Inverse Scattering Level Set Algorithm for Retrieving the Shape and Location of Multiple Targets

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Abstract— **The level set technique is an inverse scattering problem solution concern the shape of unknown targets. In this work, the level set algorithm will demonstrate the capability of reconstructing 2D and 3D PEC and dielectric targets. In addition to its robustness, the level set technique has the advantage of retrieving several objects from a single initial guess with no *a priori* information.**

I. Introduction

In inverse scattering, the goal is to infer the characteristics of unknown targets illuminated by electromagnetic waves. The collected scattered fields at the receiver points are processed to retrieve the information about the scattering targets. Inverse scattering solutions are applicable in several areas such as see-through-wall imaging, medical imaging, ground penetrating radar, non-destructive testing etc. (e.g. [1]-[2]). The inverse problems are non-unique and non-linear, which make them challenging, in many applications. Retrieving the shape and location of targets is the focus of this work with the assumption that their electrical properties are *a priori* known.

The level set method has proven to be topologically flexible and robust, among the several shape reconstruction techniques. In this work, several algorithms based on level set technique are developed for retrieving the shape and location of multiple complex targets in two- and three-dimensions (2D and 3D). The level set algorithm has proven to be successful in reconstructing multiple targets immersed in air or hidden behind a wall. The transverse electric (*TE*) and transverse magnetic (*TM*) excitations for the 2D cases are investigated. Successful shape reconstruction results are demonstrated in both cases even when the synthetic data is corrupted with Gaussian noise with SNR as low as 5dB.

II. Methodology

In the level set method, the evolving interface is represented as the zero level of a higher dimensional function ϕ . Therefore at each time t, we have the following expression for the evolving interface [3]:

$$\Gamma(t) = \left\{ \bar{r} \,\middle|\, \phi(\bar{r}, t) = 0 \right\} \tag{1}$$

Where $\bar{r} = (x, y)$ is the 2D position vector, when the moving interface is a contour (1D), and $\bar{r} = (x, y, z)$ is the 3D position vector, when the moving interface is a surface (2D). Differentiating (1) with respect to the evolving time t, yields [3]:

$$\frac{\partial \phi}{\partial t} + \nabla \phi \cdot \bar{V}(\bar{r}) = 0 \tag{2}$$

The symbol of $\bar{V}(\bar{r}) = \dfrac{d\bar{r}}{dt}$ represents the velocity vector. Eq. (2) can be rewritten as:

$$\frac{\partial \phi}{\partial t} + F(\bar{r}) \left\| \nabla \phi \right\| = 0, \quad \phi_0 = \phi(\bar{r}, t = 0) \tag{3}$$

The symbol $F(\bar{r})$ represents the component of the velocity vector in the normal direction to the evolving surfaces. The level set function Φ_0 in (1) is initialized to the signed distance function corresponding to the chosen initial guess of unknown objects.

The objective of the algorithm is to minimize the error between the scattered fields of the evolving objects and those of unknown object(s). As a result, the forward scattering problem is solved many times during the reconstruction algorithm for calculating the cost function and calculation of the deformation velocity. The Method of Moments (MoM) is used in this work as the forward solver. The cost function is defined as [4]

$$F(C_t) = \sum_{i=1}^{N_{inc}} \sum_{j=1}^{N_{meas}^i} \left| E^s(\theta_i^{inc}, \theta_{ij}^{meas}) - E^s{}_{meas}(\theta_i^{inc}, \theta_{ij}^{meas}) \right|^2 \tag{4}$$

Where θ_i^{inc} is the i^{th} incident angle and θ_{ij}^{meas} is the j^{th} measurement direction angle when the incident angle is θ_i^{inc} .

The unknown object is illuminated by N_{inc} incident directions. The symbol N^{i}_{meas} represents the number of scattering directions under i^{th} incidence. The symbol $E^{s}(\theta_{i}^{inc},\theta_{ij}^{meas})$ represents the simulated scattered field and the symbol $E^{s}{}_{meas}(\theta_{i}^{inc},\theta_{ij}^{meas})$ represents the measured scattered field. The cost function is minimized using data at all incident and scattering directions and frequencies. In order to achieve convergence, the deformation velocity is chosen such that the cost function has a negative derivative with respect to the evolving time (decreasing function). The appropriate form of the deformation velocity is obtained using the reciprocity theorem and the work of Roger et al [5]. Generally, the deformation velocity depends on forward and adjoint scattering problems [6].

