`

Breast Shape Reconstruction using Microwave Techniques and the Level Set Method

Mohammad Reza Hajihashemi and Magda El-Shenawee

Department of Electrical Engineering
University of Arkansas, Fayetteville, AR 72701, USA
mhajihas@uark.edu , magda@uark.edu

Abstract: In this paper, a level set algorithm is employed for shape reconstruction of conducting objects in two-dimensions. Minimum *a priori* information is required about the unknown objects, which is a main advantage of the level set method. Therefore several objects can be retrieved from one single initial guess. As a preliminary attempt, the breast shape is reconstructed using the proposed method. The obtained results demonstrate the efficiency and robustness of the algorithm.

Keywords: Level set, shape reconstruction, frequency hopping and breast cancer.

1. Introduction

The Purpose of inverse electromagnetic scattering is to retrieve the constitutive parameters and the shape of scattering objects using measurement data collected at a distance from the scatterers. The unknown objects are illuminated by electromagnetic waves. This problem has many applications such as target identification, geophysics, seismic exploration, remote sensing, atmospheric science, ground penetrating radar (GPR) and medical applications [1].

In the general case, the inverse scattering problem is non-linear, ill-posed and non-unique problem due to the multiple scattering effect within the objects. A general way for solving the nonlinear inverse problem is via an iterative and optimization approach. Solving the inverse problem numerically often involves solving the forward scattering problem many times, therefore the problem is computationally demanding. Furthermore, the solution to the inverse problem should be stable since small changes in the measured scattered field may result in large changes in the reconstructed profile [2].

One category of inverse scattering techniques is shape reconstruction of unknown objects. In shape reconstruction problems, we often assume that the constitutive parameters of scattering objects and the surrounding medium are known. The objective is to retrieve the number of scatterers, their shapes and positions.

2. Methodology

Level set methods have received growing attention as algorithms for inverse scattering problems and shape reconstruction due to their flexibility to handle topological changes and to compute reconstruction with minimum *a priori* information.

The use of level set methods for capturing a moving interface became popular following the seminal work of Osher and Sethian [3]. This technique has been used in diverse fields such as fluid dynamics, image processing and crystal growth. The main idea is embedding the evolving interface (curve or surface) as the zero level of a higher order function. The Level set method is a simple and versatile method for computing and analyzing the motion of an interface in two or three dimensions implicitly. The main advantage of the level set method is that the contours represented by the level set may break or merge naturally during the evolution, where topological changes are automatically handled without user interference.

We assume that the interface is represented implicitly as the zero level of a higher order function such as ϕ, as shown in Fig.1 . Therefore at each time t, the interface is defined as

$$\Gamma(t) = \left\{ \vec{x} \,\middle|\, \phi(\vec{x},t) = 0 \right\}. \quad (1)$$

The following relationship for tracking the motion of the interface using the level set representation is given as:

$$\frac{\partial \phi}{\partial t} + F \left\| \nabla \phi \right\| = 0 \ , \ \phi_0 = \phi(x,y,t=0) \quad (2)$$

where F is the normal component of deformation velocity given by:

$$F = \vec{V}(\vec{x}) \cdot \hat{n} = \vec{V}(\vec{x}) \cdot \frac{\nabla \phi}{|\nabla \phi|}. \quad (3)$$

Upon obtaining F and the initial position of the interface ϕ_0, the evolution of the interface can be tracked. Equation (2) is known as the Hamilton-Jacobi equation. It is common to choose the level set implicit function ϕ to be the signed distance function corresponding to the interface, where steep gradients and rapidly changing features are mainly avoided [4].

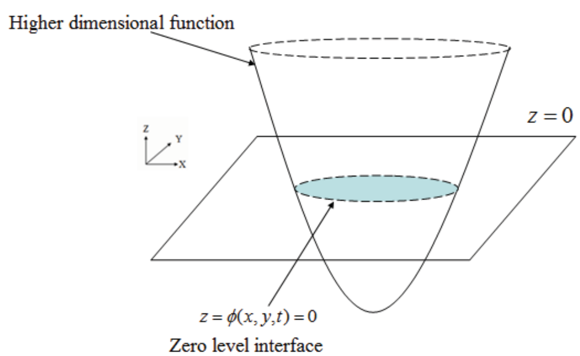


Fig. 1. Level set function evolution.

