The Level Set Technique for Microwave Imaging of 3D Dielectric Objects

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Introduction

The problem of unknown object(s) identification using electromagnetic waves has many potential applications in diverse fields of science and engineering. The electromagnetic waves are used for illumination of the unknown objects and the scattered waves are analyzed for retrieving the profile of the target objects. Therefore developing efficient inverse scattering techniques benefits many applications such as target identification, geophysics, seismic exploration, remote sensing, atmospheric science, Ground Penetrating Radar (GPR) and medical applications such as cancer and hypothermia detection. In shape reconstruction problems, the constitutive parameters of the unknown objects are generally assumed to be known, which mitigates the complexity of the inverse scattering problem. The main idea of the level set method is to embed the evolving interface (contour or surface) as the zero level of a higher-dimensional function. Using the level set framework, topological changes are handled automatically and several objects are reconstructed from a single initial guess simultaneously [1],[2].

Methodology

If the evolving surface is represented as the zero level of a three-dimensional function $\Phi(\cdot)$, the following PDE known as the Hamilton-Jacobi Equation is derived for tracking the motion of the evolving surfaces [1]:

$$\frac{\partial}{\partial t}\Phi(x,y,z,t) + V(\bar{r})\left\|\nabla\Phi(x,y,z,t)\right\| = 0 \tag{1a}$$

$$\Phi_0 = \Phi(x,y,z,t=0) \tag{1b}$$

Where $V(\cdot)$ is an appropriate form of the deformation velocity pointing in the normal direction to the evolving surfaces. The level set function Φ_0 in (1) is initialized to the signed distance function corresponding to the arbitrary initial guess of the unknown objects (e.g. a sphere in this work).

The objective of the inversion algorithm is to minimize the cost function corresponding to the error between the scattered field of the evolving objects and the unknown target object(s). Therefore the forward scattering problem is solved many times during the reconstruction algorithm for calculating the cost function and calculation of the deformation velocity. The Method of Moments (MoM) is

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used in this work for these purposes. Since the level set technique is based on the implicit representation of the evolving object(s), employing the surface-based integral equation methods is challenging using this approach. To overcome the difficulty, the marching cubes method which is a computer graphics technique [3] is employed for generation of the triangular mesh required by the MoM forward solver during the inversion. Following a similar approach for 2D conducting objects and by using the reciprocity theorem [4], the appropriate deformation velocity making a decreasing cost function for any point $\bar{r} \in S$ on the surface of the evolving object is obtained as:

$$V(\bar{r}) = \left[\sum_{i=1}^{N_I} \sum_{m=1}^{N_M(i)} -\alpha \ \mathrm{Re}\left(\bar{E}(\bar{r}).\bar{E}'(\bar{r}) \right) \right] \tag{3}$$

Where $\bar{E}(\bar{r})$ and $\bar{E}'(\bar{r})$ are the forward and adjoint electric fields correspondingly [5]. The symbol N_I represents the number of incident waves, $N_M(i)$ is the number of measurements under the i^{th} incidence and α is a positive factor. The deformation velocity at any point in the computational domain is chosen to be that of the nearest point on the moving surface.

Numerical Results

A. Reconstruction of a dielectric torus

In the first example, the level set algorithm is tested for the reconstruction of a dielectric torus made of a material with the permittivity of $\varepsilon_r = 4.5$ and loss tangent of $\tan(\delta) = 0.0111$. The torus radius and the tube radius are 7cm and 2cm, respectively. Twenty six theta-polarized plane waves illuminate the object and scattered waves are collected at 26 scattering directions per incidence with the step of $\Delta\theta = \Delta\phi = \frac{\pi}{4}$. The initial guess is chosen to be a sphere with the radius of 10 cm. Three frequencies are used in the shape reconstruction algorithm $f_1=1$ GHz, $f_2=2$ GHz and $f_3=3$ GHz. The normalized cost function and the reconstruction results are shown in Fig. 1 and Fig. 2, respectively.

B. Reconstruction of two dielectric ellipsoids

In the second example, the level set algorithm is tested for reconstruction of two dielectric ellipsoids with the permittivity of $\varepsilon_r = 5.0$. The dimensions of each ellipsoid are $a=8$cm and $b=c=2$cm. The same initial guess and the same configuration of the incident and scattering directions are used for this example. The normalized cost function and the reconstruction results are shown in Fig. 3 and Fig. 4, respectively.

C. Reconstruction of a dielectric cone

In the last example, a dielectric cone with the permittivity of $\varepsilon_r = 6.5$ and the loss tangent of $\tan(\delta) = 0.0111$ is retrieved satisfactorily using the level set algorithm. The cone radius and height are chosen to be 9cm and 7cm, respectively. The normalized cost function and the reconstruction results are shown in Fig. 5 and Fig. 6, respectively.

