Inverse Scattering Computational Algorithm for the Reconstruction of Random Rough Surface Profiles

Magda El-Shenawee[1] and Eric Miller[2]
[1]Department of Electrical Engineering
University of Arkansas, Fayetteville AR 72701
magda@uark.edu
[2]Department of Electrical and Computer Engineering
Northeastern University, Boston, MA 02115
elmiller@ece.neu.edu

Abstract

A fast algorithm for reconstructing the profile of random rough surface is presented. This algorithm uses the three-dimensional (3-D) electromagnetic waves scattered from rough ground surfaces to retrieve the unknown surface profiles. The proposed algorithm is based on merging a 3-D fast forward solver and an efficient optimization technique. The preliminary results show a potential success of the inversion algorithm.

I. INTRODUCTION

In general, the presence of rough ground surface is considered a major clutter in subsurface sensing applications. The ground response is more significant than the target signature due to the small size of targets compared with ground perturbation or ground illuminated region. Ground roughness causes considerable distortion in ground penetrating radar response (GPR), which makes it difficult to process received GPR signals using conventional signal processing techniques.

Previous work to reconstruct a two-dimensional (2-D) rough surface was published, in which the surface was assumed a perfect electric conductor [1]. More recent work to reconstruct one-dimensional rough surface (1-D) was published, in which the rough surface was assumed to be dielectric [2]. Both approaches had used the Kirchoff approximation to compute the electromagnetic waves scattered from the rough surface. The focus of the current work will be on developing a fast computational inversion algorithm for reconstructing 2-D dielectric rough surfaces. The algorithm is based on combining a fast forward solver with an efficient searching technique. It is also based on using the electromagnetic waves (GPR-type data) scattered from the rough ground surface to retrieve the surface height variation. The previously well developed steepest descent fast multipole method (SDFMM) for targets buried under a 2-D dielectric rough ground surfaces will be used here as the fast forward solver [3], [4]. In addition, an efficient optimization search technique based on the algorithm of Fletcher and Powell [5], for rapidly convergent descent method in minimizing a cost function, will be employed. This cost function represents the mean square error between synthetic data and simulated data of the scattered electric fields (GPR-type data). Several key issues will be examined here; the effect of the

0-7803-8302-8/04/$20.00 ©2004 IEEE

incident and scatter polarization and directions, the location of receivers (e.g. far-zone or near-zone), computational expenses of the algorithm, cost function type, and mathematical model of the rough surface and its unknown parameters.

II. METHODOLOGY

The reconstruction algorithm begins with assuming certain mathematical model of the rough surface that can approximately define the surface height variation. The surface model includes some unknown parameters to be recovered during the reconstruction process. The second step of the reconstruction is defining a suitable cost function to be minimized. In this work, the cost function $C(\theta)$ is defined as:

$$C(\theta)=\sum_{i=1}^{N_r}\left|\overline{E}_i^{True}-\overline{E}_i^{Sim}\right|^2 \quad (1)$$

in which, $\overline{E}_i^{True}$ and $\overline{E}_i^{Sim}$ represent the scattered electric fields for true-data (GPR-type data) and simulated data, respectively. The subscript i represents the receiver's number, where N_r is the total number of receivers above the ground. These electric fields are considered for single frequency, single polarization of the incident electromagnetic waves. The vector θ represents the unknown parameters to be recovered to reconstruct the rough surface profile. The number of elements of vector θ varies according to the assumed surface mathematical model. The cost function in (1) will be minimized until an acceptable error is achieved. The estimated vector $\hat{\theta}$ will be obtained according to: $\hat{\theta}=\arg_\theta(\min(C(\theta)))$. However, in order to minimize the cost function $C(\theta)$ and obtain the estimated vector $\hat{\theta}$, a rapid and efficient steepest decent approach will be used here (the algorithm developed by Fletcher and Powell [5]). In this work, for faster and more efficient computations, the elements of the unknown vector θ will be restricted to certain limits. In other words, upper and lower bound constraints are *a priori* provided to the optimizer, i.e. $\theta_{LB}\le\theta\le\theta_{UB}$. The iterative inversion technique to search for the unknown parameter vector θ is given by [5]:

$$\hat{\theta}_{k+1}=\hat{\theta}_k+\alpha_k d_k \quad (2)$$

in which k is the iteration index, α_k is the k-step, and the vector d_k is the vector that minimizes the quadratic equation (see details in Fletcher and Powell algorithm [5]). The optimization algorithm involves evaluating the gradient of the cost function with respect to each unknown parameter. This scenario necessitates using a fast forward solver in the inversion algorithm, such as the SDFMM.