3. The Level Set Technique for Shape Reconstruction in Electromagnetics

In this section, we describe how we can use the level set framework for shape reconstruction problems. We discuss the problem of shape reconstruction of infinite conducting cylinders with arbitrary cross sections. This is a two-dimensional electromagnetic problem. We assume that incident waves are Transverse Magnetic (TM) plane waves where the electric field is parallel to the cylinder axis. We assume that no *a priori* information is given about the number of scattering objects and/or their topology. As mentioned before, contours in the level set join and break naturally without user interfere. In general, for solving the inverse scattering problems, we should minimize the cost function corresponding to the error between scattered field of evolving objects and the synthetic measurements.

Assume that the unknown objects are illuminated by N_I incident waves and the number of measurements for i^{th} incident wave is $N_M(i);\ i = 1,2,..,N_I$. Then the cost function (error function) has the following form in the least square sense [5]

$$f(C_\tau) = \sum_{i=1}^{N_I} \sum_{m=1}^{N_M(i)} \left| E^s(\theta_i, \phi_{m,i}) - E^s{}_{meas}(\theta_i, \phi_{m,i}) \right|^2 \quad (4)$$

where θ_i is the i^{th} incident angle and $\phi_{m,i}$ is the m^{th} measurement direction angle when the incident angle is θ_i. The fields E^s and $E^s{}_{meas}$ are the calculated and synthetically measured ones, respectively. The contour C_τ, represents the moving objects in time evolution τ. For solving the shape reconstruction problem, we should decrease this cost function in an iterative manner. Without going to the proof, a form of deformation velocity which makes the cost function a decreasing function (with negative derivative with respect to the evolution time) is given by [5]:

$$F_\tau(x,y) = -\alpha \operatorname{Re}\left[e^{-i\frac{\pi}{4}} \sum_{i=1}^{N_I} \sum_{m=1}^{N_M(i)} \left(E^s(\theta_i,\phi_{m,i}) - E^s{}_{meas}(\theta_i,\phi_{m,i}) \right)^* .J_{\theta_i}(x,y)\,.J'_{\phi_{m,i}}(x,y) \right] \quad (x,y) \in C_\tau \qquad (5)$$

where α is some positive normalization coefficient, * sign means complex conjugate, J_{θ_i} is the induced current under incident angle θ_i and $J'_{\phi_{m,i}}$ is the adjoint induced current which is obtained by solving the adjoint problem. For solving the adjoint problem, a plane wave should be reflected back to the objects with incident angle $\pi + \phi_{m,i}$ and the induced current must be calculated [5].

For calculation of deformation velocity, we should solve two scattering problems associated with the forward and adjoint problems. The following flowchart shows the algorithm for the shape reconstruction.

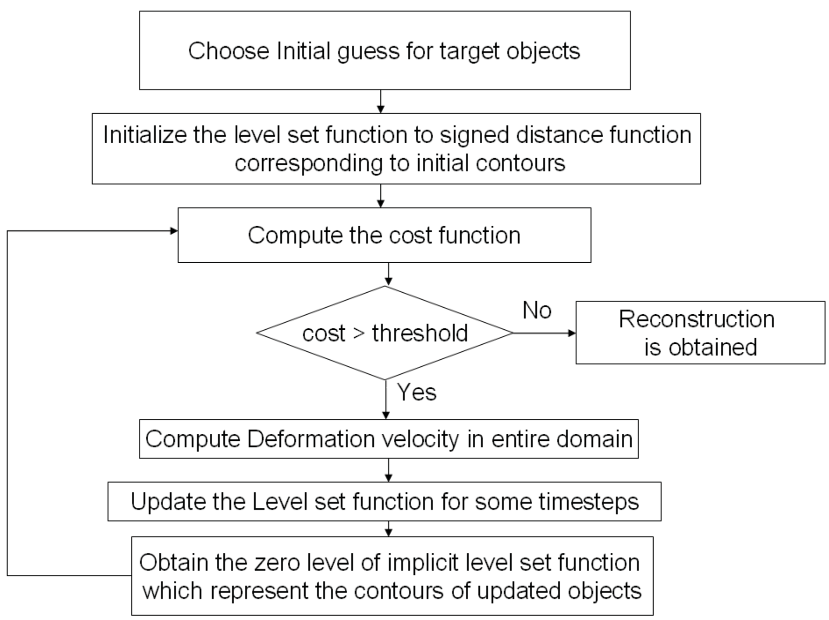


Fig. 2. Shape reconstruction algorithm using the level set method.

