ON THE REFLECTION OF ELASTIC WAVES IN A POROELASTIC HALF-SPACE SATURATED WITH NON-VISCOUS FLUID

by

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Abstract

The effects of the stiffness and Poisson’s ratio of the solid-skeleton and boundary drainage are investigated for in-plane elastic wave incidences in a poroelastic half-space saturated with non-viscous fluid. Biot’s theory of elastic wave propagation in a fluid-saturated porous medium is briefly summarized and applied to the models. Determination of the required poroelastic material constants is proposed. Both drained and undrained boundaries of the half-space are considered and formulated with appropriate boundary conditions. The amplitude coefficients, surface displacements, surface strains, rotations, and stresses are calculated and discussed. It is found that the stiffness, Poisson’s ratio of the solid-skeleton and the boundary drainage affect the reflection response of the half-space significantly.
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1 Introduction

1.1 Literature review

The dynamic response of a fluid-saturated porous medium is of interest to applications in geophysics, petroleum engineering, geotechnical engineering, and earthquake engineering. Biot (1956a) formulated the wave equations for a fluid-saturated porous medium assuming an elastic solid-skeleton and compressible fluid in the pores. He showed that two P-waves and one S-wave co-exist in this model.

The dynamic behavior of a poroelastic medium depends on the material constants, including the elastic moduli and the dynamic mass coefficients. The determination of the poroelastic material constants has been discussed by Biot and Willis (1957), Berryman (1980) and Bourbié et al. (1987). The elastic moduli have been measured experimentally by Fatt (1959) and Yew and Jogi (1976).

Deresiewicz and coworkers published a series of papers discussing the effects of free plane boundaries on wave propagation in a poroelastic medium (Deresiewicz, 1960, 1961, Deresiewicz and Rice, 1962). They studied amplitude ratios, phase velocities, and attenuation of the reflected waves.

In geotechnical earthquake engineering, the drainage of a poroelastic medium boundary is an important factor for the initiation of liquefaction. Studies of boundary conditions for physical boundaries of poroelastic media have been carried out by Deresiewicz and Skalak (1963), Lovera (1987) and de la Cruz and Spanos (1989). These authors employed different approaches, but conservation of mass and continuity of momentum were the common principles.

1.2 Objective and organization of this report

The objective of this report is to study (1) the effects of the stiffness and Poisson ratio of the solid-skeleton, and (2) the effects of drained and undrained boundaries on the reflection of waves from a poroelastic half-space saturated with non-viscous fluid (non-dissipative case).

In studies of wave propagation, the elastic body waves are usually decomposed into SH, P, and SV-waves. Because the SH-wave equation of poroelastic medium for
non-dissipative case is identical to the one for an elastic medium (Deresiewicz, 1961), a poroelastic half-space has the same response as an elastic medium. Therefore, the investigation of incident SH-wave is not included in this report.

In Chapter 2, Biot’s theory of wave propagation in fluid-saturated porous medium is briefly reviewed. The relations among the material constants are discussed and suggested for numerical analysis. The boundary conditions are adopted from the description of open-boundary and sealed-boundary by Deresiewicz and Skalak (1963).

In Chapter 3 and Chapter 4, the cases of incident plane P-waves and SV-waves are investigated for the amplitude coefficients, surface displacements, surface strains, rotations and stresses. The effects of the stiffness, Poisson’s ratio of the solid-skeleton and the effects of boundary drainage are discussed in detail.

In Chapter 5, a case study of strong motion recorded at Port Island during the 1995 Kobe Earthquake is briefly examined. We applied the model analyzed in Chapter 3 and Chapter 4 to come up with a rough preliminary interpretation on this unique record of amplified vertical motions, but reduced horizontal motions. General applications of the results of this study and brief conclusions are also stated.

It is hoped that the present study will provide useful theoretical estimates of some fundamental properties of porous media for engineering applications.
2 Review of the theory of wave propagation in a fluid-saturated porous medium

2.1 Biot’s theory of wave propagation in a fluid-saturated porous medium (low frequencies)

2.1.1 Stress-strain relations for fluid-saturated porous medium

Biot (1956a) considered an element of a solid-fluid system represented by a cube of unit size in which the stress tensors are separated into two parts – solid and fluid. The solid-skeleton is assumed to be isotropic and elastic, and the fluid is allowed only dilatational deformation. This particular solid-fluid system has the following stress-strain relations.

(1) The stress tensor on the solid part of the cube is

\[
\begin{bmatrix}
  \tau_{xx} & \tau_{xy} & \tau_{xz} \\
  \tau_{yx} & \tau_{yy} & \tau_{yz} \\
  \tau_{zx} & \tau_{zy} & \tau_{zz}
\end{bmatrix}
\] (2-1)

(2) The stress tensor on the fluid part of the cube is

\[
\begin{bmatrix}
  \sigma & 0 & 0 \\
  0 & \sigma & 0 \\
  0 & 0 & \sigma
\end{bmatrix}
\] (2-2)

where \( \sigma \) is proportional to the hydraulic pressure on the unit cube according to

\[
\sigma = \hat{n}p \quad (\hat{n} = \text{porosity})
\] (2-3)

(3) The stress-strain relations are established as

\[
\begin{bmatrix}
  \tau_{xx} \\
  \tau_{yy} \\
  \tau_{zz} \\
  \sigma \\
  \tau_{xy} \\
  \tau_{yz} \\
  \tau_{zx}
\end{bmatrix} =
\begin{bmatrix}
  P & \lambda & \lambda & Q & 0 & 0 & 0 \\
  \lambda & P & \lambda & Q & 0 & 0 & 0 \\
  \lambda & \lambda & P & Q & 0 & 0 & 0 \\
  Q & Q & Q & R & 0 & 0 & 0 \\
  0 & 0 & 0 & 2\mu & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 2\mu & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 2\mu
\end{bmatrix}
\begin{bmatrix}
  \gamma_{xx} \\
  \gamma_{yy} \\
  \gamma_{zz} \\
  \epsilon \\
  \gamma_{xy} \\
  \gamma_{yz} \\
  \gamma_{zx}
\end{bmatrix}
\] (2-4)

where \( \tau_{xx}, \tau_{yy}, \tau_{zz}, \tau_{xy}, \tau_{yz}, \tau_{zx} \) = stresses of the solid-skeleton
\[ \gamma_{xx}, \gamma_{yy}, \gamma_{zz}, \gamma_{xy}, \gamma_{yz}, \gamma_{zx} = \text{strains of the solid-skeleton} \]
\[ \sigma = \text{stress of the pore fluid} \]
\[ \varepsilon = \text{dilatational strain of the pore fluid} \]
\[ \lambda, \mu, P, Q, R = \text{elastic moduli for the solid-fluid system} \ (P = \lambda + 2\mu). \]

