1. INTRODUCTION

Numerical evaluation of brightness temperature for malignant breast cancer tumors and its behavior as a function of frequency.

The basic idea of using microwave radiometry in breast cancer detection involves measuring the natural electromagnetic radiation or emission from the female breast at microwave frequencies. This electromagnetic radiation changes considerably with the presence of malignant breast cancer tumors [4–8]. In particular, the thermal activity of the female breast is a measure of the tumor growth rate, which can provide information even beyond the physical parameters of the tumor such as its size, material, and location [6]. In [4], the authors presented results based on 618 normal female patients to show the temperature symmetry between the left and right breasts. Therefore, the temperature deviation between the left and right breasts is often used to diagnose the breast cancer in one breast [4–6].

As presented and shown in [4–6], medical microwave radiometry has a number of advantages, such as early diagnosis of the cancer even before the tumor mass contrast is formed, the non-invasiveness of the technique, the absolute harmlessness for patients of all ages, and the range (3–10 cm) of tumor burial depth. However, there are some barriers in this technique: mainly, the required sensitivity of the receiving antenna due to the small number of received thermal signals relative to the surrounding noise signals [4–6].

The objective of this work is to computationally evaluate the brightness temperature as a function of the frequency, tumor size, burial depth, and location. The output of this study can be used to understand the thermal radiation phenomenon, which will aid in designing enhanced radiometers for early detection of breast cancer. The multiple interaction model combined with the fast computational technique, the steepest descent fast multipole method (SDFMM), [9–10] is used here to compute the reflectivity, and hence the brightness temperature, due to the presence of a malignant tumor in the breast.

The geometry of the problem is depicted in Figure 1, where the breast surface is assumed flat and the tumor is modeled as a sphere of radius $a$ and buried at depth $d$ measured from its center. The figure shows the position of the radiometer (applicator) in the near zone to the breast and it shows also the multiple interactions mechanism between the tumor and the breast surface. For simplicity, the thickness of breast skin layer and any interior breast inhomogeneities are not accounted for in this model. Curve-fitted data for electrical properties of the malignant tumor and normal breast tissues are plotted in Figure 2 versus the frequency range.

**Figure 1** Cross section of a spherical tumor buried beneath a flat air-breast interface showing the multiple interactions mechanism with $n = 2$
1–10 GHz. These results are based on published experimental measurement data [11].

2. FORMULATION

The brightness temperature $T_B$ of an isothermal medium is given by [1, 3]:

$$ T_B(\theta, \varphi, f, \varepsilon, e, v, g) = T_r(1 - \Gamma), $$

where the angles $\theta$ and $\varphi$ are radiometer observation elevation and azimuth angles, respectively, $f$ is the radiometer observation frequency, $v$ is the polarization of the radiometer (horizontal $h$ or vertical $v$), $\varepsilon$ is the dielectric constant of the medium, $\varepsilon_i$ is the dielectric constant of the buried object, and $g$ represents the object geometrical dimensions. The total reflectivity of the medium with buried object is represented by $\Gamma$, while the physical temperature of the medium and the object is expressed by $T_r$ [2].

The power reflection coefficient or reflectivity $\Gamma$ and the transmissivity $Y$ are defined as the normal components of the time-average Poynting’s vectors given by [1]:

$$ \Gamma = \frac{\vec{\varepsilon} \cdot \vec{S}_w}{-\vec{\varepsilon} \cdot \vec{S}_w}, \quad (2a) $$

$$ Y = \frac{\vec{\varepsilon} \cdot \vec{S}_w}{\vec{\varepsilon} \cdot \vec{S}_w}, \quad (2b) $$

where $\vec{S}_w = \text{Re}(\vec{E} \times \vec{H}^*)/2$ is the time-average Poynting vector. The subscripts $r$, $t$, and $i$ represent reflected, transmitted, and incident waves, respectively, and $\varepsilon$ is a unit vector normal to the flat interface, as shown in Figure 1. In the case of semi-infinite medium with no buried object, reflectivity and transmissivity can be obtained in closed forms [1]. In this case, they become functions of the reflection and transmission coefficients upon illuminating the semi-infinite flat interface with plane waves [1, 2].

