Level Set Algorithm for Shape Reconstruction of Multiple Conducting Cylinders with Arbitrary Cross-Sections

Mohammad Reza Hajihashemi* and Magda El-Shenawee
Department of Electrical Engineering
University of Arkansas, Fayetteville, AR 72701, USA
mhajihas@uark.edu , magda@uark.edu

Abstract: In this paper, a level set algorithm is implemented for shape reconstruction of multiple perfect conducting objects in two-dimensions. Incident waves are assumed to be TM (Transverse Magnetic) plane waves. The presented results show the reconstruction of multiple cylinders up to five, with different cross-sections. The frequency hopping approach has shown a good convergence improvement.

1. Introduction

In inverse electromagnetic scattering problem, one tries to retrieve the dielectric parameters and the shape of unknown objects using measurement data. The unknown objects are illuminated by electromagnetic waves and the received waves are used to retrieve unknown information. Inverse problem has many applications in diverse fields such as target identification, geophysics, seismic exploration, remote sensing, atmospheric science, ground penetrating radar (GPR) and medical applications [1].

In the general case, the inverse scattering problem is non-linear, ill-posed and non-unique problem as a result of multiple scattering effect within the objects. For this reason a general way for solving the nonlinear inverse problem is via an iterative and optimization approach. Solving the inverse problem numerically often involves solving the forward scattering problem many times, therefore the problem is computationally intensive. Moreover, the solution to the inverse problem should be stable since small changes in the measured scattered field may result in large changes in the reconstructed profile [2].

In shape reconstruction problems, we often assume that the constitutive parameters of scattering objects and the surrounding medium are known. The objective is to retrieve the number of scatterers, their shapes and positions. Since the Level Set methods are topologically flexible, they are proven to be powerful in shape reconstruction problems.

2. Methodological Analysis

Level set methods are considered recently by many researchers in shape reconstruction problems [3]. This is due to their flexibility to handle topological changes in an automatic way. Therefore contours may break or join during the propagation of level set function. The main idea of the level set method is to embedding the evolving interface as the zero level of a higher order function.

978-1-4244-2042-1/08/$25.00 ©2008 IEEE

We assume that the interface is represented implicitly as the zero level of a higher order function ϕ, Such that at each time t, the interface is defined as:

$$\Gamma(t) = \left\{ (x, y) \,\middle|\, \phi(x, y, t) = 0 \right\} \quad (1)$$

By getting derivative with respect to evolving time t, we reach to the following equation for tracking the motion of the interface which is known as the Hamilton-Jacobi equation [3]:

$$\frac{\partial \phi}{\partial t} + F \left\| \nabla \phi \right\| = 0 \quad (2.a)$$

$$\phi_0 = \phi(x, y, t = 0) \quad (2.b)$$

where F is the normal component of the deformation velocity.

3. Level Set Method for Inverse Scattering of Conducting Objects

The objective of this work is obtaining the shape reconstruction of multiple infinite conducting cylinders with arbitrary cross sections. The illuminating waves are assumed to be transverse magnetic (TM) plane waves where the electric field is parallel to the cylinder's axis. The expression of deformation velocity on the surface of moving objects is based on obtaining the forward and adjoint induced surface currents as discussed by R. Ferrayé *et al* in [4].

In order to be able to reconstruct the shape of the unknown object(s), the contour(s) of the evolving object(s) should deform in the normal direction such that the error function, between measurement data corresponding to true objects and scattered fields of evolving objects, decreases in each iteration. The deformation velocity introduced by R. Ferrayé *et al* in [4] is used in this work.

The shape reconstruction algorithm is combined with the frequency hopping technique to improve the convergence of algorithm. Consequently, the cost function is minimized at all frequencies where the synthetic data is obtained. The frequency hopping plays an important role in the inversion algorithm as discussed in [4] and [5]. In the work of [4], three objects were reconstructed which is increased to five objects with more complex shapes in this work.

4. Numerical Results

In the first example, the reconstruction of two conducting cylinders is considered. The synthetic data are collected at 6 frequencies in the range of 100 MHz to 5 GHz. We have employed 36 incident angles uniformly around the object and 18 measurements per each incident, which results in 648 synthetic data at each frequency. The initial guess object is a cylinder with radius 10 cm away from the unknown objects as shown in Fig. 1-a. The reconstructed profiles at different stages are shown in Fig 1(a) -1(c). Solid contours represent the true object while the filled contours represent the evolving objects. For each stage, the iteration and the corresponding frequency are shown. A satisfactory reconstruction is obtained in iteration 5385 and frequency of 5 GHz, as shown in Fig. 1-c. The normalized cost function is shown in Fig. 5.a. The pikes in the cost function correspond to the frequency jumps during the evolution of the shape reconstruction algorithm from one frequency to the following one.

