Shape Reconstruction Using the Level Set Method for Microwave Applications

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***Abstract*—A shape reconstruction algorithm based on contour deformation using the full-band level set method is presented. Transverse magnetic plane waves are used for excitation. The results of full-band level set scheme are compared with those of narrowband scheme. A proposed measurement configuration is shown to produce more accurate results.**

***Index Terms*—Frequency hopping, level set method, shape reconstruction.**

I. Introduction

THE purpose of inverse electromagnetic scattering is to retrieve the constitutive parameters and the shape of scattering objects using measurement data upon illumination with electromagnetic waves. This problem has many applications such as target identification, geophysics, seismic exploration, remote sensing, atmospheric science, ground penetrating radar (GPR), and medical applications [1].

Due to multiple-scattering effects within the objects, the inverse scattering problem is nonlinear, ill-posed and nonunique. In general, for iteratively solving the nonlinear inverse problem, the forward scattering problem will be solved many times which makes it computationally intensive. The basic idea of inverse problems is to minimize the error between the scattered fields of the evolving objects and the measurement data in each inversion iteration.

In this work, we assume that the constitutive parameters of the scattering objects and the surrounding medium are known. The objective is to retrieve the number of scattering objects, their shapes and locations using the level set method.

In the shape reconstruction algorithm of perfectly conducting objects reported in [2], a narrowband level set method was used for tracking the motion of evolving two-dimensional (2-D) objects. In [2], the level set function was updated only in a neighborhood of the evolving interface. Although this process seems more efficient from the CPU time viewpoint, it may lead to some difficulties.

The first problem could arise in calculating the derivatives on the edge of the narrowband region. The second problem could occur in rebuilding active grids points and in reinitializing the level set function.

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This reinitialization requires solving another PDE to obtain the signed distance function from the distorted level set function. This process sometimes causes the movement of zero-level interface towards an improper direction.

In the current work, we have used a full-band (global update) level set instead of the narrowband one, where the level set function propagates in the entire computational domain. Therefore the reinitialization of the level set function is unnecessary here.

We have found that in the shape reconstruction using the level set method, the CPU time is mostly consumed in the forward scattering problem and in the calculation of the deformation velocity. Overall, the narrowband scheme requires less CPU time compared with the full-band scheme. However the full-band scheme produces more accurate reconstruction especially in multiple objects cases. A comparison between the full-band and narrowband level set schemes will be presented in this work.

The conventional method of moments (MOM) solver is employed here. The solver is used to calculate the scattered far fields from the true object for obtaining the synthetic measurement data and the scattered far fields from the evolving object for obtaining their deformation velocity.

II. Methodology

Level set methods have recently received a growing attention for shape reconstruction problems. This is due to their flexibility to handle topological changes with minimum *a priori* required information as discussed in [2]–[6].

The main idea of the level set method lies in representing the evolving interface (curve or surface) as the zero level of a higher order function. It is a versatile and simple method for tracking the motion of an interface in two or three dimensions in an implicit manner. The main advantage of the level set method is the ability of breaking the initial object into several ones and/or merging several objects into a larger one during the evolution process. These topological changes are automatically handled using the level set method [3]–[6].

We assume that the interface is represented implicitly as the zero level of a higher order function ϕ. At each time t, the interface is defined as [3]

$$\Gamma(t) = \{(x,y) \,|\, \phi(x,y,t) = 0\}. \quad (1)$$

Upon obtaining the derivative with respect to time, we have the following relationship for tracking the motion of the interface known as the Hamilton–Jacobi equation [3], [4]

$$\frac{\partial \phi}{\partial t} + F\|\nabla\phi\| = 0 \quad (2.a)$$

$$\phi_0 = \phi(x,y,t=0) \quad (2.b)$$

where F is the normal component of the deformation velocity.

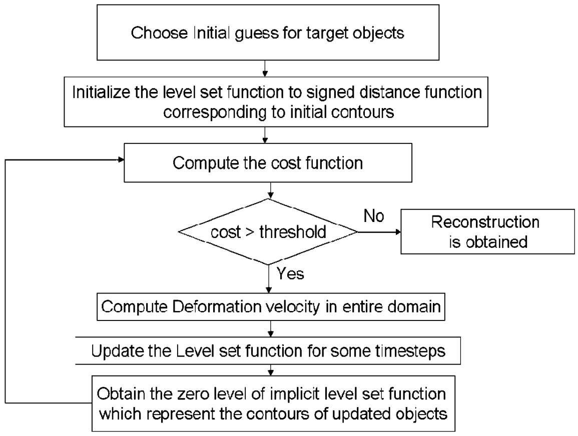

Fig. 1. The level set shape reconstruction algorithm.

