Parametric Investigation of Hilbert Based Artificial Magnetic Conductors

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Abstract: A parametric study of the design of Hilbert based Artificial Magnetic Conductors (AMCs) is conducted. The effect of using lossy conductors (such as copper), varying the dimensions of the inclusions and the substrate heights is investigated. This work is conducted via the development of a circuit model and the multiple reflection phenomenon. It is found that the equivalent inductance and capacitance in the circuit model vary linearly with the dimensions of the Hilbert inclusions for the cases considered here. It is also found that the attenuation in the reflection coefficient magnitude is more profound in the *y*-polarization than in the *x*-polarization. The results show that the magnitude increases with the decrease in Hilbert order. Finally, a configuration showing a polarization independent Hilbert based AMCs is implemented.

Keywords: Metamaterials, Artificial Magnetic Conductor, Hilbert fractals, polarization, resonant circuit.

1. Introduction

Recent years have witnessed the development of several materials with artificial electromagnetic properties not available naturally. These materials have been generally grouped under the category *Metamaterials*. Most metamaterials are created by the proper arrangement of metallic inclusions with accurately calculated shapes. By designing the inclusions to be much smaller than the wavelength, the incident waves sense the metallic inclusions as homogeneous structure with the desired artificial electromagnetic properties. In almost all metamaterials, the desired artificial properties can only be obtained within a certain frequency band.

This paper focuses on a special class of metamaterials called Artificial Magnetic Conductors (AMCs) or high impedance surfaces [1-2]. These surfaces possess the feature of reflecting plane waves with a reflection coefficient of +1 (0° phase) rather than -1 (180° phase) obtained from conventional Perfect Electric Conductors (PECs). Since originally introduced in [2], AMCs have found several applications in antenna design [3], phase shifters [4], and TEM waveguides [5]. For example the work in [2] focused on replacing PECs with AMCs to reduce the antenna size to approximately half its original size.

It is generally known that operation at lower frequencies requires structures of larger size. To obtain a more compact size, Hilbert fractal inclusions were introduced in [1] to realize AMCs. As the order of the Hilbert inclusion increases, the length of the inclusion increases while a constant area is maintained. Hence, using Hilbert inclusions can realize AMCs with smaller size in comparison to several other realizations [6]. In addition to this, Hilbert inclusions can be easily realized using planar fabrication techniques.

The main drawback in using Hilbert inclusions as AMCs is introducing asymmetric structure which responds differently to polarizations [1]. Also, as stated in [7], AMCs in general tend to be highly lossy when implemented with actual conductors instead of PECs. These issues will be investigated in this paper. In addition a simple circuit based methodology for the design and optimization of Hilbert based AMCs is developed.

2. Methodology and Numerical Results

In this section, a parametric study of Hilbert based AMCs is presented. The parameters include: the order of the Hilbert inclusion, effect of lossy conductors, the size of the unit Hilbert inclusion (S_H) and the height *(d)* of the Hilbert inclusions above the supporting ground plane. The results are obtained through simulations using ANSOFT Designer software.

A. Effect of Hilbert Order and Lossy Conductors

An infinite array of Hilbert inclusions of order ranging from 1 to 3 is used to realize AMCs. This is achieved by backing the inclusions with a ground conducting plane. Copper ($\sigma = 5.8\times10^7$ S/m) is chosen as the material for both the Hilbert inclusions and the ground plane during the simulations. In the three cases, the dimension of the square Hilbert inclusions (S_H) are set equal to 1 mm$\times$1mm. The distance between each two Hilbert inclusions in the array is set to ($S_H/(2^n-1)$) where n is the Hilbert order. The width of the metallic strip used in the Hilbert inclusions is set to 0.01 mm. The Hilbert array is positioned at a distance d = 1 mm above the ground plane. The medium between the ground plane and the Hilbert array is assumed to be air. The response of the Hilbert based AMCs for orders 1, 2, 3 is plotted versus frequency as shown in Figures 1, 2, 3, respectively. Both the magnitude and the phase of the reflection response are shown in each figure for the *x*- and *y*- polarizations.