A. Reconstruction of two targets

In the first example, the level set algorithm is employed to retrieve a 2D gun-shaped target with noise contaminated data. The targets are assumed to be infinitely conducting cylinders with gun-shape cross section. *TM*-polarized plane waves are used to illuminate the targets where the electric field is parallel to the cylinders axes. Two levels of noise are investigated, corresponding to SNR=10 dB and SNR=5 dB.

The obtained results after 4740 iterations are shown in Fig. 1 and Fig. 2, respectively. According to the obtained results, the level set has satisfactory performance even with noisy data with SNR=5 dB.

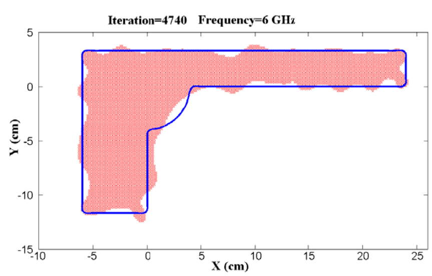

Fig. 1 Reconstruction of a gun-shaped target with SNR=10 dB.

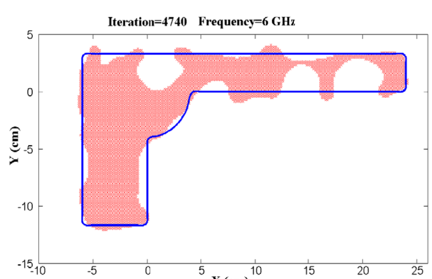

Fig. 2 Reconstruction of a gun-shaped target with SNR=5 dB.

B. Reconstruction of five complex targets using full and half measurements data.

We have proposed a new measurement system where scattering data are collected only in one side of the targets, according to $\theta_{inc}-\frac{\pi}{2}<\theta_{sc}<\theta_{inc}+\frac{\pi}{2}$. The symbols θ_{inc} and θ_{sc} represent the incident and scattering directions, respectively. The reconstruction results for five complex targets are given in [4]. The obtained results showed that collecting the scattered fields only at locations half space around the target (half measurements) produced better reconstruction results compared with the case when the datat are collected from all around the target (full measurements). More details are given in [4].

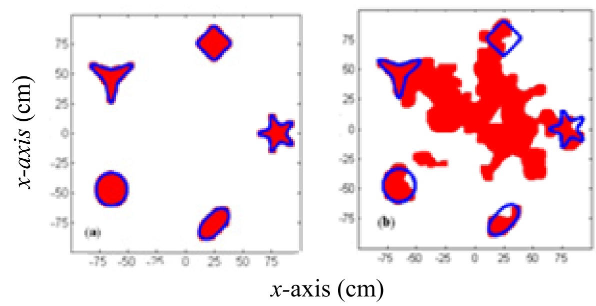

Fig. 3 (a) Half measurements versus (b) full measurements system [4]

C. Comparison between TM and TE polarizations

In another example, the performance of the algorithm is investigated under two different polarizations; *TM* versus *TE* polarization. The reconstruction under the *TE* polarization is more challenging compared with that using *TM* polarization [7]. The cost function and the retrieved profiles for reconstruction of two rectangular cylinders are shown in Fig. 4 and Fig. 5, respectively. It is observed that after 2000 iterations, two separate targets are retrieved under *TM* polarization while it is still evolving under *TE* polarization.