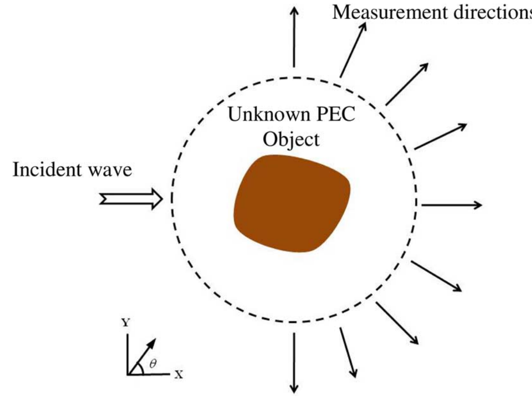

Fig. 2. Proposed incident and measurement configuration.

It is common to choose the level set implicit function ϕ as the signed distance function of the interface where steep gradients and rapidly changing features can be avoided [5].

III. Electromagnetic Shape Reconstruction

This work is focused on the problem of shape reconstruction of infinite perfect conducting cylinders with arbitrary cross sections. The incident waves are assumed transverse magnetic (TM) plane waves where the electric field is parallel to the cylinder's axis. The reconstruction is based on obtaining the forward and adjoint induced surface currents on the moving objects and consequently the expression of their deformation velocity [2].

No *a priori* information needs to be provided about the number of scattering objects or their topology. To retrieve the shape of the unknown object(s), the contour(s) of the evolving object(s) should deform in the normal direction such that the error function between measurements and scattered fields decreases in each iteration. The deformation velocity introduced by Ferrayé *et al.* in [2] is used in this work.

It should be mentioned that the cost function is minimized at all frequencies where the synthetic data is obtained. The frequency hopping plays an important role in the inversion algorithm as discussed in [2] and [6]. The flow chart of the algorithm is shown in Fig. 1.

We have implemented the shape reconstruction algorithm, using the full-band scheme for the external shape of the breast for cancer application, the star shape object and the handgun object. The obtained results proved the capability of the algorithm for those applications even when the signal to noise ratio was 5 dB (not presented due to space limit). The shape reconstruction algorithm is implemented using the SUN server 64 bits system with 2 GB of RAM.

IV. Numerical Results

A. Reconstruction of Two Elliptical Cylinders

In the first application, the reconstruction of two elliptical cylinders using the level set method and the frequency hopping technique is accomplished. Nine frequencies are used in the range from 3 to 18 GHz. The initial guess is a circle with radius of 10 cm which is centered at the origin. Each ellipse has dimensions of $a = 5$ cm and $b = 2.5$ cm. The separation distance between the two ellipses is 10 cm from center to center.

For collecting synthetic data, 36 incident plane waves uniformly illuminate the objects with the scattered waves measured in 18 directions following the configuration in Fig. 2 and described by (3). This configuration results in 676 synthetic data at each frequency.

The shape reconstruction algorithm in this case is implemented using both the full-band and the narrowband schemes. In the narrowband scheme, the level set function is updated in a neighborhood of the evolving interface (eight grid cells in this case). This requires updating the active grid points and reinitialization of the level set function in the algorithm of the narrowband scheme. The full-band scheme requires updating each grid cell in the whole computational domain. All the simulation parameters such as the time step, the number of iterations at each frequency, the incident and measurement angles are kept the same for both schemes.

The results obtained using the full-band scheme is shown in Fig. 3(a)–(c), while the result obtained using the narrowband scheme is shown in Fig. 3(d). In these results, the full-band scheme required 420 CPU minutes while the narrowband scheme required 315 CPU minutes. Fig. 3(c) clearly shows that the full-band scheme provides more accurate reconstruction results compared with the narrowband results shown in Fig. 3(d). This is due to the fact that solving the appropriate PDE for reinitialization of the distorted level set function to obtain the signed distance function may lead to numerical errors and improper movement of the zero-level interface [3], [4].

Although the full-band scheme is $\sim 30\%$ slower than the narrowband scheme, its final reconstructed profile is more accurate as shown in Fig. 3.