### 2.1.2 The wave equations

To develop the wave equations for the low frequency range, three assumptions are made (Biot, 1956a): (1) the relative motion of the fluid in the pores is laminar flow which follows Darcy’s law (Reynolds number < 2000); (2) the elastic wavelength is much larger than the unit solid-fluid element; (3) the size of the unit element is large compared to the pores. Based on the above assumptions and stress-strain relations, the wave equations of fluid-saturated porous medium are

\[ \mu \nabla^2 \tilde{u} + \nabla[(\lambda + \mu)e + Q\varepsilon] = \frac{\partial^2}{\partial t^2} (\rho_{11}\tilde{u} + \rho_{12}\tilde{U}) + \hat{b} \frac{\partial}{\partial t} (\tilde{u} - \tilde{U}) \]  
(2-5a)

\[ \nabla[Qe + Re] = \frac{\partial^2}{\partial t^2} (\rho_{12}\tilde{u} + \rho_{22}\tilde{U}) - \hat{b} \frac{\partial}{\partial t} (\tilde{u} - \tilde{U}) \]  
(2-5b)

where \( \tilde{u} \) = displacement vector for the solid-skeleton
\( \tilde{U} \) = displacement vector for the pore fluid
\[ e = \text{div}(\tilde{u}), \quad \varepsilon = \text{div}(\tilde{U}) \]
\[ \rho_{11}, \rho_{12}, \rho_{22} = \text{dynamic mass coefficients} \]
\[ \hat{b} = \text{dissipative coefficient} = \hat{n}^2 \frac{\tilde{\mu}}{\hat{k}} \]

\( \hat{n} = \text{porosity}, \quad \hat{k} = \text{permeability}, \quad \tilde{\mu} = \text{absolute viscosity} \).

Next, we apply Helmholtz decomposition to the displacement vector

\[ \tilde{u} = \nabla(\phi) + \text{curl}(\tilde{\psi}) \]  
(2-6a)

\[ \tilde{U} = \nabla(\Phi) + \text{curl}(\tilde{\Psi}) \]  
(2-6b)

where \( \phi \) and \( \Phi \) are P-wave potentials and \( \psi \) and \( \Psi \) are S-wave potentials for solid and fluid, respectively.

Substituting Eq.(2-6) into Eq.(2-5), and rearranging the terms, the following two sets of equations with respect to P-wave and S-wave potentials are obtained
for P-wave potentials:

\[
\begin{aligned}
P \nabla^2 \phi + Q \nabla^2 \Phi &= \frac{\partial^2}{\partial t^2} (\rho_{11} \phi + \rho_{12} \Phi) + \hat{b} \frac{\partial}{\partial t} (\phi - \Phi) \\
Q \nabla^2 \phi + R \nabla^2 \Phi &= \frac{\partial^2}{\partial t^2} (\rho_{12} \phi + \rho_{22} \Phi) - \hat{b} \frac{\partial}{\partial t} (\phi - \Phi)
\end{aligned}
\]  
(2-7)

and for S-wave potentials:

\[
\begin{aligned}
\mu \nabla^2 \psi &= \frac{\partial^2}{\partial t^2} (\rho_{11} \psi + \rho_{12} \tilde{\psi}) + \hat{b} \frac{\partial}{\partial t} (\psi - \tilde{\psi}) \\
0 &= \frac{\partial^2}{\partial t^2} (\rho_{12} \psi + \rho_{22} \tilde{\psi}) - \hat{b} \frac{\partial}{\partial t} (\psi - \tilde{\psi})
\end{aligned}
\]  
(2-8)

### 2.1.3 Solutions for P-waves

In the present study, we consider a non-dissipative case in which the porous medium is saturated with non-viscous fluid (\(\mu = 0\)). For such a case, the last terms in Eq.(2-7) and Eq.(2-8) drop out.

If the wave potentials have harmonic time variations, the potentials can be expressed as

\[
\begin{aligned}
\phi &= \phi(x, y, z)e^{-i\omega t}, \quad \Phi = \Phi(x, y, z)e^{-i\omega t} \\
\psi &= \psi(x, y, z)e^{-i\omega t}, \quad \Psi = \Psi(x, y, z)e^{-i\omega t}
\end{aligned}
\]  
(2-9a, 2-9b)

Substituting Eq.(2-9a) into Eq.(2-7), and eliminating \(\Phi\), the P-wave equations for the solid-skeleton become

\[
A \nabla^4 \phi + \omega^2 B \nabla^2 \phi + \omega^4 C \phi = 0
\]  
(2-10)

where \(A = PR - Q^2\),

\[
B = \rho_{11} R + \rho_{22} P - 2 \rho_{12} Q
\]

\[
C = \rho_{11} \rho_{22} - \rho_{12}^2
\]

Eq.(2-10) can be decomposed into

\[
(\nabla^2 + k^2_{\alpha,j}) \phi_j = 0, \quad (j = 1, 2)
\]  
(2-11)

where

\[
 k_{\alpha,j} = \frac{\omega}{V_{\alpha,j}} \quad (j = 1, 2)
\]  
(2-12a)

are the P wavenumbers, and
are the P-wave velocities

From Eq. (2-11), it is seen that two P-waves exist in the medium.

The general solution for the solid-skeleton is

$$\phi = \phi_1 + \phi_2$$

(2-13)

To determine the wave potential for the fluid component, substituting Eq. (2-9a) into Eq. (2-7), the following is obtained

$$\Phi = \Phi_1 + \Phi_2 = f_1\phi_1 + f_2\phi_2$$

(2-14)

where

$$f_j = \frac{A/V_{a,j}^2 - \rho_{11}R + \rho_{12}Q}{\rho_{12}R - \rho_{22}Q}$$

(2-15)

**2.1.4 Solutions for S-waves**

Substituting Eq. (2-9b) into Eq. (2-8) and eliminating \(\Psi\), the S-wave equation for the solid-skeleton becomes

$$(\nabla^2 + k_\beta^2)\psi = 0$$

(2-16)

where

$$k_\beta = \frac{\omega}{V_\beta}$$

(2-17a)

is the S-wave wavenumber, and

$$V_\beta = \sqrt{\frac{\mu\rho_{22}}{C}}$$

(2-17b)

is the S-wave velocity.

The S-wave potential for fluid the component can be also obtained

$$\Psi = f_3\psi$$

(2-18)

where

$$f_3 = -\frac{\rho_{12}}{\rho_{22}}$$

(2-19)
2.2 Validity of Biot’s theory

2.2.1 Theoretical considerations

Biot’s theory is based on the principles of continuum mechanics, and assumes linear behavior of the materials. However, porous medium involves discontinuities between the solid and pores, for which the macroscopic laws of mechanics are not always applicable. For example, if the incident wavelengths are short enough to “feel” the pores, then the diffraction of waves occurs and Biot’s theory is no longer valid.

