Level Set Shape Reconstruction Algorithm for *TE* polarization

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Abstract: In this paper, the level set shape reconstruction algorithm is implemented to retrieve multiple perfect conducting objects illuminated by Transverse Electric field (*TE*) where the magnetic field is parallel to the cylinders' axes. The results show good shape reconstruction for single and double objects.

1. Introduction

The objective of inverse electromagnetic scattering or microwave imaging is to retrieve the shape of unknown objects using measurement data. The unknown scattering objects are illuminated by electromagnetic waves and the scattered waves are analyzed to reconstruct the unknown objects. Inverse problem has many applications in diverse fields such as geophysical survey, seismic exploration, atmospheric science, nondestructive testing, ground penetrating radar (GPR), quantum field theory and medical applications [1]. In general, in order to solve the inverse scattering problems, the forward scattering problem should be solved many times. This is due to the fact that inverse problems are non-linear, ill-posed and non-unique as a result of multiple scattering effects within the objects. Therefore the problem is computationally demanding. In this work, the constitutive parameters of scattering objects and the surrounding medium are assumed to be known. The goal is to retrieve the number of scatterers and their shapes. Since the Level Set methods are topologically flexible, they are an appropriate option in these kinds of inverse problems.

2. Methodology

We assume that the interface is represented implicitly as the zero level of a higher order function ϕ, such that at each time t, the interface is defined as [2],[3]:

$$\Gamma(t)=\left\{(x,y)\,\middle|\,\phi(x,y,t)=0\right\} \tag{1}$$

Upon getting the derivative with respect to evolving time t, we reach to the following equation for tracking the motion of the interface which is known as the Hamilton-Jacobi equation [3]:

$$\frac{\partial\phi}{\partial t}+F\left\|\nabla\phi\right\|=0 \tag{2.a}$$

$$\phi_0=\phi(x,y,t=0)$$

where F is the normal component of the deformation velocity.

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It can be shown using the reciprocity theorem [4] in similar approach for the *TM* polarization presented in [5], that the appropriate deformation velocity is:

$$F_\tau(r) = -\alpha \operatorname{Re}\left[e^{-j\frac{\pi}{4}} \sum_{i=1}^{N_I} \sum_{m=1}^{N_M(i)} \overline{\left(H_\tau^{sc}(\theta_i,\phi_{m,i}) - H^{sc}{}_{meas}(\theta_i,\phi_{m,i}) \right)} \left(k_0^2 J_{\theta_i}(r) J'_{\phi_{m,i}}(r) - \frac{dJ_{\theta_i}}{ds} \frac{dJ'_{\phi_{m,i}}}{ds} \right) \right] \tag{3}$$

Where N_I the number of incident waves, $N_M(i)$ is the number of measurements of the i^{th} incidence, θ_i is the i^{th} incident angle and $\phi_{m,i}$ is the m^{th} measurement angle associated with θ_i. The symbol $J_z^{\theta_i}$ represents the induced forward current, $J_z'^{\phi_{m,i}}$ is the induced adjoint current and $\frac{d}{ds}$ indicates to the derivation with respect to the curvilinear abscissa. The symbols H^s and $H^s{}_{meas}$ represent the calculated and the measured scattered magnetic field patterns, respectively. The symbol α is a positive normalization coefficient. The currents derivatives in (3), makes the shape reconstruction algorithm, more challenging compared with the *TM* case.

3. Numerical Results

The shape reconstruction algorithm is combined with the frequency hopping technique to improve the convergence of the algorithm. The frequency hopping plays an important role in inversion algorithm as discussed in [2] and [5].

The first example shows the reconstruction of a star-shaped object using the *TE* level set algorithm. Four frequencies in the range 2 GHz- 5 GHz are employed. The normalized cost function is shown in Fig. 1. The reconstruction results at different iterations stages are shown in Fig. 2.a-d. The obtained results show successful reconstruction of the star-shaped object. The second example shows the reconstruction of two ellipses using the *TE* level-set algorithm. The reconstruction results at different iterations are shown in Fig. 3.a-d. The normalized cost function is shown in Fig. 4.