Fig. 4 shows the cost function which is normalized with respect to the synthetic measurement data at each frequency. The frequency hopping is demonstrated in the cost function plot, where the cost at each frequency is not normalized to the previous value. This explains the jumps in the cost function plot where the frequency value is noted. The results in Fig. 4 demonstrate an oscillatory behavior at higher frequencies which could indicate to an inherent limitation in the accuracy of the forward solver at these frequencies.

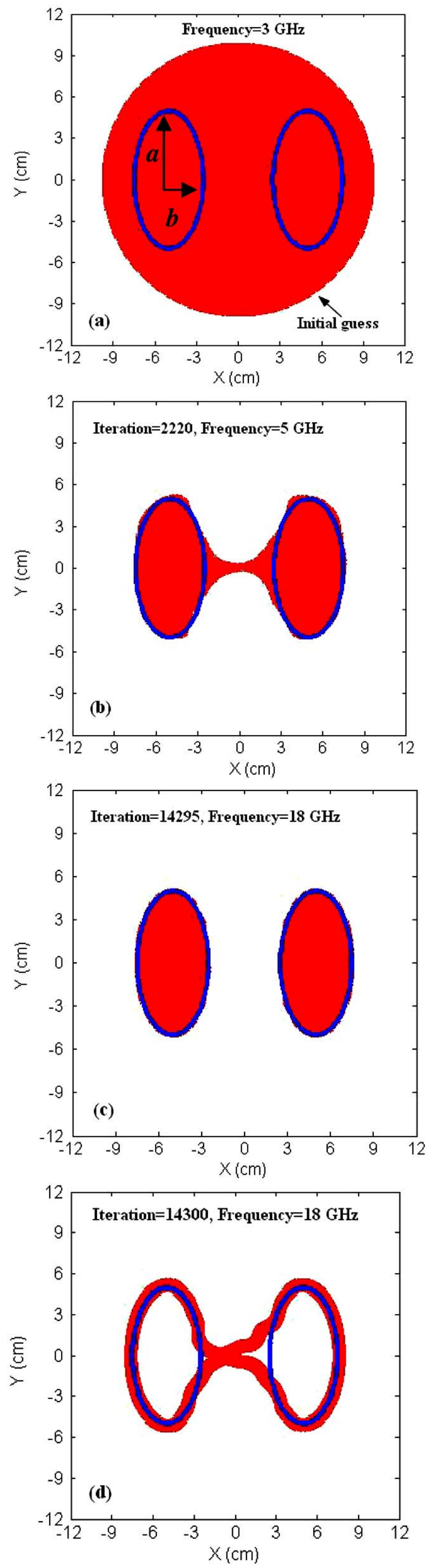

Fig. 3. Reconstruction of two elliptical cylinders: (a) initial guess, (b) after 2220 iterations using full-band scheme, (c) after 14295 iterations using full-band scheme, and (d) after 14300 using narrowband scheme.

Another important factor in the reconstruction algorithm is the incident and measurement configuration of Fig. 2. The algorithm reported in [2] is based on a uniform incidence of plane waves all around the object with measuring the scattered waves

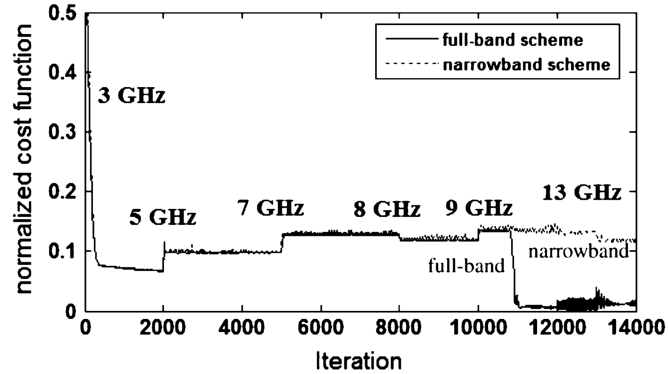

Fig. 4. The cost function of two elliptical cylinders.

in all directions. In the current work, the measurement angles are uniformly arranged as

$$\theta_i - \frac{\pi}{2} \le \theta_m < \theta_i + \frac{\pi}{2} \quad (3)$$

Where θ_i and θ_m are the incident and measurement directions, respectively. Using the configuration in (3), we observed that the deformation velocity is more well-behaved and toward the unknown objects. We tested this configuration in the case of the handgun shape, the two cylinders and the five cylinders. All reconstruction results have confirmed our observation. The results of Fig. 5 show the accuracy of using the configuration in (3) compared with the full measurement configuration for the reconstruction of five cylinders. Fig. 5(a), shows the result of using (3) where 36 incident angles and 18 measurements per incidence are employed. While Fig. 5(b), employs 26 incident angles and 26 uniform measurement angles per incidence.