4. Numerical Results

For the first case, a star-shaped object is reconstructed using the proposed algorithm. The level set reconstruction algorithm is combined with the frequency hopping technique to retrieve the object details. Seven frequencies are used from 0.5 GHz to 12 GHz. The size of the object is about 30 cm. We have used 20 incident angles uniformly around the object and 20 measurements per each incident, which results in 400 synthetic data at each frequency. The initial guess object is a cylinder with radius 10 cm. The normalized cost function is shown in Fig. 3 and the reconstruction results at different stages are shown in Fig. 4. The pikes in the cost function correspond to the frequency jumps during the evolution of the shape reconstruction algorithm. In these figures, solid contours represent the true object while the filled contours represent the evolving objects. For each stage, the iteration and the corresponding frequency are shown.

The Reconstruction started with a wavelength that is two times equal to the true object size. This low frequency (0.5 GHz), determines the overall object size and location. If we start with a relatively high frequency, deformation velocity is not feasible to retrieve the object as it has many oscillations and instabilities. After 300 iterations, the cost function is almost constant which means the first frequency is unable to retrieve finer details of the object. Very high frequencies cannot contribute much to the reconstruction, since the forward solver is not accurate enough at these frequencies. From iteration 2500, the moving contours are almost stagnated. A quite satisfactory reconstruction is obtained at iteration 3000 and frequency of 10 GHz, as shown in Fig. 4-d.

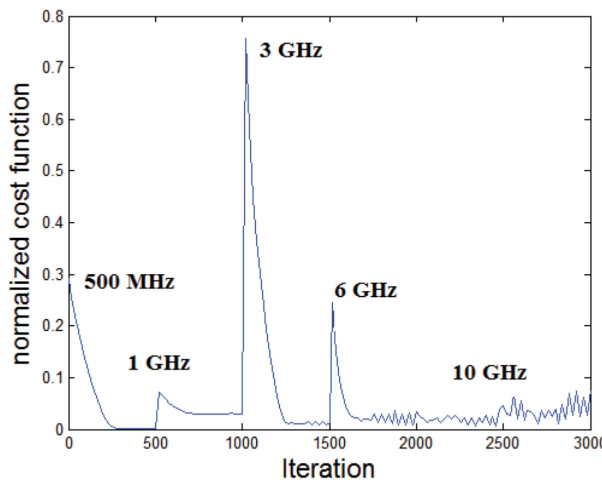
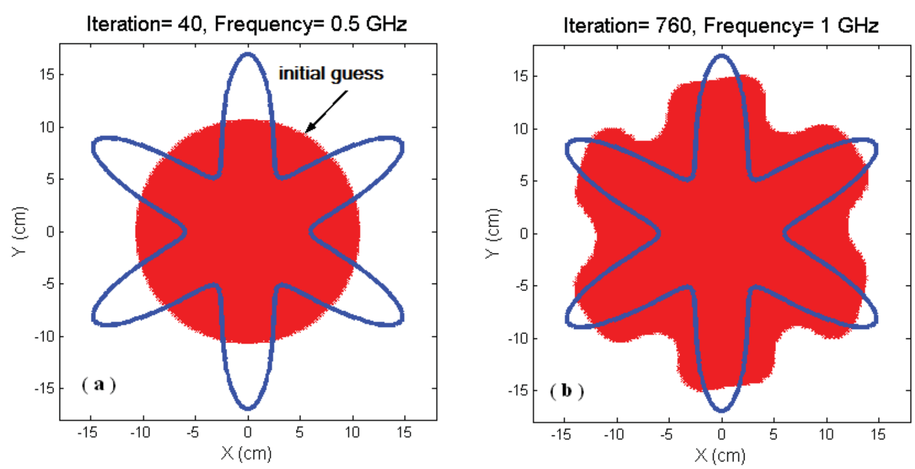


Fig. 3. Normalized cost function of star-shaped object reconstruction.