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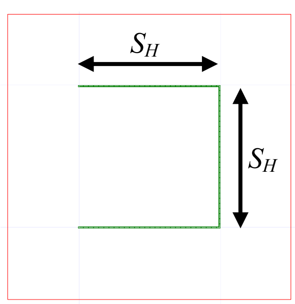

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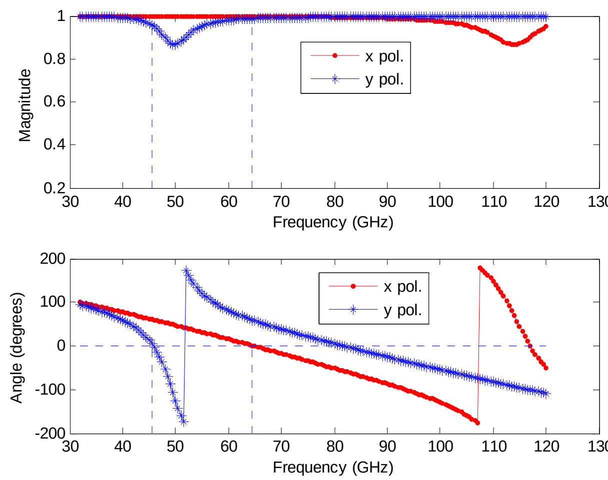

Fig. 1: a) Hilbert of order 1 unit element
b) The response of the Hilbert AMC of order 1.

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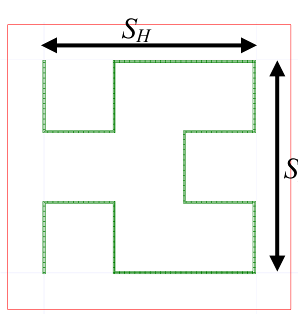

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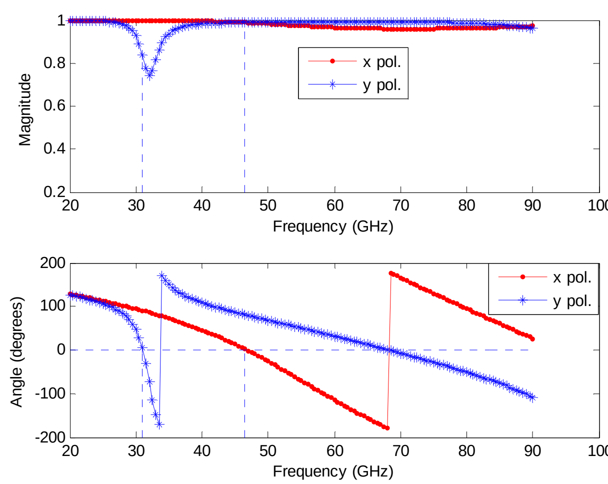
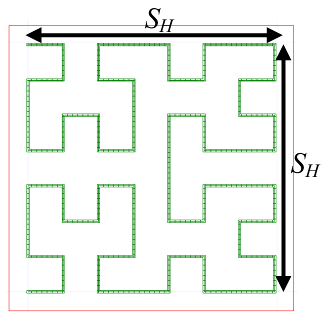


Fig. 2: a) Hilbert of order 2 unit element
b) The response of the Hilbert AMC of order 2.

a)



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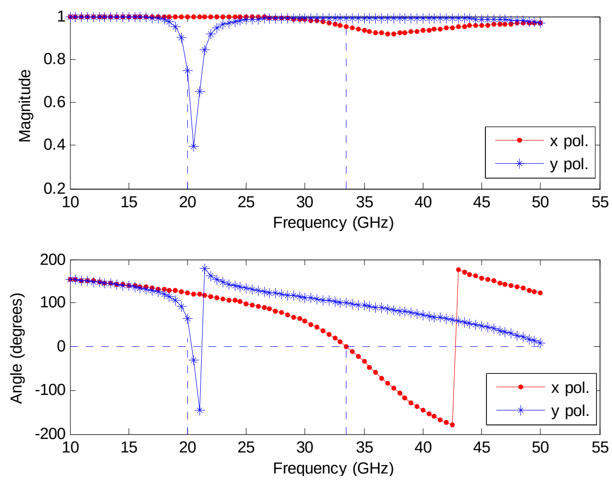


Fig. 3: a) Hilbert of order 3 unit element
b) The response of the Hilbert AMC of order 3.