Bourbié et al. (1987) suggested that the minimum homogenization volume for porous rock is about three times the pore diameter. The range of pore sizes in porous rock is from $10^{-3}$ mm to 1 mm, and for the study of earthquake strong motion, the continuum mechanics representation is applicable, because the short wavelengths of earthquake shaking, e.g. 10 m (assuming frequency = 30 Hz, $V_\beta = 300$ m/s) are still much longer relative to the minimum homogenization size of rock.

Darcy’s law for pore fluid is another integral part of Biot’s theory. If the frequency of the incident waves exceeds a certain value, the laminar flow in the pores breaks down. Biot (1956a) showed that this frequency may be related to the fluid viscosity and the size of pores as

$$f_i = \frac{\pi \hat{\nu}}{4d^2}$$

(2-20)

where $d$ is pore diameter and $\hat{\nu}$ is kinematic viscosity. In the case of water, $\hat{\nu} = 1.3 \times 10^{-6}$ m$^2$/s, and we find that the maximum $f_i = 10^4$ Hz for $d = 10^{-2}$ mm, and $f_i = 25$ Hz for $d = 2 \times 10^{-1}$ mm.

To apply Biot’s theory to geotechnical engineering, it is important to determine the range of its validity with respect to the grain size of the soil. In general, the soil consists of a collection of solid particles with various sizes. To determine the pore size of certain soil, the distribution of grain sizes needs to be analyzed. The “effective” pore diameter for soil is assumed to be one-fifth of $D_{10}$ in permeability studies (Cedergran, 1989), where the $D_{10}$ is the grain size that corresponds to 10% of the sample passing by weight. Permeability can be used to determine the mobility of pore fluid in the soil. For
poor drainage or impervious soils (clay or silt), relative fluid flow does not occur. In that case, Biot’s theory is not applicable.

Fig. 2.1 shows the effective pore size, permeability and frequency $f_i$ with respect to soil grain size. It is seen that the effective pore size ($D_{10}/5$) range in which Biot’s theory is applicable is from 0.01 to 0.2 mm. Therefore, soils consisting of sands and gravels, with grain size $D_{10} = 0.05$ to 1 mm are eligible to apply Biot’s theory.

Fig. 2.1 Soil grain size with corresponding (a) soil classification, (b) the effective pore size, (c) permeability and drainage properties (Terzaghi and Peck, 1948), (d) frequency $f_i$, (e) the range of effective pore size where Biot’s theory can be applied, and (f) the main range of soils may have grain size $D_{10}$ where Biot’s theory can be applied.

### 2.2.2 An example of the geographical distribution of sands

The preceding discussion suggests that the candidate sites for use of Biot’s theory in geotechnical engineering must meet two criteria: (1) the soils mainly to consist of sand or gravel deposits, and (2) the soil stratum to be fluid-saturated. Generally, the areas with sand deposits near rivers, lakes, reservoirs, or seashores are the most applicable sites for
strong motion response analysis by Biot’s theory. Of civil engineering interest, geotechnical structures such as earth dams or man-made landfills at ports (for example, Port Island in Kobe, and Terminal Island in Los Angeles) are areas to study the dynamics of fluid-saturated media.

To illustrate the geographical extent of the areas where physical conditions may exist for use of Biot’s theory in interpretation of incident seismic waves, we cite examples from Trifunac and Todorovska (1998). Fig. 2.2 shows the geographical distribution of surficial sand deposits in Los Angeles-Santa Monica region of Southern California (after Tinsley and Fumal, 1985). It is seen from Fig. 2.2 that large areas of metropolitan Los Angeles are covered by sand deposits. In those areas, when the water table is essentially at ground surface (Tinsley et al., 1985), the physical conditions will exist for use of Biot’s theory in interpretation of observed strong motion amplitudes.

![Fig. 2.2 Geographical distribution of surficial sand deposits in Los Angeles – Santa Monica region (shaded areas, after Tinsley and Fumal, 1985).](image)
2.3 Material constants for fluid-saturated porous media

In Eq.(2-5), two sets of material constants, elastic moduli and dynamic mass coefficients, are required in the model. Determination of the material constants for porous media is discussed in the following section.

2.3.1 Elastic moduli

Biot and Willis (1957) proposed a method to calculate the elastic moduli of porous media from experimental measurements. For an isotropic system, the four elastic moduli can be determined from the shear modulus, compressibility of the solid-skeleton (jacketed compressibility), compressibility of the solid-fluid system (unjacketed compressibility), and compressibility of the pore fluid (fluid infiltrating coefficient) as follows

\[ \mu = \mu_s \]  
\[ \lambda = \lambda_s \frac{Q^2}{R} \]  
\[ R = \frac{\hat{n}^2}{\gamma + \delta - \delta^2 / \kappa_s} \]  
\[ Q = \frac{\hat{n}(1-\hat{n} - \delta / \kappa_s)}{\gamma + \delta - \delta^2 / \kappa_s} \]

where
\[ \hat{n} = \text{porosity} \]  
\[ \lambda_s = \frac{2\nu_s}{1-2\nu_s} \]  
\[ \mu_s = \text{Lamé constant for the solid-skeleton} \]  
\[ \nu_s = \text{Poisson’s ratio for the solid-skeleton} \]  
\[ \kappa_s = -\frac{e}{p} = \frac{1}{K_s} = \text{compressibility of the solid-skeleton} \]  
\[ \delta = -\frac{e}{p} = \text{compressibility of the solid-fluid system} \]
\[ \gamma = - \frac{\hat{n}(\varepsilon - e)}{p} = \hat{n}(\frac{1}{K_f} + \frac{e}{p}) \]

\[ = \hat{n}(\kappa_f - \delta) = \text{compressibility of the pore fluid} \]

\[ K_s = \frac{2(1+\nu)}{3(1-2\nu)} \mu_s = \text{bulk modulus of solid-skeleton} \]

\[ K_f = \text{bulk modulus of fluid} \]

\[ \kappa_f = \text{fluid compressibility} \]

### 2.3.2 Dynamic mass coefficients

In Biot’s theory (1956a), the framework of the mass components in a unit cubic solid-fluid system is given as

\[ \rho_1 = \rho_{11} + \rho_{12} = (1 - \hat{n})\rho_s \]

\[ \rho_2 = \rho_{12} + \rho_{22} = \hat{n}\rho_f \]

\[ \rho = \rho_1 + \rho_2 = (1 - \hat{n})\rho_s + \hat{n}\rho_f \]

where

\[ \rho = \text{unit mass of the solid-fluid aggregate} \]

\[ \rho_1 = \text{solid mass in the unit cube (unit mass of solid-skeleton)} \]

\[ \rho_2 = \text{fluid mass in the unit cube} \]

\[ \rho_s = \text{density of the solid material} \]

\[ \rho_f = \text{density of the fluid} \]

To determine Biot’s dynamic mass coefficients, Berryman (1980) proposed the following relations.

\[ \rho_{11} = (1 - \hat{n})(\rho_s + \tau_r \rho_f) \]

\[ \rho_{22} = \hat{n}\tau_a \rho_f \]

where \( \tau_r, \rho_f = \text{the induced mass due to oscillation of the solid particle in the fluid} \)

\[ \tau_a \geq 1 = \text{toruosity parameter} \]