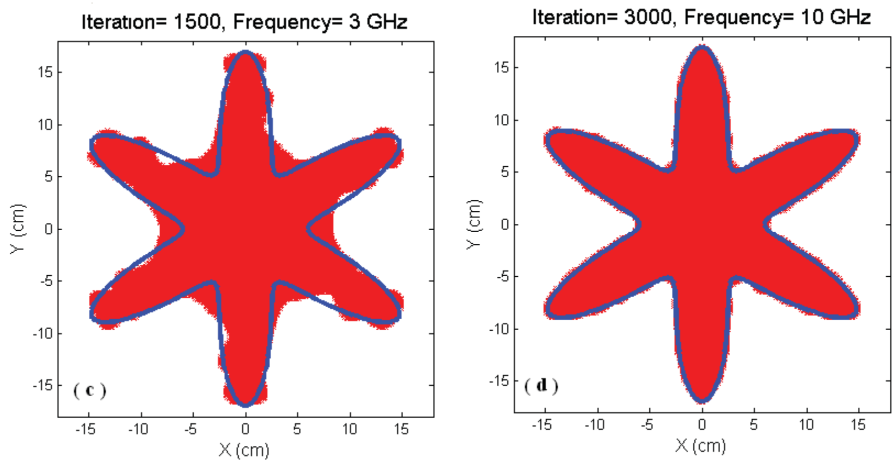


Fig. 4. Reconstruction of star-shaped object at different iterations.

In the second example, the shape reconstruction of human breast is considered. For this case, 9 frequencies are used in the range of 0.5 GHz to 15 GHz. The reconstruction starts with a small circle as the initial guess that is not even in the center of true object. Relatively low frequencies are used at the beginning to retrieve the overall abject size, then higher frequencies are used for obtaining the finer details. Thirty-six incident plane waves and 18 measurements per incident are employed. From iteration 4000 and frequency of 6 GHz, the moving object almost stagnates. Due to the resolution issue in the higher frequencies, the true object could not be reconstructed perfectly. This issue needs more investigation. The normalized cost function is shown in Fig. 5, and the reconstruction results at different stages are shown in Fig. 6.

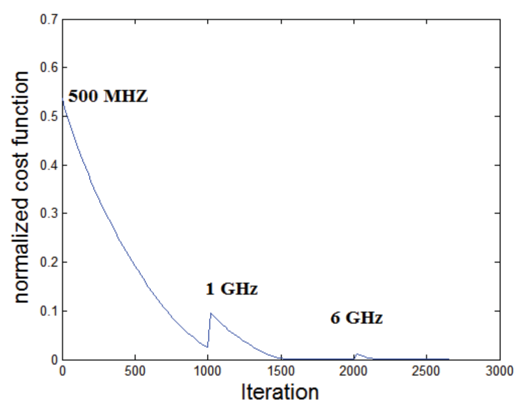
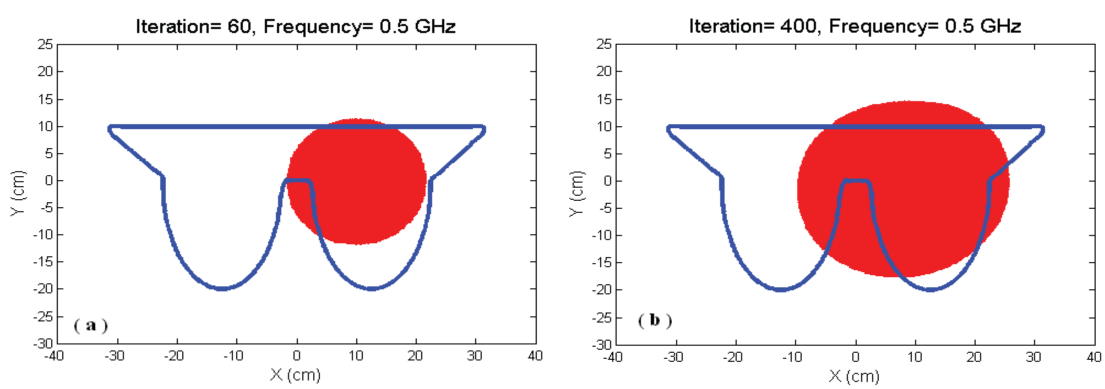


Fig. 5. Normalized cost function for shape reconstruction of human breast.