III. NUMERICAL RESULTS

The numerical results are obtained using the fast forward solver SDFMM for a surface of size $1m\times1m$. The incident electromagnetic wave is represented by Gaussian beam incident to the surface with horizontal polarization [4]. The relative dielectric constant of the surface material is assumed $\varepsilon_r=2.5-i0.18$. The far-field horizontal electric field scattered from the ground at normal incidence is calculated for the cost function, i.e., these results represent copolarized HH waves at single receiver ($N_r = 1$ in (1)). A 2-D sinusoidal rough surface is assumed with the model equation $h(x, y) = H \cos(2\pi x / L) \cos(2\pi y / L)$, where H and L are the surface maximum height and period, respectively [1]. The behavior of the cost function is plotted versus the surface parameters H and L (not shown here). For simplicity, the surface period is assumed known as L = 30cm, and the inversion algorithm is tested to obtain the unknown parameter H as shown in Fig. 1. The sinusoidal surface was successfully reconstructed with relative error less than 3%. The inversion algorithm required 76 runs of the 3-D SDFMM forward solver to achieve 10^{-6} error in the cost function. Each SDFMM run required 10 CPU minutes. The inversion algorithm is also tested on a groove-like dielectric rough surface (1-D) [2], on which a variety of strategies are demonstrated in the inversion algorithm. These strategies are the multiple-incidence strategy, the multiple-frequency strategy, and/or combination of both strategies. The numerical results of reconstructing the groove-like surface using the multiple-incidence are compared with those using single incidence, and the results are shown in Fig. 3. In this example, we assumed $\varepsilon_r = 4 - i0.01$ and eleven receivers located at 15 cm above the ground mean plane, separated by 6cm. The results show that multiple-incidence provides better reconstruction of the surface.

ACKNOWLEDGMENTS

This research was sponsored by NSF-Northeastern University CenSSIS, grant No. EEC-9986821.

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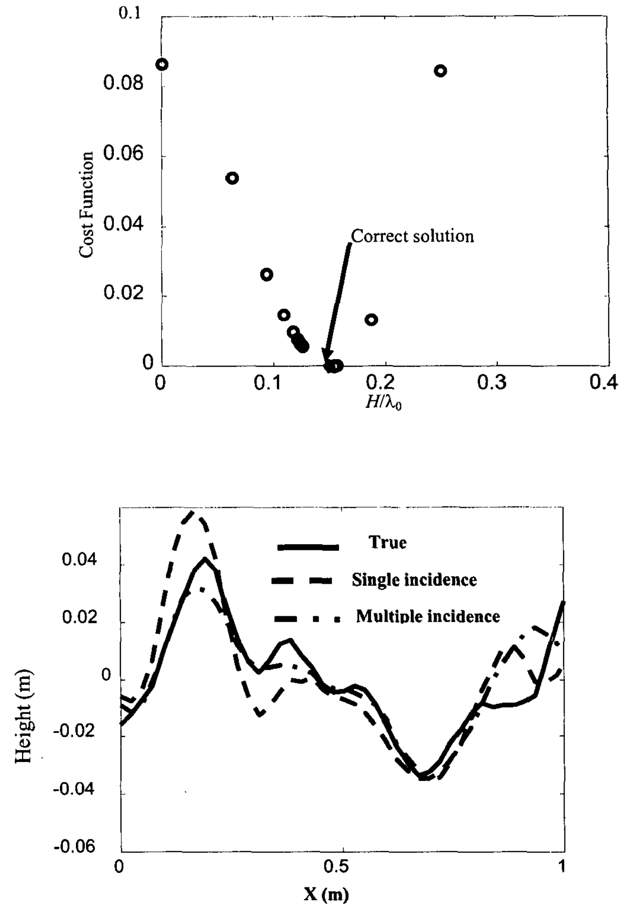

Fig. 2 Reconstruction of groove-like dielectric rough surface modeled by B-spline function with 16 parameters. Comparison between single-incidence strategy and multiple-incidence-strategy.