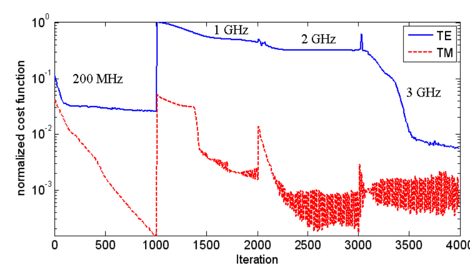

Fig. 4 Cost function for reconstruction of two rectangular targets in *TM* and *TE* polarizations

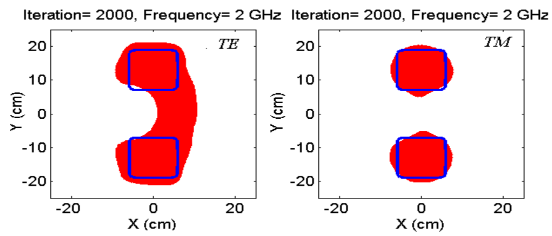

Fig. 5 Reconstruction of two rectangular targets in *TM* and *TE* Polarizations [7]

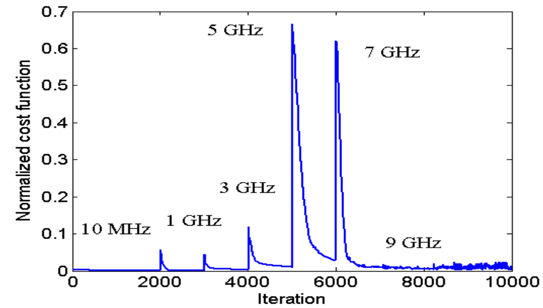

Fig.8. Normalized cost function for reconstruction of the defected pipe

C. Parallelization of the level set algorithm

To speed-up the algorithm, the MPI parallelization techniques are used to scale down the reconstruction time from several hours to a few seconds, the details of parallelization are given in [8]. The reconstruction results for imaging a star-shaped PEC target and the speedup versus the number of processors are shown in Fig. 6 and Fig. 7, respectively. The obtained results show a speedup of ~ 84 X is achieved using 256 processors on the San Diego super computer.

(a) (b) (c)

Fig. 6 Reconstruction of star-shaped target at (a) 500MHz, (b) 1GHz, and (c) 10GHz, using the MPI parallelized algorithm in [8]

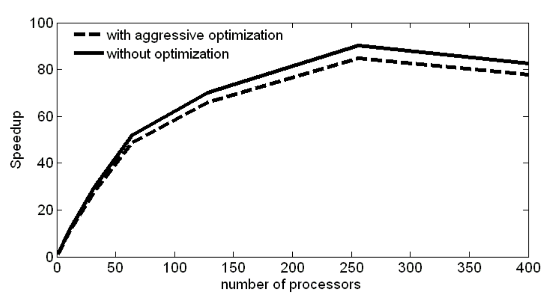

Fig. 7 Speed-up versus the number of processors

D. Reconstruction of the defected pipe behind a dielectric wall

The level set algorithm is modified to retrieve the shape of a defected pipe located behind a dielectric wall [9]. The stationary phase approximation is used to evaluate sommerfeld integrals the Green's functions in stratified media. The cost function and the final reconstructed profile are shown in Fig. 8 and Fig. 9, respectively. The obtained results show that the level set algorithm is capable of retrieving small features in hidden targets behind a dielectric wall. The reconstruction versus the distance to the wall was discussed in [9].

Initial guess

wall

True object

x-axis (cm)

Fig. 9 Reconstruction of a defected pipe hidden behind a dialect wall of thickness 20 cm, conductivity 0.011 S/m and permittivity of 4.5. The reconstruction was converged after ~10000 iterations at 9 GHz, using the level set algorithm [9]

E. Reconstruction of 3D PEC target

In another example, the results level set algorithm demonstrates retrieving the shape of a 3D perfectly conducting conical frustum as shown in Fig. 10. The cost function and the results using noiseless and noisy data are depicted in Fig. 10-12, respectively. The details of the reconstruction algorithm are given in [10].