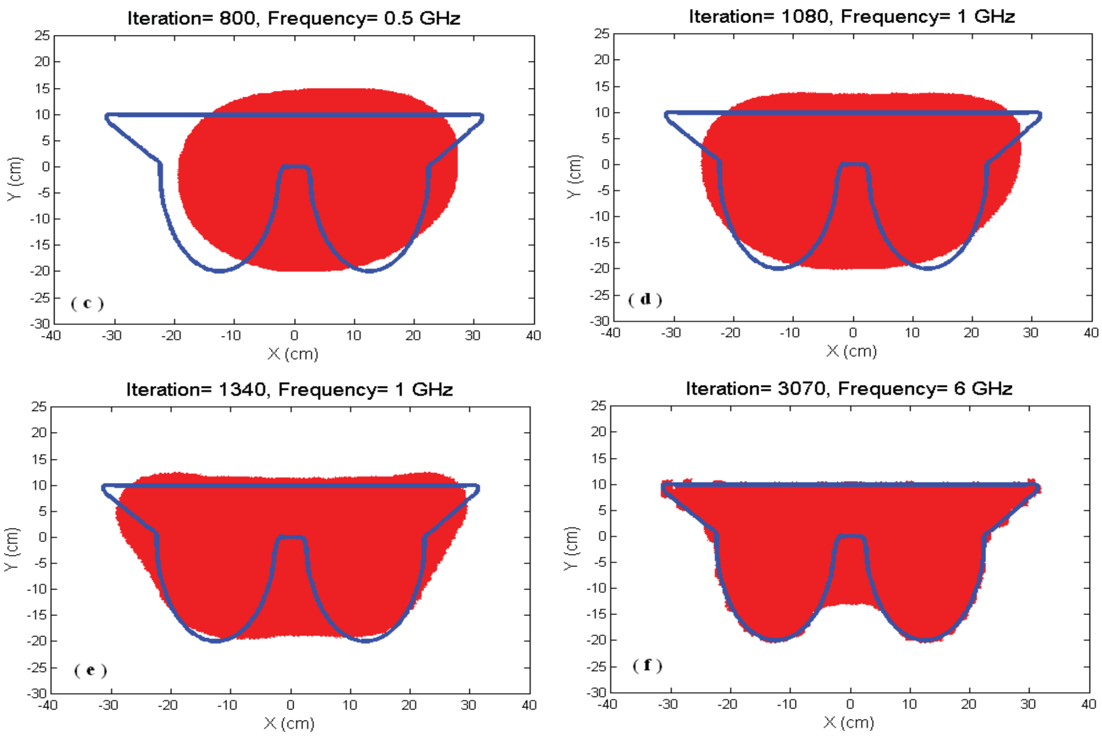


Fig. 6. Reconstruction of human breast at different stages.

5. Conclusions

A shape reconstruction algorithm based on contour deformation using the level set method is presented. This method has proved to be efficient and flexible for reconstruction of 2-D conducting cylinders with arbitrary cross-sections. More results will be presented in the conference.

6. Acknowledgements

This work is funded by the National Science Foundation Award Number ECS-0524042

References

[1] A. T. Vouldis, C. N. Kechribaris, T. A. Maniatis, K. S Nikita and N. K. Uzunoglu, "Investigating the enhancement of three-dimensional diffraction tomography by using multiple illumination planes," *Journal of Optical Society of America*, vol. 22, no. 7, pp. 1251-1262, July 2005.

[2] W. C. Chew, "Imaging and inverse problems in electromagnetics," *Advances in Computational Electrodynamics: The Finite-Difference Time-Domain Method*, A. Taflove, Ed. Norwood, MA: Artech House, 1998,ch. 12.

[3] J. Sethian*, Level Set Methods and Fast Marching Methods,* Cambridge University Press, 1999.

[4] S. Osher and R. P. Fedkiw, *Level Set Methods and Dynamic Implicit Surfaces,* Springer-Verlag 2003.

[5] R. Ferrayé, J. Dauvignac, and C. Pichot, "An Inverse Scattering Method Based on Contour Deformations by Means of a Level Set Method Using Frequency Hopping Technique," *IEEE Transactions On Antennas and Propagation*, vol. 51, no. 5, May 2003.