However, the current problem is different due to the presence of a buried object (the tumor) in the flat medium (the breast). The idea here is to computationally evaluate Eqs. (2a) and (2b) to obtain the reflectivity and transmissivity, respectively, of the whole scatterer (the medium with the buried object). In this case, the air-flat interface is modeled with a truncated square surface [9, 10] and, to eliminate the edge excitations, an incident Gaussian beam is employed. The Gaussian beam is basically a summation of plane waves tapered towards the surface edges [9, 10]. The size of the truncated surface and the incident half-beam width should be much larger than the buried object. The total brightness temperature is reformulated to account for the buried object as follows [2]:

$$ T_B(\theta, \varphi, f, \varepsilon, e, v, g) = T_r(1 - \Gamma_{surf} - P_{ref}/P), $$

where the subscripts $surf$ and $obj$ indicate the interface surface and the buried object, respectively. The total incident power is given by $P_r$, the power reflected due to the object only is $P_{ref}$, and $\Gamma_{surf}$ is the reflectivity of the flat surface with no buried object. The multiple interaction model presented in [10] is used here to compute the equivalent surface currents on the air-medium interface. These currents can be decomposed into two quantities; one is due to the external excitation and the second is due to the multiple interactions with the tumor (see Fig. 1). The deviation in brightness temperature due to the presence of the tumor is expressed as [2]:

$$ \Delta T_B(\theta, \varphi, f, \varepsilon, e, v, g) = -T_r(P_{ref}/P). $$

The surface currents due to interactions with the tumor are used to compute the reflected power $P_{ref}$ and hence to obtain the deviation in brightness temperature [12]. In practice, this deviation represents the differential in temperature between the left and right breasts used to detect the cancer [4–6].

3. NUMERICAL RESULTS

In example 1, the reflectivity $\Gamma$ and transmissivity $Y$ of a lossless flat medium with no buried object are computed. The relative dielectric constant is assumed to be $\varepsilon_r = 2.55$ and the truncated surface is assumed to be 240 cm $\times$ 240 cm. The incident and observed waves are assumed to be in normal direction for all cases in this section. The objective of this example is to validate the numerical computations of reflectivity and transmissivity obtained using the SDFMM with those obtained using the closed forms for semi-infinite medium excited with plane waves [1]. The ratios of the time-average power densities in Eqs. (2a) and (2b) are computed using the SDFMM with resolution of 1.2 cm [9, 10, 12]. Briefly, the calculated surface currents on the flat medium are used to radiate electric and magnetic fields at 3 cm above and below the interface. No dependency on this distance is observed in the reflectivity or transmissivity for lossless medium. For lossy medium, the fields are compensated with the attenuation factor. However, smaller distances affected the accuracy of the near-field calculations [12]. The medium reflectivity and transmissivity are plotted in Figure 3 versus the $x$ direction at $y = 120$ cm. Excellent comparison is shown within a square spot of 150 cm $\times$ 150 cm centered at $x = 120$ cm, $y = 120$ cm, which is the center of the Gaussian beam footprint. As shown in Figure 3, the beam width is $2W = 96$ cm, which implies, as expected, that plane waves can be assumed within the spot area of $2W \times 2W$. The reflectivity and transmissivity are added up to unity, as shown in Figure 3. Moreover, at normal incidence, the results for the vertical or horizontal polarizations are similar.

In Figure 4, the same data of the previous example is used and a comparison is shown for reflectivity and transmissivity, but in this case for a slightly lossy medium. The relative dielectric constant here is $\varepsilon_r = 3.55 - j0.4$ (loamy soil with 5% moisture.
Because the computed time-average power densities are obtained at 3 cm below the interface (as previously explained), the field values in this case are multiplied by $\exp(2a/\alpha_l)$, where $\alpha$ is the medium attenuation constant and $l = 3$ cm. The computed results show excellent agreement with the closed forms in [1] and also within the same spot area $2W \times 2W$, similar to the previous lossless case.

In Figure 5, the reflectivity and transmissivity of normal breast tissues with relative dielectric constant $\varepsilon_r = 10 - j1.2$ are computed (see Fig. 2 and [11]). Notice that the normal breast tissues have almost the same dielectric constant in the microwave frequency range 1–10 GHz. The results are plotted versus the $x$ direction per wavelength and at $y = 4 \lambda_0$. The modeled surface dimensions are $8 \lambda_0 \times 8 \lambda_0$, where $\lambda_0$ is the free space wavelength. Also in this example, the computed values of the transmitted fields are multiplied by $\exp(2a/\alpha_l)$, with $l = 0.1 \lambda_0$, the near field resolution is $0.04 \lambda_0$, and the half-beam width is $W = 1.6 \lambda_0$. The reflectivity and transmissivity, upon compensation with the attenuation factor, are added up to unity as shown in Figures 4 and 5.

The reflectivity in Figure 5 is larger than that in Figures 3 or 4, which is due to the larger dielectric constant of the medium, in this case $10 - j1.2$, as shown in Figure 2. As expected, the transmissivity in Figure 5 is smaller than that in Figures 3 or 4, due to the larger conductivity of the medium in this case. Notice the slight oscillations in the transmissivity and reflectivity in Figure 5, which are due to the increase of edge reflections in this case. This could be decreased by increasing the Gaussian beam tapering, that is, decreasing $W$ [9, 10].