The value of \( \tau_r \) has to be calculated from a microscopic model of the solid-frame moving in the fluid. For spherical solid particles, \( \tau_r = 0.5 \) (Berryman, 1980). Substituting Eq.(2-26) into Eq.(2-25), the following is obtained
\[
\rho_{11} = (1 - \hat{n}) \rho_s + \hat{n}(\tau_a - 1) \rho_f
\]  \hspace{1cm} (2-27a)
\[
\rho_{12} = -\hat{n}(\tau_a - 1) \rho_f
\]  \hspace{1cm} (2-27b)
\[
\rho_{22} = \hat{n} \tau_a \rho_f
\]  \hspace{1cm} (2-27c)

and
\[
\tau_a = 1 + \tau_r \left( \frac{1 - \hat{n}}{\hat{n}} \right)
\]  \hspace{1cm} (2-28)

Eq.(2-28) implies that \( \tau_a = 1 \) as \( \hat{n} = 1 \), for pure fluid medium, and \( \tau_a \to \infty \) as \( \hat{n} \to 0 \), for solid medium.

### 2.3.3 Limits of the material constants and simplified formulae

If the medium is pure fluid, then \( \hat{n} = 1 \), \( \mu = \lambda = Q = 0 \), \( R = K_f \), and \( \rho_{11} = \rho_{12} = 0 \), \( \rho_{22} = \rho_f \). In this case, the Eq.(2-5a) disappears, and Eq.(2-5b) gives the dynamic equation of motion of the fluid under the assumption of small displacements as

\[
R \ \text{grad}[\text{div} \hat{\mathbf{U}}] = \rho_f \frac{\partial^2 \hat{\mathbf{U}}}{\partial t^2}
\]  \hspace{1cm} (2-29)

If the medium is pure solid, then \( \hat{n} = 0 \), \( Q = R = 0 \), and \( \rho_{12} = \rho_{22} = 0 \), \( \rho_{11} = \rho_s \.

In this case, the Eq.(2-5b) disappears, and Eq.(2-5a) gives the Helmholtz wave equation for elastic solid as

\[
(\lambda + \mu) \text{grad}[\text{div} \mathbf{u}] + \mu \nabla^2 \mathbf{u} = \rho_s \frac{\partial^2 \mathbf{u}}{\partial t^2}
\]  \hspace{1cm} (2-30)

For dynamic analyses in soil mechanics, it is often assumed that the compressibility of the solid-fluid system can be neglected \( \delta \to 0 \), relative to the compressibility of the solid-skeleton and the fluid. Therefore, Eq. (2-22) through Eq.(2-24) can be simplified as

\[
\lambda = \lambda_s + \frac{Q^2}{R} = \frac{2\nu_s}{1 - 2\nu_s} \mu_s + \frac{(1 - \hat{n})^2}{\hat{n}} K_f
\]  \hspace{1cm} (2-31)

\[
Q = \frac{\hat{n}(1 - \hat{n})}{\gamma} = (1 - \hat{n})K_f
\]  \hspace{1cm} (2-32)

\[
R = \frac{\hat{n}^2}{\gamma} = \hat{n}K_f
\]  \hspace{1cm} (2-33)
If we assume the solid-fluid system is formed by spherical solid particles ($\tau_r = 0.5$), the material constants can be simplified as in table 2.1.

Table 2.1 The simplified formulas of material constants for porous medium.

<table>
<thead>
<tr>
<th>Elastic moduli</th>
<th>Dynamic coefficients of mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu = \mu_s$</td>
<td>$\rho_{11} = (1 - \hat{n}) \rho_s + \hat{n} (\tau_\alpha - 1) \rho_f$</td>
</tr>
<tr>
<td>$\lambda = \frac{2\nu_s}{1 - 2\nu_s} \mu_s + \frac{(1 - \hat{n})^2}{\hat{n}} K_f$</td>
<td>$\rho_{12} = -\hat{n} (\tau_\alpha - 1) \rho_f$</td>
</tr>
<tr>
<td>$Q = \frac{\hat{n}(1 - \hat{n})}{\gamma} = (1 - \hat{n}) K_f$</td>
<td>$\rho_{22} = \hat{n} \tau_\alpha \rho_f$</td>
</tr>
<tr>
<td>$R = \frac{\hat{n}^2}{\gamma} = \hat{n} K_f$</td>
<td>$\tau_\alpha = \frac{1}{2} \left( 1 + \frac{2}{\hat{n}} \right)$</td>
</tr>
</tbody>
</table>

2.4 Wave velocities in fluid-saturated porous media

The wave velocity is one of the most important parameters in the study of wave propagation. The relations among different wave velocities, depending on the combinations of material constants will influence the nature of the reflections at the boundaries.

For porous medium with 30% porosity, for example, and consisting of spherical solid particles ($\rho_s/\rho_f = 2.7$), the wave velocities for various $\mu/K_f$ ratios and Poisson’s ratios of the solid-skeleton can be computed using material constants shown in Table 2.1. The $\mu/K_f$ ratio represents the stiffness of the solid material with respect to the fluid bulk modulus. The Poisson’s ratio may represent the consolidation status of the solid-skeleton. In general, consolidated materials have small Poisson’s ratios, and unconsolidated materials have large Poisson’s ratios. The wave velocities for different combinations of material constants are shown in Table 2.2.

From Table 2.2, it is seen that the fast P-wave velocities are always larger than the S-wave velocities in the porous media, same as in elastic media. The slow P-wave velocities are larger than the S-wave velocities only for soft, unconsolidated solids (e.g. $\mu/K_f \leq 0.1$ and $\nu_s = 0.4$).

If the porous medium is water-saturated ($K_f = 2200$ Mpa), the $\rho_s/\rho_f$ ratio equals the specific gravity of the solid (the ratio between the unit masses of solid and water).
The shear wave velocities, $V_\beta$, and the overall nature of the material for different $\mu/K_f$ ratios are illustrated in Table 2.3.

Table 2.2 Wave velocity ratios for different Poisson’s ratios of solid-skeleton and $\mu/K_f$ ratios in porous medium (assuming porosity = 0.3, $\rho_s/\rho_f$ = 2.7).

<table>
<thead>
<tr>
<th>$\mu/K_f$</th>
<th>$V_s$</th>
<th>$V_\alpha/V_\beta$</th>
<th>$V_\alpha/V_\beta$</th>
<th>$V_\alpha/V_\beta$</th>
<th>$V_\alpha/V_\beta$</th>
<th>$V_\alpha/V_\beta$</th>
<th>$V_\alpha/V_\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.1</td>
<td>1.58</td>
<td>0.29</td>
<td>2.29</td>
<td>0.64</td>
<td>5.99</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>1.71</td>
<td>0.29</td>
<td>2.37</td>
<td>0.67</td>
<td>6.01</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>1.80</td>
<td>0.30</td>
<td>2.44</td>
<td>0.69</td>
<td>6.04</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>0.3</td>
<td>1.94</td>
<td>0.30</td>
<td>2.53</td>
<td>0.72</td>
<td>6.07</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
<td>2.50</td>
<td>0.30</td>
<td>2.96</td>
<td>0.80</td>
<td>6.22</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Table 2.3 Shear wave velocities, $V_\beta$, for different $\mu/K_f$ ratios in water-saturated porous medium (porosity = 0.3, solid specific gravity = 2.7).