The same frequencies and incidence waves are employed for reconstruction of three conducting cylinders as shown in Fig.2. The corresponding cost function is shown in Fig. 5.b. A satisfactory reconstruction is obtained in iteration 5010 as shown in Fig. 2.c. In the reconstruction of four and five cylinders, shown in Fig. 3 and Fig. 4 respectively, the number of iterations at low frequencies is increased from 1500 to 3000 as shown in Fig. 5.c and Fig. 5.d.

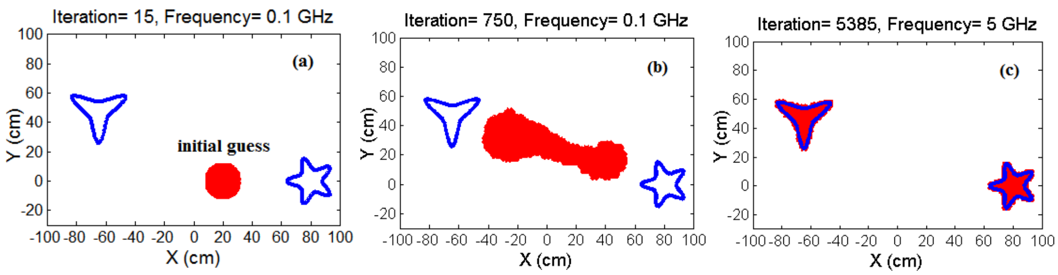

Fig. 1. Reconstruction of two cylinders, (a)initial guess, (b) after 750 iterations, (c) final result

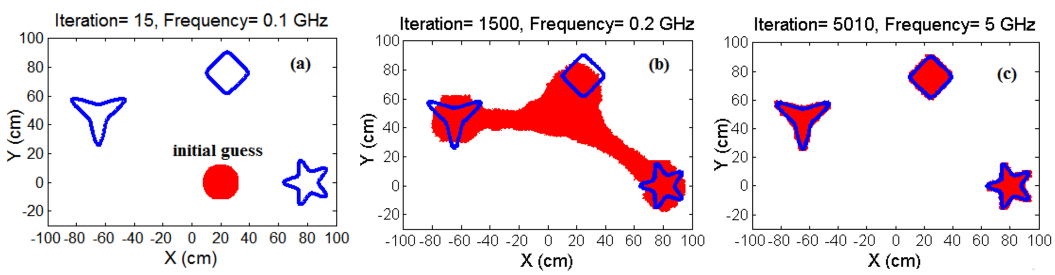

Fig. 2. Reconstruction of three cylinders, (a)initial guess, (b) after 1500 iterations, (c) final result

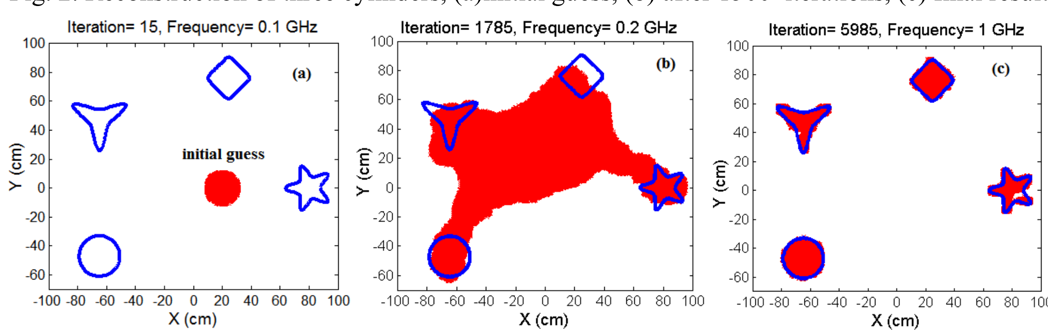

Fig. 3. Reconstruction of four cylinders, (a)initial guess, (b) after 1785 iterations, (c) final result

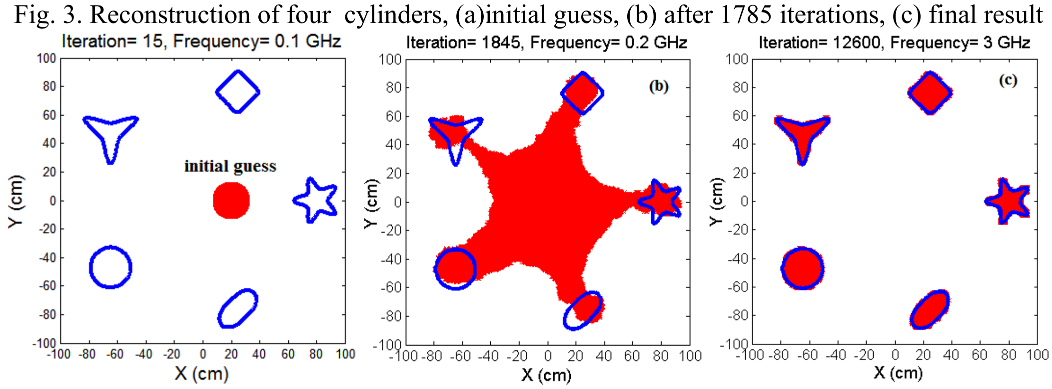

Fig. 4. Reconstruction of four cylinders, (a)initial guess, (b) after 1845 iterations, (c) final result