From Figures 1-3, the 0° phase frequency (f_0) of the reflection response and the bandwidth defined as the frequency interval from -90° to + 90° in the phase [1] can be determined. For a material to be characterized as ideal AMCs, the 0° phase frequency (f_0) of the reflection response should have an amplitude of 1. Yet the use of lossy conductors causes the magnitude at f_0 to be lower than this value as shown in Figures 1-3. It is important to note that only the first f_0 frequency, the lowest frequency, will be considered since the main advantage of the Hilbert inclusions as AMCs is operating at low frequency with respect to the Hilbert size. For example, in Figure 3 b), for the y-polarization the phase became zero at f_0 = 20.3 GHz with magnitude of 0.54 and at f_0 = 50 GHz with magnitude ~ 1.

It is clear from the above results that the magnitude attenuation is more profound in the *y*-polarization than the *x*- polarization, and also the attenuation increases with Hilbert order. It is also observed in Figures 1 and 2 that the minimum of the magnitude occurs at a frequency closer to that of the 180° phase, which affects the magnitude of one. However, increasing the height *d* increases the bandwidth which could in return increase the magnitude of the reflection response at f_0. When *d* becomes 2 mm instead of 1 mm in the second Hilbert order, the magnitude increases from 0.83 to 0.98 (results are not presented here). In addition, when PECs were used instead of copper, f_0 remained unchanged while the magnitude became ~ 1.

In summary, the AMCs frequency f_0, bandwidth and the magnitude of the reflection response at f_0 are listed in Table 1 for the *x*- and *y*- polarization.

Table 1: AMCs Characteristics for *x*- and *y*- polarizations.

Hilbert Order	f_0 (GHz) *x*- polarization	f_0 (GHz) *y*- polarization	Bandwidth *x*-polarization	Bandwidth *y*- polarization	Magnitude *x*- polarization	Magnitude *y*- polarization
1	64.8	45.8	85.3	34.5	1	0.95
2	46.8	31.1	56.0	15.8	0.99	0.83
3	33.5	20.3	32.2	5.9	0.952	0.54

B. Circuit Modeling

Analyzing AMCs using the ANSOFT electromagnetic simulators consumes a significant amount of CPU time (e.g. generating Figure 2 required 7.5 hours on a Pentium 4, 2.8GHz, 2.5GB RAM PC). Designing AMCs with certain desired characteristics will consume even more time since several design iterations will be needed to optimize the response. In order to reduce this computational cost, a simple circuit model for the Hilbert based AMCs is developed based on the multiple reflection phenomenon. In this section, only PECs Hilbert inclusions are used. The basic idea is to substitute the array of Hilbert inclusions (without the ground plane) by an equivalent *L-C* circuit and then compensate for the ground plane using the closed form equation of multiple reflections.

The scattering parameters of the shunt *L-C* circuit shown in Figure 4a) are: $S_{11} = -j\omega C/\left(2\left(1-\omega^2 LC\right)+j\omega C\right)$ and $S_{21} = 2\left(1-\omega^2 LC\right)/\left(2\left(1-\omega^2 LC\right)+j\omega C\right)$. To acquire the equivalent *L-C* values, the ANSOFT Designer software is used to obtain the reflection *R* and transmission *T* responses of only the infinite Hilbert array (without the ground plane) as shown in Figure 4b). Upon plotting S_{11} and S_{21} versus frequency, the same responses of Figure 4 are obtained. The results of Figure 4 are for the y-polarization Hilbert of order 1 with S_H = 1 mm.