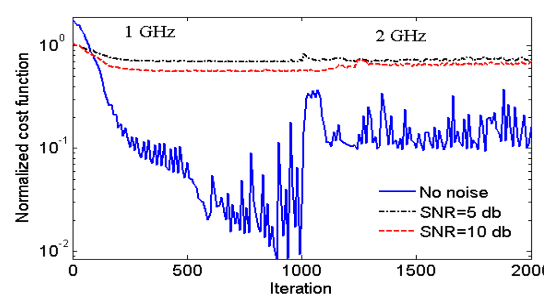

Fig. 10. Normalized cost function for reconstruction of a conical frustum

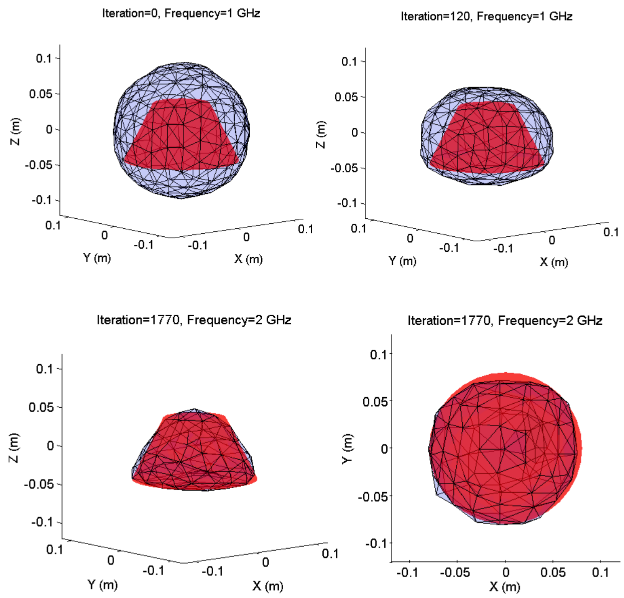

Fig. 11 a-d. Reconstruction of a conical frustum at different stages, (a) initial guess, (b) after 120 iterations at 1 GHz (c) after 1770 iterations at 2 GHz, (d) after 1770 iterations at 2 GHz (top view).

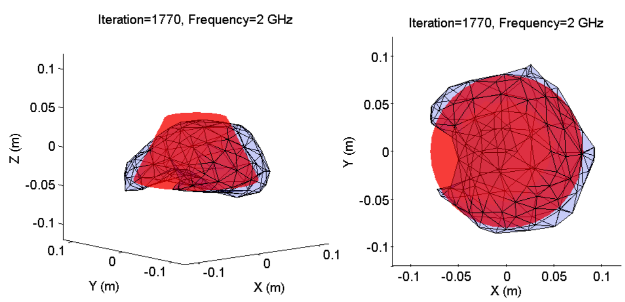

Fig. 12 a-b. Final reconstruction result of a conical frustum (a) after 1770 iterations at 2 GHz using SNR=10 dB, (b) after 1770 iterations at 2 GHz using SNR=10 dB (top view).

F. *Reconstruction of two elliptical dielectric targets*

In this example, the level set shows it capability to reconstruct dielectric spheroids as presented in Fig. 13. The formulation of the deformation velocity is different for dielectric targets with details given in [10]. The dimensions of each ellipsoid are a=8cm and b=c=2cm. The dielectric material of the objects has permittivity of $\varepsilon_r = 5.0$. The cost function and the retrieved surfaces after several iteration numbers are shown in fig. 13 and Fig. 14, respectively.

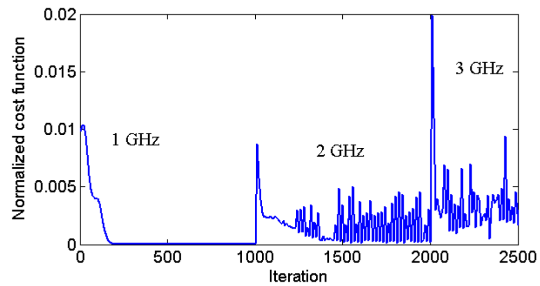

Fig. 13. Normalized cost function for reconstruction of two ellipsoids

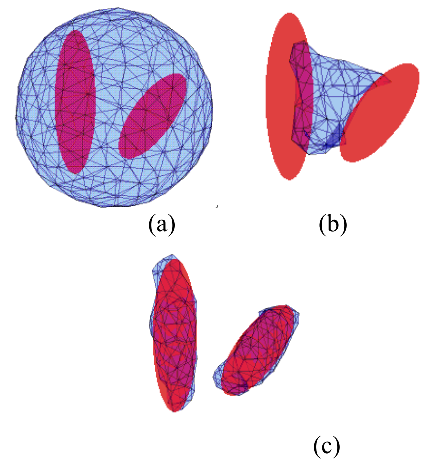

Fig. 14. The reconstruction of two ellipsoids at different iterations (a) initial guess, (b) 1GHz, (c) 3GHz [11]