In Figures 3–5, a validation is demonstrated by computing the reflectivity and transmissivity using the SDFMM. In Figure 6, the time-average reflected power density $S_{\text{ar}}$ due to only the malignant tumor is plotted versus the $x$ direction at $y = 4 \lambda_0$. The malignant tumor is modeled by a sphere of radius $a = 5$ mm and is buried at depth of $d = 2$ cm, measured from its center as depicted in Figure 1. The dielectric constant of the malignant tumor varies with frequency as shown in Figure 2, while the dielectric constant of the normal breast tissues is assumed constant as $\varepsilon_r = 10 - j1.2$. The computed results show excellent agreement with the closed forms in [1] and also within the same spot area $2W \times 2W$, similar to the previous lossless case.

In Figure 5, the reflectivity and transmissivity of normal breast tissues with relative dielectric constant $\varepsilon_r = 10 - j1.2$ are computed (see Fig. 2 and [20]).
The frequency ranges from 1–5 GHz in steps of 200 MHz, which implies that there are 21 curves plotted in Figure 6. Notice that the dimension of the plane-wave square-spot area, as shown in Figures 3–5, ranges from 96 cm to 19.2 cm. This area is considered much larger than the diameter of the tumor, which is 1 cm in this case. The purpose of Figure 6 is to show that the waves reflected due to the tumor are sensed within the spot area of $2W/11003W$ for all considered frequencies. The tumor reflected waves are spherical waves as discussed in [2].

The emphasis of the following examples is to demonstrate the behavior of thermal emissivity or tumor brightness temperature versus frequency. Therefore, in Figure 7, the reflectivity due to the tumor is plotted versus the frequency from 1–5 GHz in steps of 200 MHz. The tumor radius and burial depth are 5 mm and 2 cm, respectively. The tumor reflectivity is obtained by computing the term $(P_{obj}/P_i)$ in Eq. (3b), which is obtained by integrating the time-average reflected power density $S_{tr}$ due to the tumor and the time-average incident power density $S_{in}$ of the Gaussian beam over the radiometer cross section area $A$, where $A = 2W \times 2W$.

As explained in Figures 3–5. As an example, the radiometer cross section was 1 cm $\times$ 2.3 cm at 3.3 GHz in [4]. As expected, the total reflected power $P_{obj}$ depends on the cross section area $A$ of the receiving antenna (the radiometer or applicator) as shown in Figure 7(a). For simplicity, the radiometer cross section is assumed to be a square with dimension ranging from 0.25 $\lambda_0$ to 2 $\lambda_0$. The results clearly show the oscillatory behavior of tumor reflectivity versus frequency, with peaks occurring at 3 GHz and 4.5 GHz. Also, the results show that the smaller the observing area $A$, the larger the magnitude of the sensed tumor reflectivity. Upon multiplying the reflectivity of Figure 7(a) with the physical temperature $T_s$, the deviation in brightness temperature due to the presence of the tumor is shown in Figure 7(b). The physical temperature of breast tissues vary with a woman’s age, for example, at 50 years of age, the average temperature of the breast is assumed to be 33.7°C (or 306.85 K), as reported in [5]. Naturally, the tumor’s physical temperature is different from the surrounding normal breast tissues because they vary in material, as shown in Figure 2. However, in this work it is assumed that the physical temperature $T_s$ in Eqs.

Figure 7 (a) The reflectivity of malignant tumor versus frequency. The tumor radius is $a = 5$ mm and it is buried at depth of 2 cm measured from its center (see Fig. 1); (b) Brightness temperature deviation $\Delta T_b$ due to the presence of malignant tumor of radius $a = 5$ mm buried in normal breast tissues for same data of Fig. 7a.

Figure 8 (a) Brightness temperature deviation $\Delta T_b$ due to the presence of malignant tumor in normal breast tissues. The radiometer cross section is assumed as $A = 0.25 \lambda_0 \times 0.25 \lambda_0$; (b) Deviation in brightness temperature $\Delta T_b$ due to the presence of malignant tumor of radius $a = 5$ mm buried in normal breast tissues for same data of Fig. 7a.
The conducted numerical evaluation for a breast cancer tumor’s brightness temperature shows clear oscillatory behavior versus the frequency. The results show that the peaks of brightness temperature occur at the same frequencies regardless of the tumor’s burial depth, however, the oscillating frequencies depend on the tumor’s size and material. These observations can be used to enhance the radiometer design parameters such as the operating frequency or multiple frequencies, the bandwidth, and the radiometer-receiving cross-section area. This statement is considered a future work in collaboration with the NASA Langley Research Center.

REFERENCES

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