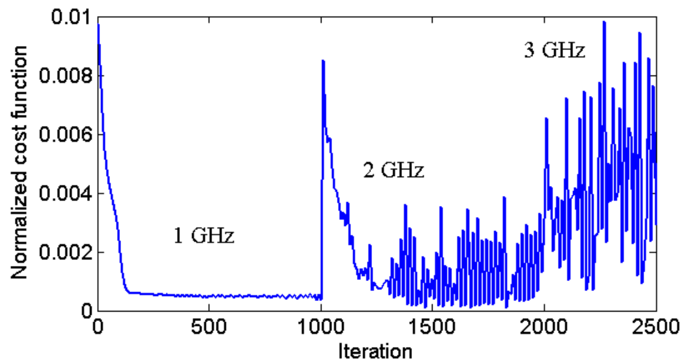

Fig. 1. Normalized cost function for reconstruction of a dielectric torus

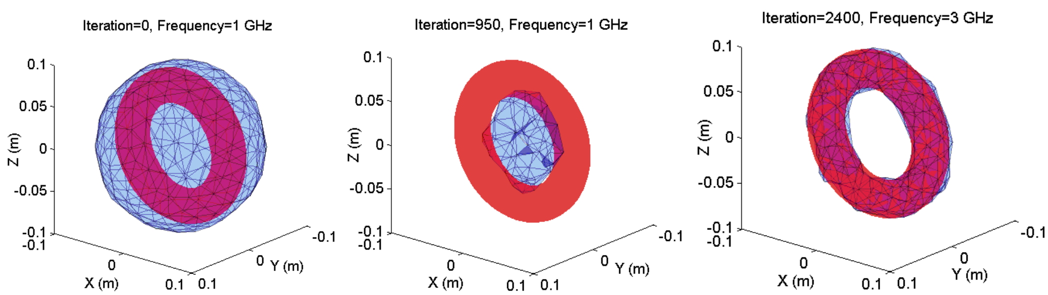

Fig. 2. The reconstruction of dielectric torus at different iterations

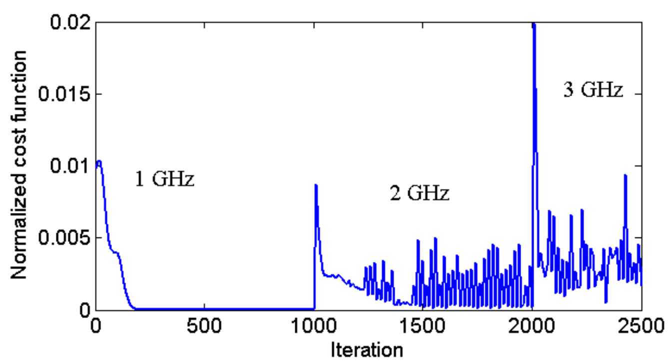

Fig. 3. Normalized cost function for reconstruction of two ellipsoids

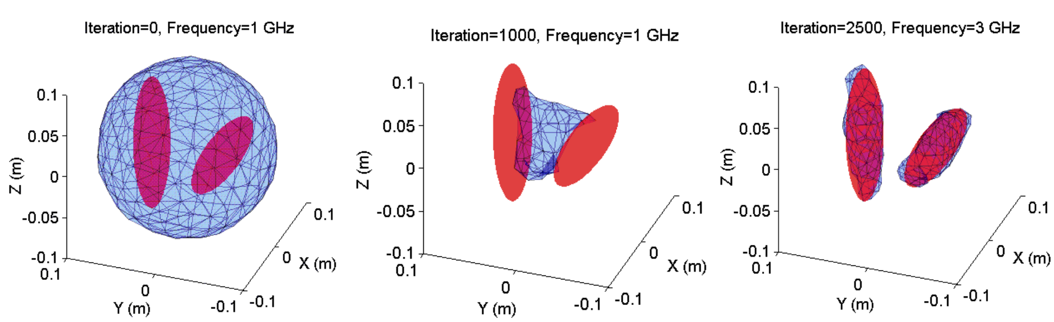

Fig. 4. The reconstruction of two ellipsoids at different iterations

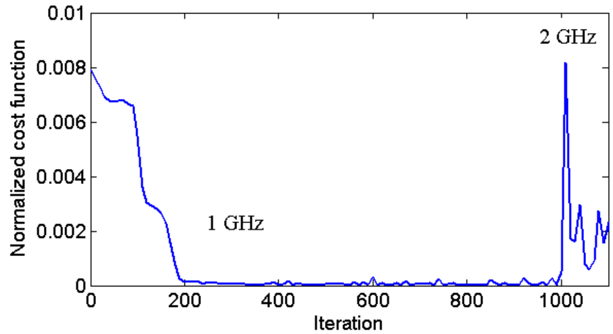

Fig. 5. Normalized cost function for reconstruction of a dielectric cone

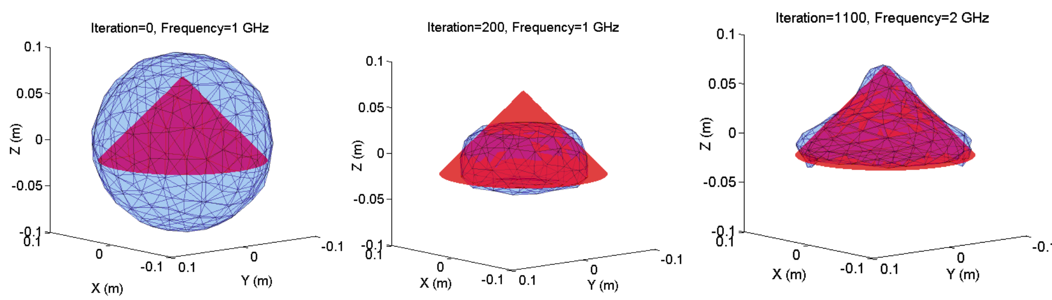

Fig. 6. The reconstruction of a dielectric cone at different iterations

Conclusions

The level set algorithm is employed for reconstruction of multiple homogeneous three-dimensional dielectric objects. The marching cubes method is used for generating the triangular mesh for the MoM forward solver. The presented results show the capability of the level set algorithm with successful retrieval of multiple objects using a single initial guess.

Acknowledgements

This work is funded by Entergy Co. and partially by the National Science Foundation Award Number ECS-0524042

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