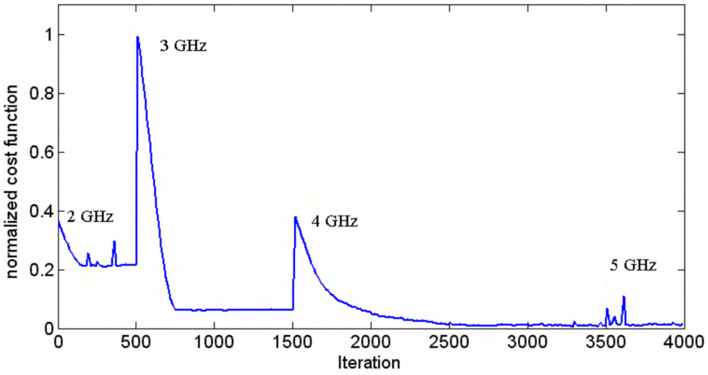

Fig. 1. The normalized cost function of the star-shaped object

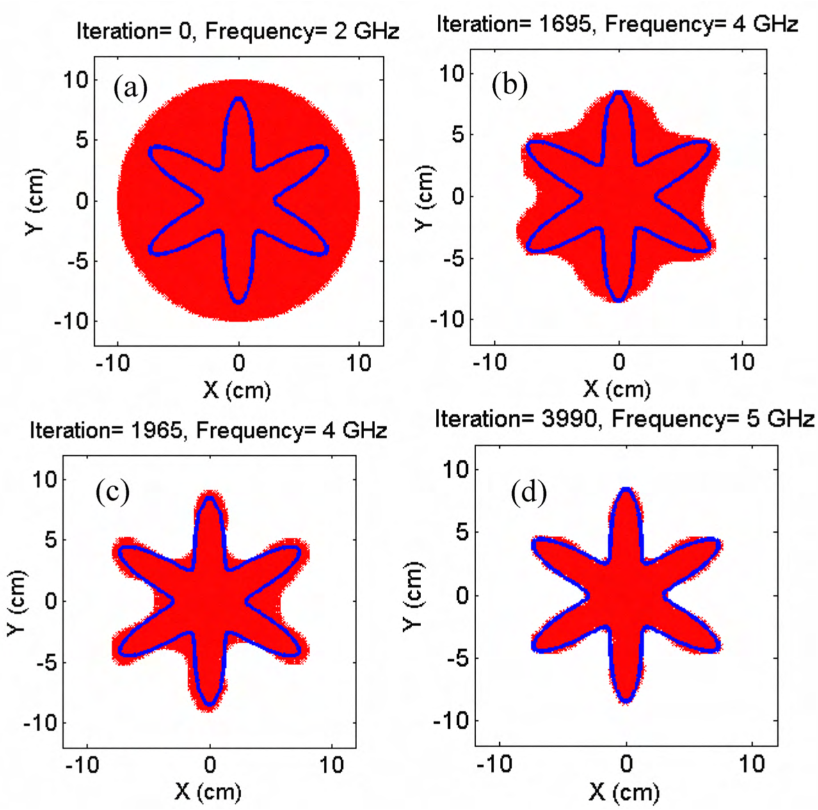

Fig. 2. Reconstruction of a star-shaped object, (a) initial guess, (b) after 1695 iterations, (c) after 1965 iterations, (d) final result

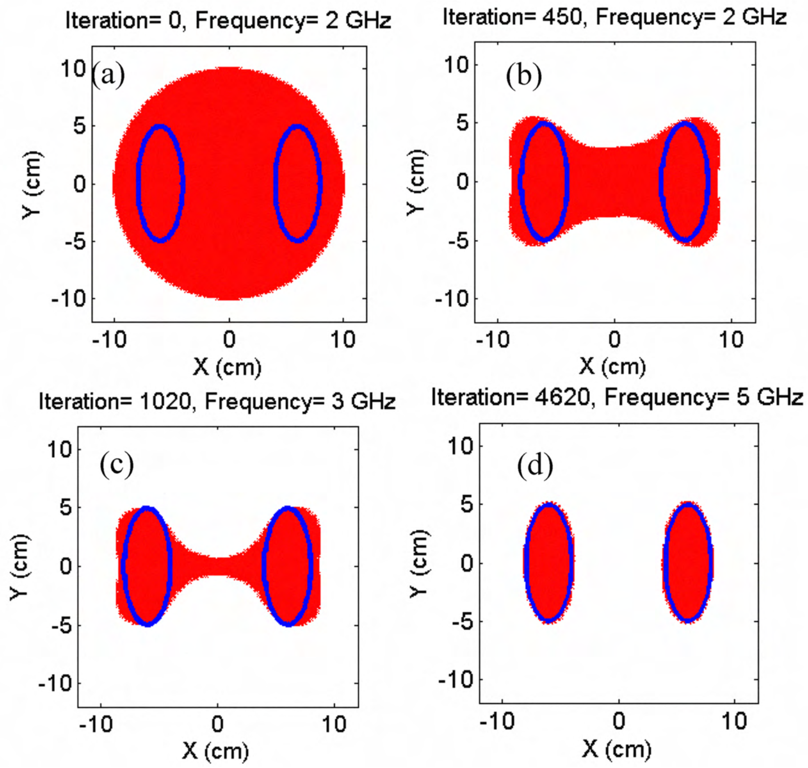

Fig. 3. Reconstruction of two ellipses (a) initial guess (b) after 450 iterations, (c) after 1020 iterations, (d) final result

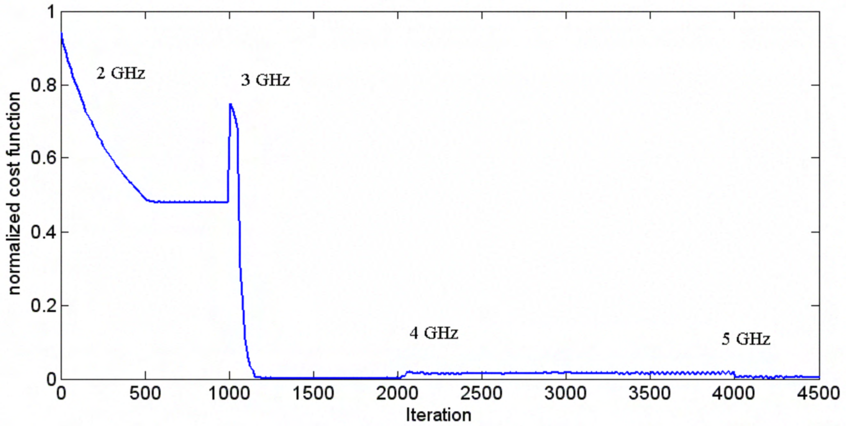

Fig. 4. The normalized cost function of two ellipses

5. Conclusions and Discussions

The *TE* level-set shape reconstruction algorithm is implemented to retrieve multiple perfectly conducting cylinders with arbitrary cross-sections. The shape reconstruction is more challenging in compare to the *TM* case. The challenges in the *TE* level-set algorithm are due to the presence of current derivatives in the deformation velocity. Additional smoothing techniques such smoothing splines and moving averages are employed for calculating these derivatives.

6. Acknowledgements

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