<table>
<thead>
<tr>
<th>$\mu/K_f$</th>
<th>Shear wave velocity $V_\beta$</th>
<th>Field materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>$\approx$ 3000 m/sec</td>
<td>Porous rock</td>
</tr>
<tr>
<td>1</td>
<td>$\approx$ 1000 m/sec</td>
<td>Porous rock</td>
</tr>
<tr>
<td>0.1</td>
<td>$\approx$ 300 m/sec</td>
<td>Stiff soil</td>
</tr>
<tr>
<td>0.01</td>
<td>$\approx$ 100 m/sec</td>
<td>Soft soil</td>
</tr>
</tbody>
</table>

2.5 Boundary conditions for porous media

In a wave propagation problem, employing appropriate boundary conditions leads to specific solution. To establish boundary conditions for porous media, the solid-fluid interaction of the aggregate must be considered in addition to the elastic behavior of the solid material. Deresiewicz and Skalak (1963), Lovera (1987), and de la Cruz and Spanos (1989) proposed boundary conditions for two different fluid-saturated porous media in contact. These conditions were derived starting with two principles – the conservation of mass and the continuity of momentum.
The effects of a free plane boundary on wave propagation in a poroelastic half-space have been investigated by Deresiewicz (1960), where the boundary conditions for the free plane boundary include: (1) zero stresses of the solid-skeleton in both normal and tangential directions of the plane; and (2) zero pore fluid pressure on the plane. In this case, the pores are open to the air and the pore fluid can be drained from the solid-fluid aggregate.

The case of sealed-boundary in which pore fluid is trapped inside the aggregate is also important to study for geotechnical engineering applications, because the pore fluid pressure may build up and induce liquefaction. According to the work of Deresiewicz and Skalak (1963), the boundaries for a free surface can be illustrated by the simplified diagrams in Fig. 2.3. Fig. 2.3(a) represents the open-boundary case in which the pore fluid is not restricted. In Fig. 2.3(b), the boundary is sealed with a thin membrane so that the pore fluid is restricted inside the aggregate. To study the effects of sealed-boundary, the boundary conditions given by Deresiewicz and Skalak (1963) were adopted and compared with the ones for open-boundary case in Table 2.4.

![Fig. 2.3](image-url)  
**Fig. 2.3** Simplified diagrams of the porous-air boundaries: (a) open-boundary; (b) sealed-boundary. In the porous medium, the gray area represents the solid-skeleton and the white parts are the pores.

**Table 2.4** Boundary conditions for open-boundary and sealed-boundary.

<table>
<thead>
<tr>
<th>Open-boundary</th>
<th>Sealed-boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{nn} = 0$</td>
<td>$\tau_{nn} + \sigma = 0$</td>
</tr>
<tr>
<td>$\tau_{nt} = 0$</td>
<td>$\tau_{nt} = 0$</td>
</tr>
<tr>
<td>$\sigma = 0$</td>
<td>$u_n - U_n = 0$</td>
</tr>
</tbody>
</table>

Note: subscript $n$: normal, $t$: tangential vector on the boundary surface
3 Response of a fluid-saturated porous half-space to an incident plane P-wave

3.1 The model

Consider a fluid-saturated porous half-space subjected to a plane P-wave with incidence angle $\theta_{\alpha 1}$ as shown in Fig. 3.1. We assume the wave has harmonic time dependence and, for brevity, we omit the $e^{-i\omega t}$ terms.

The incident P-wave potential is
\[
\phi^i = a_0 e^{ik_{\alpha 1} (x \sin \theta_{\alpha 1} + y \cos \theta_{\alpha 1})}
\]  
(3-1a)

and the reflected wave potentials are
\[
\phi^r = a_1 e^{ik_{\alpha 1} (x \sin \theta_{\alpha 1} + y \cos \theta_{\alpha 1})}
\]  
(3-1b)
\[
\phi^r = a_2 e^{ik_{\alpha 2} (y \sin \theta_{\alpha 2} - x \cos \theta_{\alpha 2})}
\]  
(3-1c)
\[
\psi^r = b e^{i\beta (x \sin \theta_{\beta} + y \cos \theta_{\beta})}
\]  
(3-1d)

The boundary conditions will be used to determine the unknown amplitude coefficients. To investigate the ground responses for drained and undrained boundaries, both open and sealed cases will be considered in the following.
3.1.1 Open-boundary case

This case has been studied by Deresiewicz (1960). Applying the conditions for open-boundary in Table 2.4, we can state the boundary conditions in Cartesian coordinates as

\[
\begin{bmatrix}
\tau_{yy} \\
\tau_{xy} \\
\sigma
\end{bmatrix}
\bigg|_{y=0}
= \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\] (3-2)

Substituting Eq.(3-1) into Eq.(3-2), the equations can be simplified as in Eq.(3-3), where the notation used is summarized in Appendix A.

\[
\begin{bmatrix}
G_{11,1} & G_{12} & -G_{12} \\
-G_{21,1} & -G_{21,2} & G_{22} \\
G_{61,1} & G_{61,2} & 0
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
b
\end{bmatrix}
= -a_0
\begin{bmatrix}
G_{11,1} \\
G_{21,1} \\
G_{61,1}
\end{bmatrix}
\] (3-3)

From Eq.(3-3), the three real amplitude coefficients are obtained.

3.1.2 Sealed-boundary case

Similar to the solution of the open-boundary case, the following boundary conditions are obtained from Table 2.4.

\[
\begin{bmatrix}
\tau_{yy} + \sigma \\
\tau_{xy} \\
U_y - U_y
\end{bmatrix}
\bigg|_{y=0}
= \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\] (3-4)

After substituting Eq.(3-1) into Eq.(3-4), the unknown coefficients can be determined from

\[
\begin{bmatrix}
G_{11,1} + G_{61,1} & G_{11,2} + G_{61,2} & -G_{12} \\
-G_{21,1} & -G_{21,2} & G_{22} \\
-(1-f_1)G_{41,1} & -(1-f_2)G_{41,2} & (1-f_3)G_{42}
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
b
\end{bmatrix}
= -a_0
\begin{bmatrix}
G_{11,1} + G_{61,1} \\
G_{21,1} \\
(1-f_1)G_{41,1}
\end{bmatrix}
\] (3-5)

3.1.3 Surface response

The following responses of the solid-skeleton at the half-space surface are calculated.
Displacements

The displacement responses of the fluid-saturated porous medium at the ground surface are obtained as

\[
\begin{bmatrix}
    u_x \\
    u_y
\end{bmatrix}
|_{y=0} = \begin{bmatrix}
    G_{31,1} & G_{31,2} & -G_{32} \\
    G_{41,1} & G_{41,2} & G_{42}
\end{bmatrix}
\begin{bmatrix}
    a_0 \\
    a_1 \\
    a_2 \\
b
\end{bmatrix} e^{ik_0x}
\] (3-6)

where \( k_0 = k_{a1} \sin \theta_{a1} = k_{a2} \sin \theta_{a2} = k_{\beta} \sin \theta_{\beta} \) is the apparent wavenumber on the half-space surface.