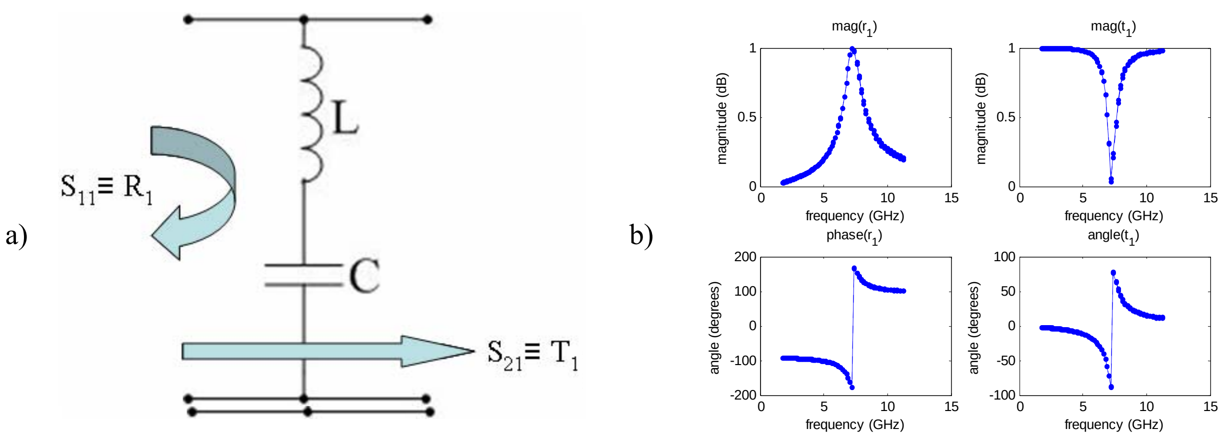


Fig. 4: a) The equivalent *L-C* circuit mode. b) The reflection and transmission responses of the y-polarization of the Hilbert array of order 1, S_H =1 mm without the ground plane.

The above process is repeated for S_H = 2 mm and S_H = 3 mm for the same polarization. The values of *L* and *C* are plotted in Figure 5 versus S_H. It is observed that the values of *L* and *C* follow a linear relationship with the dimension S_H. The same agreement is obtained from the Hilbert order 2 for the same polarization and same dimensions (not presented here).

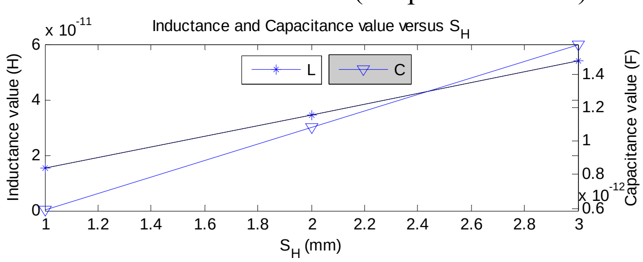


Fig. 5: The equivalent *L* and *C* values for Hilbert inclusions of order 1 versus

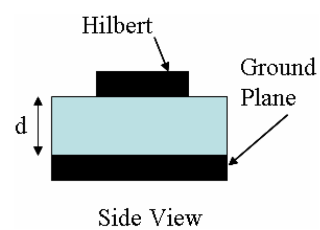


Fig. 6: The Hilbert array at height *d* above the ground plane.

At this point, Figure 5 provides values of *L* and *C* at any S_H value between 1 mm and 3 mm without using ANSOFT simulations.

C. The Multiple Reflections Formula

This section discusses how the effect of the ground plane can also be incorporated in the model. The values of S_{11} and S_{21} are obtained using the interpolated values of *L* and *C* from Figure 5. The overall response of the Hilbert inclusions with the ground plane, (R_{AMC}), is calculated using the multiple reflection formula as:

$$R_{AMC} = S_{11} - \frac{(S_{21})^2 e^{-j2\beta d}}{1 + S_{11} e^{-j2\beta d}} \quad (1)$$

where β is the wave number of the air medium between the Hilbert array and the ground plane as shown in Figure 6. Using this model, the results in Figure 7 are generated. Figure 7 shows the variation of f_0 and the bandwidth with the parameters S_H from 1 mm to 3 mm in increments of 0.2 mm and d from 3 mm to 10 mm.