B. Detection of a Crack in a Pipe Using Limited View Data

In this application, the results for obtaining the location and the shape of a crack in a conducting cylinder are presented. This particular case is important for GPR applications for detecting defects in underground pipes. The main difference in this case is that the incident and measurement directions are limited to fewer angles. Shape reconstruction using limited view is very important in practical situations where measurement data may be available only in certain directions (e.g., buried utility pipes, landmines, and breast cancer tumors).

In this work, we only considered pipes immersed in air. We have employed 20 incident angles and 20 measurements per each incident angle where $\pi < \theta_i < 2\pi$ and $0 < \theta_m < \pi$. All angles are measured with respect to the x-axis (see Fig. 2).

The radius of the defected conducting cylinder is 10 cm and is positioned at 50 cm from the initial guess as shown in Fig. 6(a). The reconstruction starts at low frequency of 10 MHz. In this example, 12 frequencies are employed up to 15 GHz as indicated in the cost function plot in Fig. 7. Low frequencies are appropriate for finding the object's location while higher frequencies are meant for retrieving the details of the shape. The same observation was reported by Ferrayé *et al.* in [2]. After 2360 iterations at 100 MHz, the location of the cylinder is retrieved as shown in Fig. 6(d). The crack shape is recovered at 15 GHz as shown in Fig. 8, where the algorithm is tested for two locations

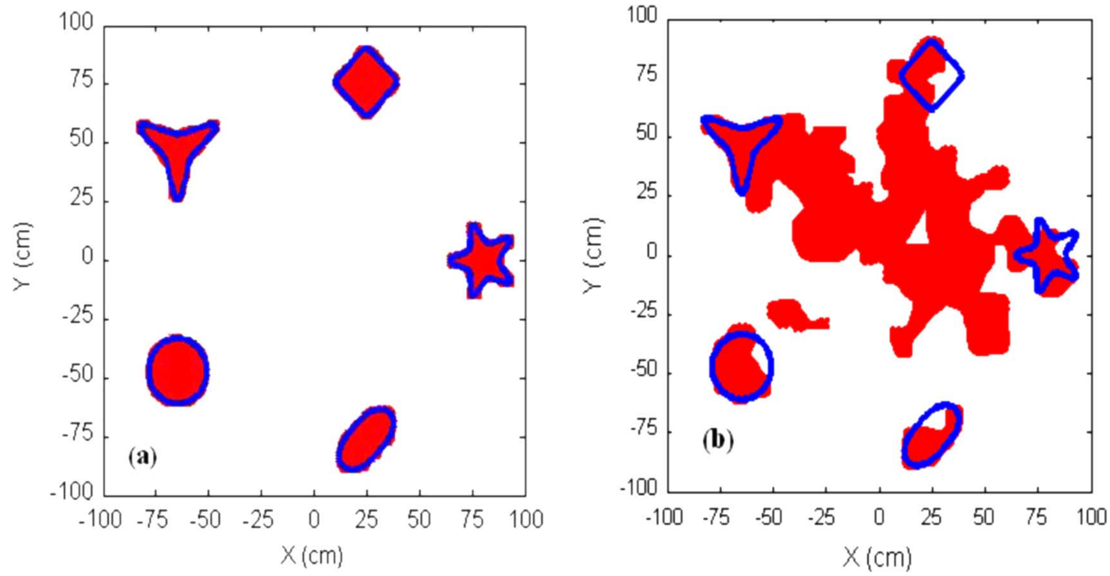

Fig. 5. Final reconstruction of a five cylinders after 12 555 iterations at 3 GHz (a) using configuration of (3); (b) full measurement configuration.