Surface strains

Trifunac (1979) and Lee (1990) calculated the surface strains from the derivatives of displacements

\[
\begin{bmatrix}
    \gamma_x \\
    \gamma_y
\end{bmatrix}
|_{y=0} = \begin{bmatrix}
    \partial u_x / \partial x \\
    \partial u_y / \partial y
\end{bmatrix}
\] (3-7)

Substituting Eq. (3-6) into Eq.(3-7), we obtain

\[
\begin{bmatrix}
    \gamma_x \\
    \gamma_y
\end{bmatrix}
|_{y=0} = \begin{bmatrix}
    G_{71,1} & G_{71,2} & -G_{72} \\
    G_{81,1} & G_{81,2} & G_{82}
\end{bmatrix}
\begin{bmatrix}
    a_0 \\
    a_1 \\
    a_2 \\
b
\end{bmatrix} e^{ik_0x}
\] (3-8)

Surface rotation

Trifunac (1982) calculated the rotation (rocking) on the ground surface from

\[
\psi_{xy} = \frac{1}{2} \left( \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \right) |_{y=0} = -\frac{1}{2} \left( \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} \right) |_{y=0}
\]

\[
= \frac{1}{2} k_{\beta}^2 e^{ik_0x}
\]

It is clear that \( \psi_{xy} \) is associated only with SV-wave motion. Therefore, the normalized rotation can be defined as (Trifunac, 1982; Lee and Trifunac, 1987)

\[
\xi_{xy} = \frac{\psi_{xy}(\lambda_{\beta} / \pi)}{ik_{a1} e^{ik_0x}} = -b(k_{\beta} / k_{a1}) e^{i\pi/2}
\] (3-10)
where $\lambda_\beta$ is the wavelength of SV-waves, and $ik_{11}e^{ika_{11}}$ is the displacement induced by an incident P-wave.

From Eq. (3-10), it is seen that the rocking is phase-shifted relative to the incident motion by $\pi/2$.

Stresses

The stresses on the ground surface can be obtained as

$$
\begin{bmatrix}
\tau_{yy} \\
\tau_{xy} \\
\tau_{xx} \\
\sigma_y
\end{bmatrix}
= \mu
\begin{bmatrix}
G_{11,1} & G_{11,1} & G_{11,2} & -G_{12} \\
-G_{21,1} & -G_{21,1} & -G_{21,2} & G_{22} \\
G_{51,1} & G_{51,1} & G_{51,2} & -G_{52} \\
G_{61,1} & G_{61,1} & G_{61,2} & 0
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
a_2 \\
b
\end{bmatrix}
e^{ika_{11}}
$$

(3-11)

3.2 An example of surface response analysis

A fluid-saturated porous half-space subjected to an incident P-wave with unit amplitude is investigated for surface response. We assume porous medium with 30% porosity consisting of spherical solid particles ($\tau_r = 0.5$, and $\rho_s/\rho_f = 2.7$). The relations among material constants in Table 2.1 can be used to calculate the surface response for various Poisson’s ratios of the solid-skeleton and $\mu/K_f$ ratios.

The surface response for the open boundary and sealed boundary cases are discussed in sections 3.3 and 3.4, respectively. Fig. 3.2 through Fig. 3.5 and Fig. 3.7 through Fig. 3.10 show the amplitude coefficients, displacements, surface strains, rotations and stresses versus incident angle for various solid materials with different Poisson’s ratios for open-boundary cases and for sealed-boundary cases, respectively. Fig. 3.6 and Fig. 3.11 show comparisons for varying solid stiffness with the same Poisson’s ratio, $\nu_s = 0.25$, for open-boundary case and sealed-boundary case, respectively. Fig. 3.12 illustrates the effects of open-boundary and sealed-boundary for the porous material with $\mu/K_f$ ratio = 0.1 and $\nu_s = 0.25$.

It is noted that the displacements and rotations are normalized by a factor of $k_{11}$ which is the displacement intensity of incident P-wave. The surface strains are
normalized by a factor of $k_{a_1}^2$ which is the strain intensity of incident P-wave. The stresses are normalized by a factor of $k_p^2$ which is the stress intensity induced by incident P-wave (Pao and Mao, 1971). All the above complex amplitudes are plotted as absolute values.

3.3 Results and discussion for open-boundary case

3.3.1 Amplitude coefficients

From parts (a), (b) and (c) in Fig. 3.2 through Fig. 3.5, it is seen that the amplitude coefficients of the slow P-waves are much smaller ($10^{-2}$), than those of the fast P-waves. The coefficient variations with respect to Poisson’s ratio are significant for the solid-dominated case (large $\mu/K_f$ ratio), and diminish with decreasing solid stiffness. Fig. 3.6(a), (b) and (c) show that the absolute values of $a_1$ and $b$ for an elastic medium are always larger than the ones for the porous medium. The effects of the incident angle decrease with decreasing solid stiffness.

3.3.2 Displacements

From parts (d) and (e) in Fig. 3.2 through Fig. 3.5, it is seen that the effects of variations in Poisson’s ratio are significant for the displacement amplitudes in the solid-dominated case. These effects diminish with decreasing solid stiffness. The peak displacement $u_x$ has maximum value 1.89 for $\mu/K_f=10$ and $\nu=0.1$, and decreases with decreasing $\mu/K_f$ ratio. For $\mu/K_f=0.01$, the peak displacement $u_x$ reduces to 0.11. The displacement $u_y$, always has amplitudes equal to two for vertical incidence and zero for horizontal incidence, which is same as in the case of an elastic medium. Fig. 3.6(d) and (e) show how the amplitudes of $u_x$ decrease with decreasing solid stiffness and are always smaller than the amplitudes for an elastic medium.

