On the Stability of Surface Shape Reconstruction Using Microwave Algorithm for 3-D Breast Tumor Based on the Adjoint-Fields Scheme

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Abstract

The main purpose of this paper is to investigate some stability issues observed in the surface shape reconstruction algorithm of 3-D irregular breast tumors. The developed algorithm is based on integrating the method of moments (MoM) for the forward solver with the adjoint-field scheme for the computations of appropriate sensitivities derivaties. The results show that the stability of the algorithm relies on the manner in which the adjoint fields are calculated.

1. Introduction

All existing imaging techniques amount to solving inverse problems that are mathematically ill-posed. This makes it extremely difficult and challenging to derive efficient reconstruction algorithms that can be applied successfully to real data that are inherently "noisy". In the application of breast cancer detection, tumors typically have a high contrast with respect to the background parameter distribution at microwave frequencies [1]. Due to the resulting strong nonlinearity of this inverse problem, straightforward linearization techniques (Born-type methods) are not justified, and therefore, novel nonlinear fast imaging techniques must be developed to properly solve the problem of the reconstruction of images with microwaves.

In this paper, we present a surface shape reconstruction algorithm that is based on a nonlinear inversion technique [2]. In particular, we propose an alternative regularization scheme, namely the reformulation of the inverse problem as a surface shape reconstruction problem. We believe that this reformulation serves as a tool for reconstructing images which are less sensitive to noise in the data than other pixel-based methods that need regularization schemes that 'over-smooth' the reconstructed images [2-5].

In our previous work [4-5], we presented a microwave imaging algorithm for shape reconstruction of 3-D irregular breast cancer tumor. The algorithm is based

1-4244-0878-4/07/$20.00 ©2007 IEEE

on integrating the *adjoint-field* scheme with the method of moments (MoM). The results showed that employing multiple frequencies, multiple sources and receivers, and multiple polarizations improves the convergence of the algorithm.

The objective of the current work is to investigate and eliminate the causes of some instability observed in [4, 5]. We believe that the algorithm is correct, and its convergence has been succesfully proved in other applications such as diffuse optical tomography or electrical impedance tomography [4]. We believe that the observed instabilities are due to implementation issues in calculating the adjoint-fields as will be clarified in the numerical results section.

2. Methodology

The adjoint-field scheme computes the descent gradient direction of the residual error (mismatch) by solving the forward problem twice; one for the direct problem and another one for the adjoint associate problem [4, 5].

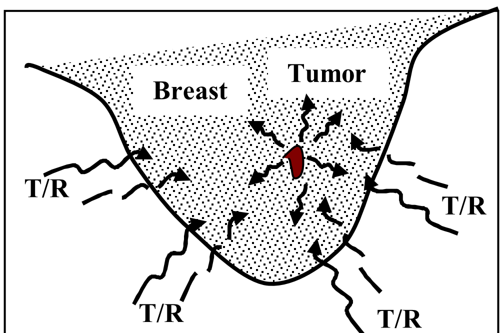

Fig. 1 Breast cross section to schematically show the back-propagation scheme.

The residual errors, the mismatch between the simulated scattered fields and the (synthetic) measurements, are calculated at all receivers for all incident frequencies and polarizations. The complex conjugate residuals at all receivers are considered new artificial sources to be back-propagated to the target. The later constitutes the second forward problem to be solved to calculate the sensitivities on the tumor surface.

A cross section of the 3-D breast and tumor is shown in Fig. 1. The configuration shows the incident waves from the transmitter T (solid arrows), the scattered waves from the tumor (reversed solid arrows) and the back-propagated waves from the receiver R to the tumor (dashed-line arrows). The descent gradient direction c is given by [2, 4-5]:

$$c = \mathrm{Re}\left(\left(\overline{E}_i + \overline{E}_S\right) \cdot \overline{E}_a \left(\hat{\varepsilon}_T^2 - \hat{\varepsilon}_B^2\right)\right) \tag{1}$$

where $\overline{E}_i$, $\overline{E}_s$ and $\overline{E}_a$ represent the incident, the scattered (original forward problem) and the adjoint- fields (adjoint-forward problem) at the surface of the tumor, respectively. The relative dielectric constants of the tumor and of the breast background ($\hat{\varepsilon}_T$ and $\hat{\varepsilon}_B$, respectively) are assumed varying with the frequency following the work in [1].

Each surface node will be updated using [4, 5]:

$$\bar{x}_{new} = \bar{x}_{old} - \gamma c \quad (2)$$

where γ is a small step size and $\bar{x}$ represents the coordinates (x, y, z) of each surface node of the tumor. In this work, γ is automatically calculated such that the average displacement of all surface nodes in a given iteration is ~ 0.5mm.