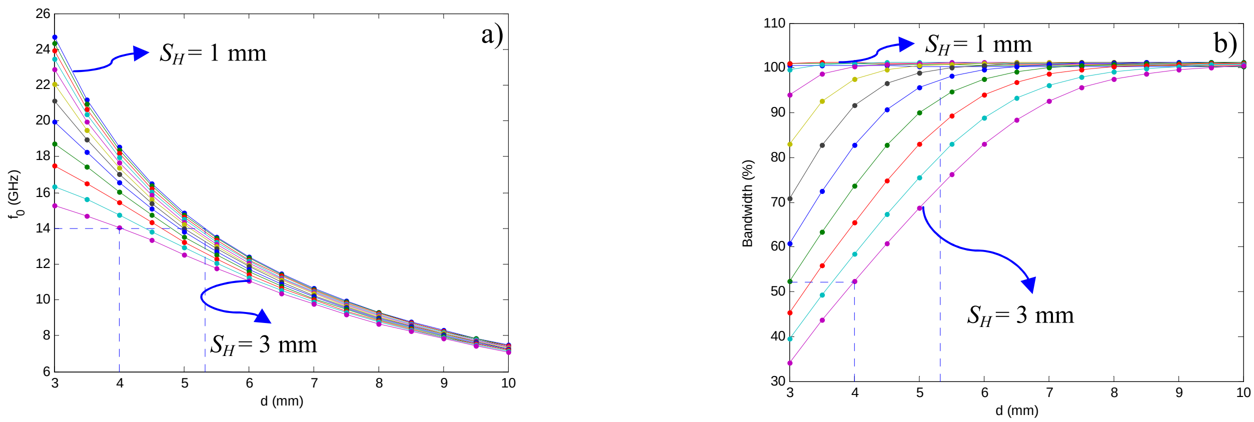
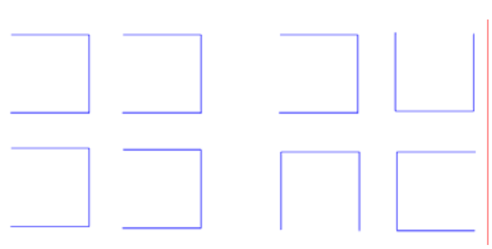


Fig. 7. a) The variation of f_0 and b) bandwidth with respect to the dielectric thickness d and S_H.

The plots of Figure 7 provide a design tool of Hilbert based AMCs. For example, if an AMC is to be designed to operate at 14 GHz, one can select $4\text{ mm} \le d \le 5.33\text{ mm}$ and $1\text{ mm} \le S_H \le 3\text{ mm}$. Depending on S_H and d, the bandwidth can be obtained from Figure 7 b).

D. Polarization Independent Hilbert Configuration

As shown in Figures 1-3, distinct differences exist between the reflection response of the x- and y-polarizations. In order to create a polarization independent structure, Hilbert inclusions will be structured as shown in Figure 8. This idea was also used in Frequency Selective Surfaces (FSS) [8]. With this arrangement, the performance of the x- and y- polarizations became about exactly the same as shown in Figure 9 obtained using ANSOFT Designer software.



a) b)

Fig. 8: a) The conventional array b) The polarization independent array for Hilbert order 1.

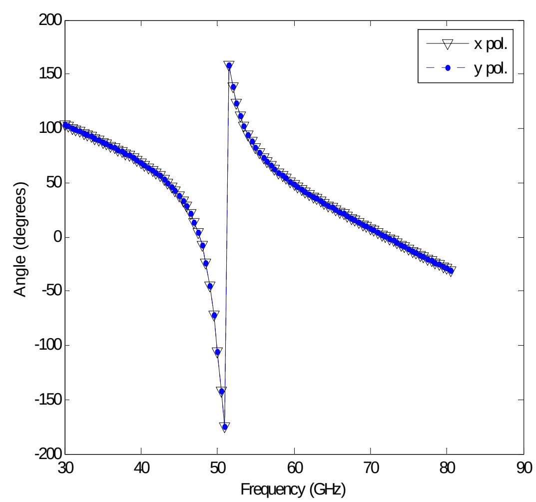


Fig. 9: The phase of the reflection response of the polarization independent configuration. (Hilbert order 1, S_H = 1 mm, and d = 1 mm)

Notice that only the phase is presented in Figure 9 since in this section PECs material was used. This indicates that the obtained magnitude is ~ 1 at all considered frequencies.

3. Conclusions

Based on the equivalent circuit model developed in this work, it is observed that the equivalent capacitance *C* and inductance *L* vary linearly with the dimension S_H for the cases considered here. This led to the development of a simple model which facilitates the design and optimization of Hilbert based AMCs.

In this paper the effect of lossy conductors, such as copper, on the performance of Hilbert based AMCs is investigated. It is found that the loss in the magnitude of the reflection response is more profound in the y-polarization than in the x-polarization. The minimum magnitude of the reflection response is located near the frequency for a phase angle of 180°. Therefore, the effect of the magnitude attenuation can be reduced if the frequency for 0° phase is shifted away from it.

A polarization independent configuration based on changing the orientation of the Hilbert inclusions is also studied.

The range of validity of the proposed equivalent circuit model is under investigation. Also, models for higher Hilbert orders are being investigated and the results will be presented at the conference meeting.

4. Acknowledgements

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