|\hat{M}|$, (b) electric current $|\hat{J}|$. Comparison between the proposed model and the conventional MoM. Data are for Example 1 (Figs.2a and b).
Figure 4. Convergence of magnetic surface current solution on (a) the upper half of the buried cylinder shown at $y=1.52\lambda_0$, (b) on the buried cylinder shown at $x=1.52\lambda_0$. Data are for Example 1.