3.3.3 Surface strains

From parts (f) and (g) in Fig. 3.2 through Fig. 3.5, it is seen that the surface strain variations for different Poisson’s ratios are significant for the solid-dominated case, and reduce with decreasing solid stiffness. The peak surface strain $\gamma_s$ has maximum value of 1.78 for $\mu/K_f=10$ and $\nu_s=0.1$, and decreases with decreasing $\mu/K_f$ ratio. For $\mu/K_f=$
0.01, the peak $\gamma$ reduces to 0.083. The peak surface strain $\gamma$ has maximum value of 0.56 for $\mu/K_f = 10$ and $\nu_s = 0.4$, and decreases with decreasing $\mu/K_f$ ratio. For $\mu/K_f = 0.01$, the peak of $\gamma$ reduces to 0.055. Fig. 3.6(f) and (g) illustrate how the surface strains decrease with decreasing solid stiffness and are always smaller than the corresponding amplitudes for elastic medium.

### 3.3.4 Rotations

Parts (h) and (i) in Fig. 3.2 through Fig. 3.5 show the normalized rotation with respect to horizontal displacement and vertical displacement versus incident angle, respectively. It is seen that the dependence of rotation on Poisson’s ratio is significant for solid-dominated case, and reduces with decreasing solid stiffness. The $\xi_{xy}/u_x$ ratio always equals 1 for vertical incidence and decreases with increasing incident angle. When $\mu/K_f = 10$ and $\nu = 0.1$, $\xi_{xy}/u_x$ has a minimum equal to 0.26 for horizontal incidence. For $\mu/K_f = 0.01$, $\xi_{xy}/u_x$ remains close to 1, which means that the incident angle does not affect the ratio of $\xi_{xy}/u_x$. The peak rotation $\xi_{xy}/u_y$ has a maximum equal to 1.263 when $\nu_s = 0.1$ and $\mu/K_f = 10$, and decreases with decreasing $\mu/K_f$ ratio. For $\mu/K_f = 0.01$, the peak $\xi_{xy}/u_y$ reduces to 0.108. Fig. 3.6(h) and (i) show how the rotations $\xi_{xy}/u_x$ approach 1 and $\xi_{xy}/u_y$ tend to 0 with decreasing solid stiffness.

### 3.3.5 Stresses

From parts (j) in Fig. 3.2 through Fig. 3.5, it is seen that the stress dependence on Poisson’s ratios is significant for a solid-dominated case, and reduces with decreasing solid stiffness. The peak stress $\tau_{xx}$ has maximum value of 1.5 as $\mu/K_f = 10$ and $\nu_s = 0.1$, and decreases with decreasing $\mu/K_f$ ratio. For $\mu/K_f = 0.01$, the peak $\tau_{xx}$ reduces to $8.0 \times 10^{-4}$. Fig. 3.6(j) illustrates how the amplitudes of $\tau_{xx}$ decrease with decreasing solid stiffness, and are always lower than the amplitude for the elastic medium.
Fig. 3.2 (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude coefficient $b_1$; (d) Displacement $u_x$; and (e) Displacement $u_y$, versus incident angle, to an incident P-wave for $R = 0.3$, $\mu / \lambda = 10$, open boundary case.
Fig. 3.2  (f) Surface strain $\gamma_x$; (g) Surface strain $\gamma_y$; (h) Rotation $\xi_{xy}/u_y$; (i) Rotation $\xi_{xy}/u_y$ versus incident angle, to an incident P-wave for $\hat{n}=0.3$, $\mu/K_r=10$, open boundary case.
Fig. 3.2 (j) Stress $\tau_{xx}$ versus incident angle, to an incident P-wave for $\tilde{n} = 0.3$, $\mu / K_r = 10$, open boundary case.

Fig. 3.3 (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude $b$ versus incident angle, to an incident P-wave for $\tilde{n} = 0.3$, $\mu / K_r = 1$, open boundary case.
Fig. 3.3  (d) Displacement $u_x$ ;  (e) Displacement $u_y$ ;  (f) Surface strain $\gamma_x$ ;  (g) Surface strain $\gamma_y$ versus incident angle, to an incident P-wave for $\bar{n} = 0.3, \mu / K_r = 1$, open boundary case.
Fig. 3.3 (h) Rotation $\xi_{xy}/u_x$; (i) Rotation $\xi_{xy}/u_y$; (j) Stress $\tau_{xx}$ versus incident angle, to an incident P-wave for $\hat{n} = 0.3$, $\mu / K_s = 1$, open boundary case.
Fig. 3.4 (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude coefficient $b$; (d) Displacement $u_x$; and (e) Displacement $u_y$ versus incident angle, to an incident P-wave for $\bar{\mu} = 0.3$, $\mu/K_t = 0.1$, open boundary case.
Fig. 3.4 (f) Surface strain $\gamma_s$; (g) Surface strain $\gamma_r$; (h) Rotation $\delta_y$ / $u_y$; (i) Rotation $\delta_y$ / $u_r$, versus incident angle, to an incident P-wave for $\tilde{\eta} = 0.3$.
Fig. 3.4  (j) Stress $\tau_{xx}$ versus incident angle, to an incident P-wave for $\bar{n} = 0.3, \mu / K_r = 0.1$, open boundary case.

Fig. 3.5  (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude $b$ versus incident angle, to an incident P-wave for $\bar{n} = 0.3, \mu / K_r = 0.01$, open boundary case.
Fig. 3.5 (d) Displacement $u_x$; (e) Displacement $u_y$; (f) Surface strain $\gamma_x$; (g) Surface strain $\gamma_y$ versus incident angle, to an incident P-wave for $\hat{n} = 0.3$, $\mu / K_f = 0.01$, open boundary case.
Fig. 3.5 (b) Rotation $\xi/\eta$; (i) Rotation $\xi_{x,0}/\eta$; (j) Stress $\tau_{x,0}$ versus incident angle, to an incident P-wave for $\eta = 0.3, \mu/\nu = 0.01$ open boundary case.
Fig. 3.6 (a) Amplitude coefficient $a$; (b) Amplitude coefficient $b$; (c) Amplitude $u_1$; (d) Displacement $u_2$; and (e) Displacement $u_3$ versus incident angle, to an incident P-wave for $n = 0.3$, $v = 0.25$, with different $\mu/\lambda$. 

$V = 0.25$

- $\mu/K = 0.01$
- $\mu/K = 0.1$
- $\mu/K = 1$
- $\mu/K = 10$

$V = 0.3$

- $\mu/K = 0.01$
- $\mu/K = 0.1$
- $\mu/K = 1$
- $\mu/K = 10$

Elastic
Fig. 3.6: (a) Surface strain $\gamma_1$; (b) Surface strain $\gamma_2$; (c) Surface strain $\gamma_3$; (d) Rotation $\theta_1$; (e) Rotation $\theta_2$; (f) Rotation $\theta_3$; (g) open boundary case with different $\mu / K$. 