G. *Reconstruction of two dielectric targets with different permittivity values*

In the last example, we have used two level set functions to reconstruct the shape of two dielectric targets having different permittivity values [10]. The two objects are a dielectric ellipsoid with the permittivity of $\varepsilon_{r1} = 5.0$ and loss tangent of $\tan(\delta_1) = 0.001$ and a dielectric cube with the permittivity $\varepsilon_{r2} = 2.2$ and loss tangent of $\tan(\delta_2) = 0.001$. The cost function and the reconstruction results are shown in Fig. 15 and Fig. 16, respectively.

In all results, the frequency hopping was employed which explains the jumps in the cost function in all results of this section. Moreover, we believe that the oscillations shown in the cost functions that occur towards the end of each frequency are numerical artefacts and can be easily eliminated upon using careful stopping criteria in the algorithm. Ongoing research to improve the accuracy of the level set algorithm for both the 2D and 3D configuration is underway. The level set algorithm still has a space for improvements especially regarding its speed-up where all results show excessive

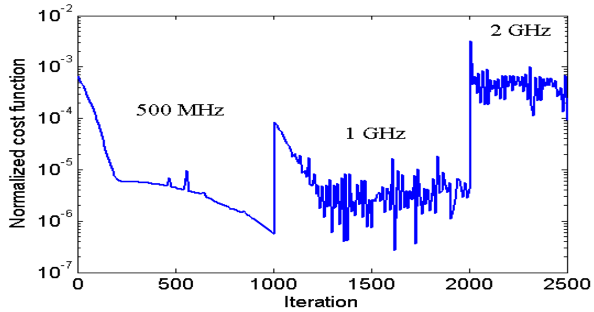

Fig. 15. Normalized cost function for the ellipsoid and the cube

number of iterations. We believe this drawback can be improved even when running the algorithm on a single CPU.

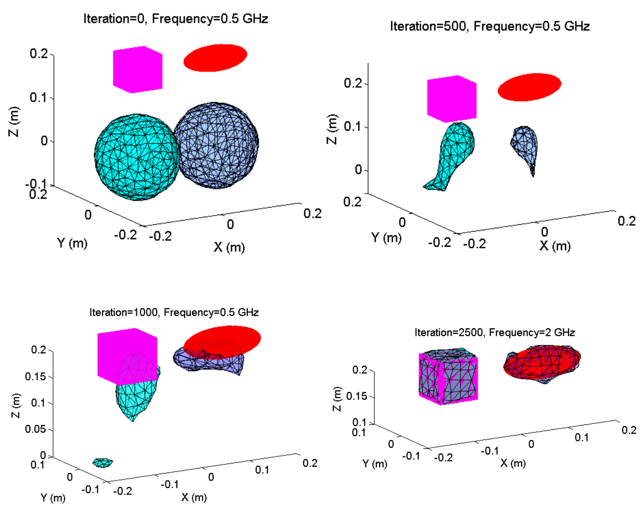

Fig. 16. Reconstruction of the ellipsoid and the cube after different iteration numbers, (a) initial guess, (b) after 500 iterations at 0.5 GHz, (c) after 1000 iterations at 0.5GHz, (d) after2500 iterations at 2 GHz.

III. Conclusions

Several examples are presented to demonstrate the capability of the level set algorithm in inverse scattering shape reconstruction problems. The examined cases include 2D and 3D conducting and dielectric targets immersed in air or hidden behind a dielectric wall. The frequency hopping plays an important role in assuring teh convergence of the algorithm. We observed that in general, the lower frequencies help to retrieve the locations of the targets and their general profiles while the higher frequencies help retrieve the finer details of their shapes. The level set algorithm was tested on experimental data as discussed in [10].

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