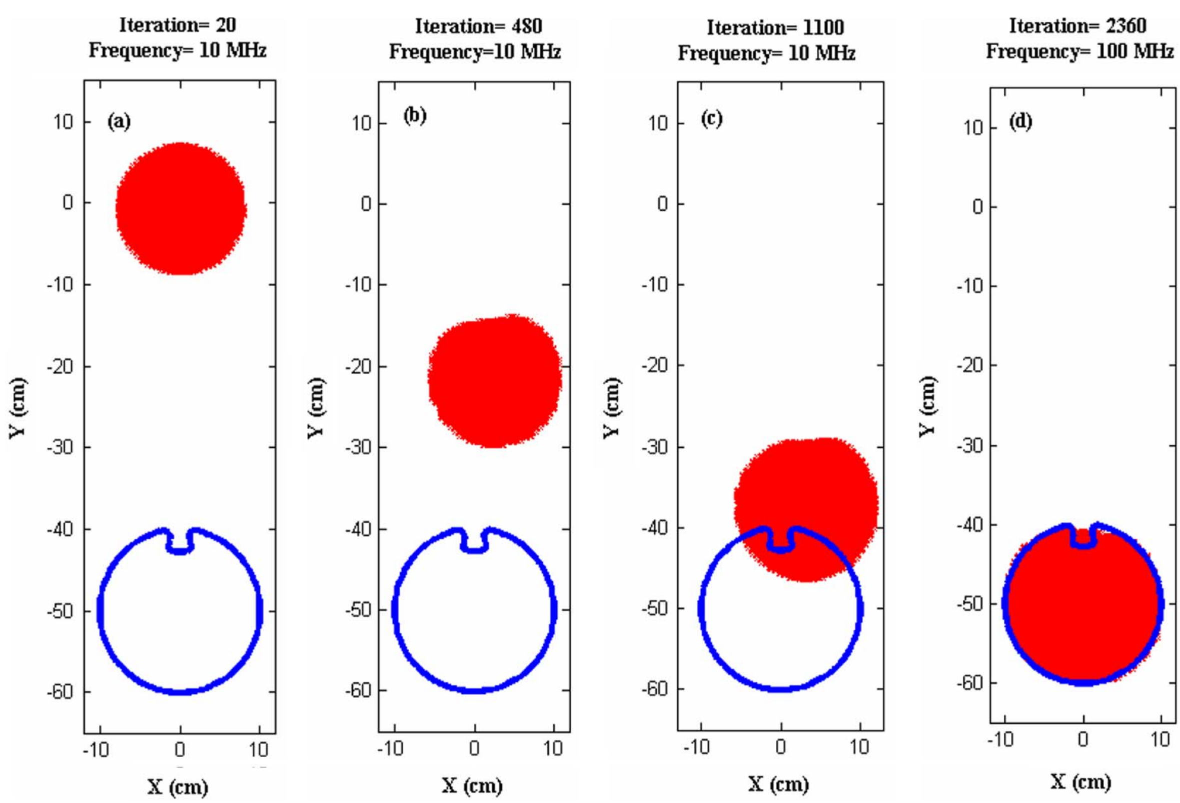

Fig. 6. Retrieving the location of a defected cylinder.

of the crack keeping the same directions of incident and measured waves. The results show some deterioration in the shape reconstruction of the crack in Fig. 8(b). However, in both cases, the results indicate to a very good agreement in recovering the location of the cylinder.

In all above examples, two smoothing techniques were implemented. The spline interpolation was used for smoothing the evolving contours while the moving average method was employed for eliminating the sharp features in the surface currents and the deformation velocity.

Note that solving the forward scattering problem using the numerical solution of the electric field integral equation requires explicit representation of the zero level interfaces (i.e., the number of scattering objects and their connectivity information). As known, the level set method is based on implicit representation of evolving interfaces which allows handling topological changes automatically. Some challenges arise in practical implementation due to the contradiction between the implicit representation of the level set method and the explicit needs of the numerical solution of the forward problem.

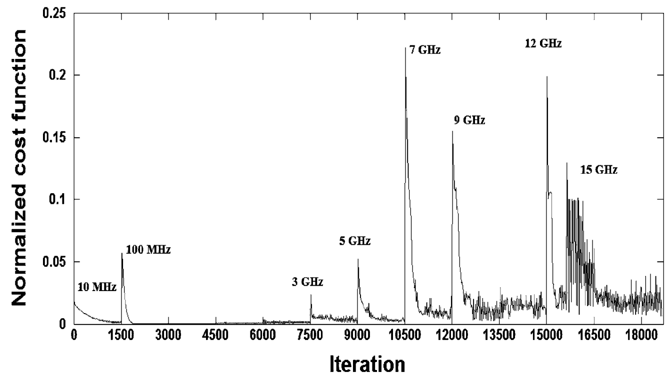

Fig. 7. Normalized cost function of a cylinder with a crack.