In our preliminary work [4, 5], we reformulated the descent direction to be directly function of the electric and magnetic surface currents obtained using the MoM. As known, the surface currents are very sensitive to the accuracy of the surface discretization into triangular patches as used here. Upon updating the surface nodes according to Eq. (2), the triangular patches could become significantly different in size, leading to less accurate MoM solution, etc. We believe that this scenario has caused the reconstruction process to significantly deteriorate unless a regularization scheme is applied as it is explained in [5].

The observed instability in [5] leads to the need of an additional regularization step after each update of the surface nodes. This regularization was based on filtering neighboring nodes by an average filter in order to obtain a smooth surface. This smoothing operation relied on a smoothing parameter that should be as small as possible but could not be estimated *a priori*.

3. Results and Discussion

To focus on the algorithm, the breast model consists only on two homogeneous regions. In addition, the skin layer is ignored with the assumption that the surrounding medium perfectly matches the normal breast tissue. Although these assumptions are not realistic for breast cancer, they simplified the scattering calculations of the problem. The electromagnetic source used here is a plane wave with vertical or horizontal polarization, while the fields are received as co- and cross-polarization. The total number of transmitter/receiver (T/R) is 40 where 5 T/R are located at each plane of constant elevation angle ($0.1\pi \leq \theta \leq 0.9\pi$), and 8 T/R at each plane of constant azimuth angle ($0 \leq \varphi \leq 2\pi$).

The computational domain is $20\times 20\times 20$ cm^3. The number of surface nodes of the tumor are 8 and 16, in the θ- and φ-directions, respectively. A synthetic tumor of irregular shape is generated using the spherical harmonics model where 10 random harmonic coefficients are employed as discussed in [3].

The results in Fig. 2a show the cost functional behavior vs the iteration number. Two initial guesses are examined here: a sphere of radius 5 mm (Fig. 2b), and a sphere of radius 1cm (Fig. 2c). The frequency hopping technique is used based on 24 frequencies for the 5mm sphere, and 12 frequencies for the 1cm sphere. These results show the success in reconstructing the shape tumor.

Acknowledgements

This research is sponsored in part by the NSF award no. ECS-0524042, the Spanish Ministerio de Educación y Ciencia (MEC) Grant nº FIS2004-03767, and the Women Giving Circle at the University of Arkansas grant no. WGC-22.

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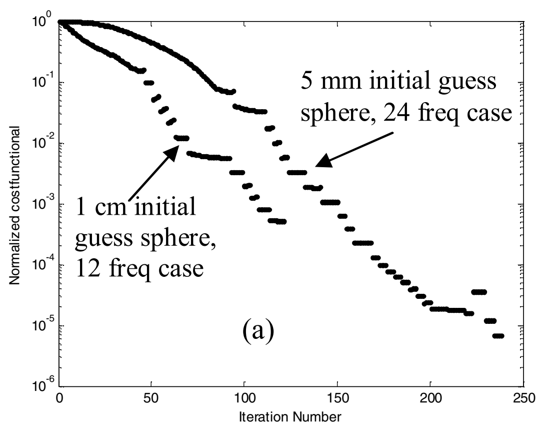

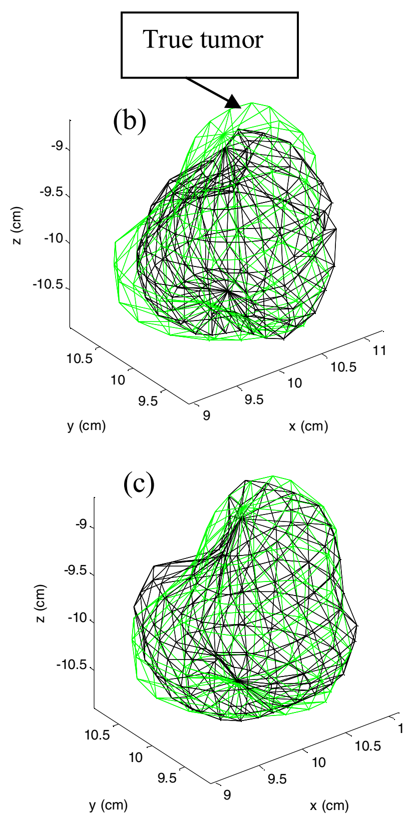

Fig. 2 Reconstruction results.
(a) costfunctional vs iteration number, (b) reconstruction with initial guess 5mm sphere and 24 frequencies 3-9.8GHz, (c) reconstruction with 1cm sphere and 12 frequencies 3-7.4GHz..