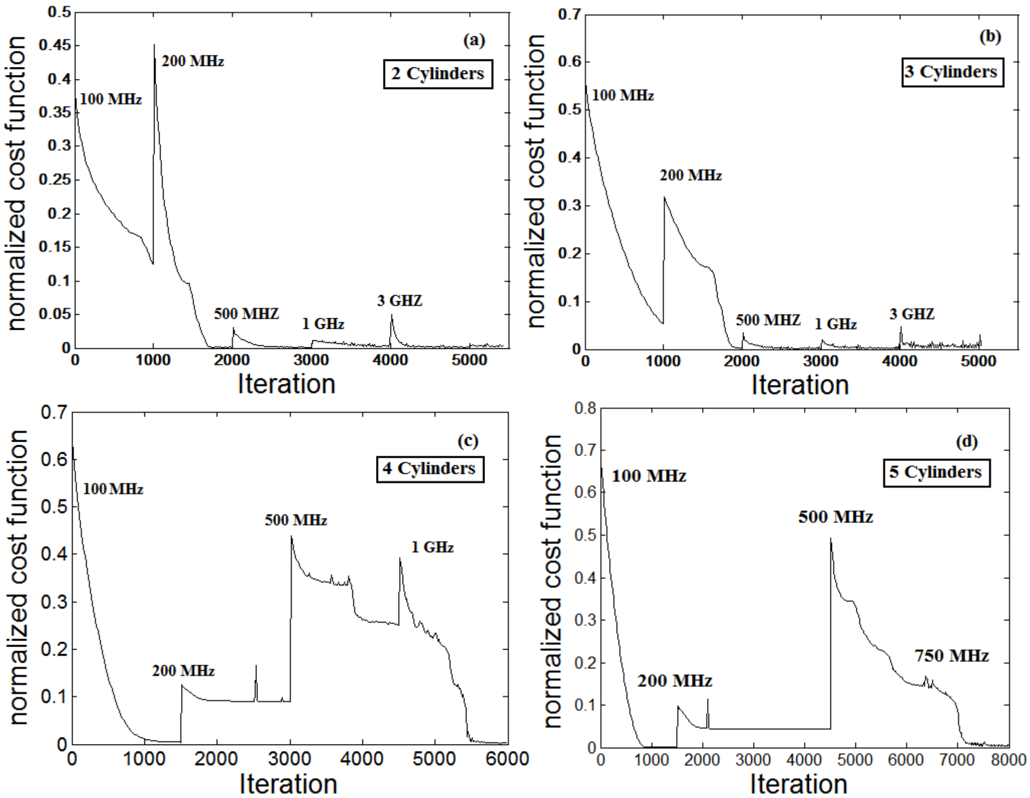

Fig. 5. Normalized cost function for reconstruction of (a) two (b) three (c) four (d) five cylinders

5. Conclusions

A shape reconstruction algorithm based on contour deformation using the level set method is presented. This method has proved to be efficient and flexible for reconstruction of 2-D multiple perfectly conducting cylinders with arbitrary cross-sections. More results will be presented in the conference.

6. Acknowledgements

This work is funded by the National Science Foundation Award Number ECS-0524042

References

[1] A. T. Vouldis, C. N. Kechribaris, T. A. Maniatis, K. S Nikita and N. K. Uzunoglu**,** "Investigating the enhancement of three-dimensional diffraction tomography by using multiple illumination planes," *Journal of Optical Society of America*, vol. 22, issue 7, pp. 1251-1262, July 2005.

[2] W. C. Chew, "Imaging and inverse problems in electromagnetics," *Advances in Computational Electrodynamics: The Finite-Difference Time-Domain Method*, A. Taflove, Ed. Norwood, MA: Artech House, 1998,ch. 12.

[3] J. A. Sethian*, Level Set Methods and Fast Marching Methods,* Cambridge Univ press, 1999.

[4] Ralph Ferrayé, Jean-Yves Dauvignac, and Christian Pichot, "An Inverse Scattering Method Based on Contour Deformations by Means of a Level Set Method Using Frequency Hopping Technique," *IEEE Transactions On Antennas and Propagation*, Vol. 51, No. 5, May 2003.

[5] M. R. Hajihashemi and Magda El-shenawee, "Breast Shape Reconstruction using Microwave techniques and the level set method," *Proceeding of ACES conference on Applied Computational Electromagnetic, 30 March -4 April* 2008*,*Niagara Falls, Canada.