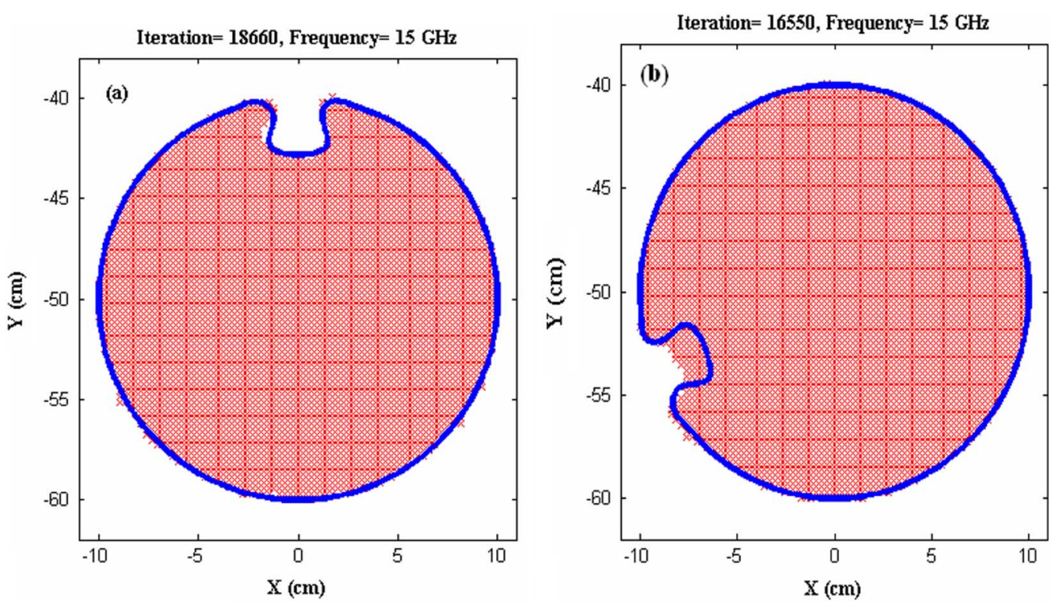

Fig. 8. Crack position in a cylinder at (a) top and (b) oblique.

The above challenge is resolved through the implementation of the marching squares method [7]. This method is a computer graphic based technique for polygonization of the zero-level interface of the level set function. This technique allows extracting the number of scattering objects and their connectivity information required by the forward solver.

V. Conclusion

The results show that the full-band level set scheme provides more accurate results compared with the narrowband scheme, especially for the multiple objects cases. A proposed measurement configuration demonstrates more accurate shape reconstruction results, which seems to be consistent for nonsymmetric scenarios. The later observation is currently undergoing more investigation.

References

[1] A. T. Vouldis, C. N. Kechribaris, T. A. Maniatis, K. S. Nikita, and N. K. Uzunoglu, "Investigating the enhancement of three-dimensional diffraction tomography by using multiple illumination planes," *J. Opt. Soc. Amer.*, vol. 22, no. 7, pp. 1251–1262, Jul. 2005.

[2] R. Ferrayé, J.-Y. Dauvignac, and C. Pichot, "An inverse scattering method based on contour deformations by means of a level set method using frequency hopping technique," *IEEE Trans. Antennas Propag.*, vol. 51, no. 5, May 2003.

[3] J. A. Sethian, *Level Set Methods and Fast Marching Methods*. Cambridge, U.K.: Cambridge University Press, 1999.

[4] S. Osher and J. A. Sethian, "Fronts propagating with curvature-dependant speed: Algorithms based on Hamilton—Jacobi formulations," *J. Computat. Phys.*, vol. 79, pp. 12–49, 1988.

[5] S. J. Osher and R. P. Fedkiw, *Level Set Methods and Dynamic Implicit Surfaces*. New York: Springer-Verlag, 2003.

[6] O. Dorn and D. Lesselier, "Level Set methods for inverse scattering," *Inverse Problems*, vol. 22, no. 4, Aug. 2006.

[7] C. Maple, "Geometric design and space planning using the marching squares and marching cubes algorithms," in *Proc. Int. Conf. Geometric Modeling and Graphics*, Jul. 2003, pp. 90–95.