$V = 0.25$, $V = 0.5$, $V = 0.75$, $V = 1.0$.
Fig. 3.6 (1) Stress $\tau_{xx}$ versus incident angle, to an incident P-wave for $n=0.3, v_s=0.25$, open boundary case, with different $\mu/K_s$. 

\[ v_s = 0.25, \mu/K_s = 0.1 \]

\[ v_s = 0.25, \mu/K_s = 1.0 \]

\[ v_s = 0.25, \mu/K_s = 10.0 \]

Incident Angle (degree)
3.4 Results and discussion for sealed-boundary case

3.4.1 Amplitude coefficients

From parts (a), (b) and (c) in Fig. 3.7 through Fig. 3.10, it is seen that the amplitude coefficients of the slow P-waves are much smaller ($10^{-2}$) than those of the fast P-waves. The coefficient variations with respect to Poisson’s ratio are significant for the solid-dominated case, and reduce with decreasing solid stiffness. It is noted that the coefficient $a_1$ does not equal to zero for vertical incidence. Fig. 3.11(a), (b) and (c) show that the peak absolute values of $a_1$ and $b$ for elastic medium are always larger than the ones for the porous medium. The effects of incident angle decrease with decreasing solid stiffness.

3.4.2 Displacements

From parts (d) and (e) in Fig. 3.7 through Fig. 3.10, it is seen that the displacement variations caused by Poisson’s ratios are significant for the solid-dominated case, and reduce with decreasing solid stiffness. The peak displacement $u_x$ has maximum value 1.95 for $\mu/K_f = 10$ and $\nu_s = 0.1$, and decreases with decreasing $\mu/K_f$ ratio. For $\mu/K_f = 0.01$, the peak displacement $u_x$ reduces to 0.124. The peak displacement $u_y$ has maximum value equal to 2.36 for $\mu/K_f = 0.01$ and $\nu_s = 0.1$, and decreases with increasing of $\mu/K_f$ ratio. The peak of $u_y$ reduces to 1.96 for $\mu/K_f = 10$ and $\nu_s = 0.1$. Fig. 3.11(d) and (e) show how the amplitudes of $u_x$, decrease with decreasing solid stiffness, and are always smaller than the amplitudes for elastic medium. In contrast, for $u_y$ components, the peak amplitudes are larger than those for the elastic case, except for $\mu/K_f = 10$.

3.4.3 Surface strains

From parts (f) and (g) in Fig. 3.7 through Fig. 3.10, it is seen that the variations in surface strain with respect to Poisson’s ratio are significant for $\gamma_x$ components for solid-dominated case, and reduce with decreasing solid stiffness. The surface strain variations are always significant in $\gamma_y$ components. The peak surface strain $\gamma_x$ has the maximum value equal to 1.85 for $\mu/K_f = 10$ and $\nu_s = 0.1$, and decreases with decreasing $\mu/K_f$ ratio. For $\mu/K_f = 0.01$, the peak of $\gamma_x$ reduces to 0.096. It is found that the surface strain $\gamma_x$ is no longer equal to zero for vertically incident P-wave. The peak surface strain $\gamma_y$ has the
maximum value equal to 8.81 for $\mu/K_f = 0.01$ and $\nu_s = 0.1$, and decreases with increasing $\mu/K_f$ ratio. The peak $\gamma_y$ reduces to 0.543 for $\mu/K_f = 10$ and $\nu_s = 0.3$. Fig. 3.11(f) and (g) show how the amplitudes of $\gamma_s$ decrease with decreasing solid stiffness, and are always lower than the amplitudes for elastic medium. For $\gamma_y$ components, unlike for the elastic medium, the surface strains are not zero for vertical incidence. In the case of soft solid-skeleton, it is found that the peak surface strains $\gamma_y$ exceed the peak surface strains for elastic medium.

3.4.4 Rotations

From parts (h) and (i) in Fig. 3.7 through Fig. 3.10, it is seen that the variations in rotation in terms of Poisson’s ratios are significant for the solid-dominated case, and reduce with decreasing solid stiffness. The $\xi_{xy}/u_x$ ratio equals 0.993 for vertical incidence and decreases to 0.275 when $\nu_s = 0.1$ and $\mu/K_f = 10$. For $\mu/K_f = 0.01$, the $\xi_{xy}/u_x$ remains 1.03 which is larger than in the open-boundary and elastic cases. The peak rotation $\xi_{xy}/u_y$ has the maximum value equal to 1.263 when $\nu_s = 0.1$ and $\mu/K_f = 10$, and decreases with decreasing $\mu/K_f$ ratio. For $\mu/K_f = 0.01$, the peak of $\xi_{xy}/u_y$ reduces to 0.108. Fig. 3.11(h) and (i) show how the peak values of $\xi_{xy}/u_x$ vary for different solid stiffnesses and tend to be insensitive to the incident angle with decreasing solid stiffness.

3.4.5 Stresses

From parts (j) and (k) in Fig. 3.7 through Fig. 3.10, it is seen that the peak stress $\tau_{xx}$ has the maximum value equal to 1.576 for $\nu_s = 0.1$ and $\mu/K_f = 10$, and decreases with decreasing $\mu/K_f$ ratio. For $\mu/K_f = 0.01$, the peak of $\tau_{xx}$ reduces to $3.40 \times 10^{-2}$ for $\nu_s = 0.1$. The pore pressure $\sigma$ has the same amplitude as the stress $\tau_{yy}$, but with opposite direction, due to the nature of boundary conditions. The peak pore pressure has the maximum value $6.52 \times 10^{-2}$ when $\nu_s = 0.1$ and $\mu/K_f = 1$, and a minimum value equal to $2.74 \times 10^{-2}$ for $\mu/K_f = 0.01$ and $\nu_s = 0.4$. Fig. 3.11(j) shows how the peak stresses $\tau_{xx}$ decrease with decreasing solid stiffness and are always lower than the corresponding stresses for the elastic medium.
Fig. 3.7 (a) Amplitude coefficient $\alpha_1$; (b) Amplitude coefficient $\alpha_2$; (c) Amplitude coefficient $\beta$; (d) Displacement $u_1$; and (e) Displacement $u_2$ versus incident angle, to an incident P-wave for $\eta = 0.3$, $\mu_1/\mu = 10$, scaled boundary case.
Fig. 3.7 (f) Surface strain $\gamma_2$; (g) Surface strain $\gamma_3$; (h) Rotation $\theta_2 / \theta_0$; (i) Rotation $\theta_3 / \theta_0$; (j) versus incident angle, to an incident P-wave for $\mu = 0.3$. $\mu_t K_0 = 10$, scaled boundary case.
Fig. 3.7 (j) Stress $\tau_{xx}$; (k) Pore pressure $\sigma$ versus incident angle, to an incident P-wave, for $\hat{n} = 0.3$, $\mu/K_\mu = 10$, sealed boundary case.

Fig. 3.8 (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude $b$ versus incident angle, to an incident P-wave for $\hat{n} = 0.3$, $\mu/K_\mu = 1$, sealed boundary case.
Fig. 3.8 (d) Displacement $u_x$; (e) Displacement $u_y$; (f) Surface strain $\gamma_x$; (g) Surface strain $\gamma_y$; versus incident angle to an incident P-wave for $\theta_0 = 0.3$, $\mu/K_r = 1$, sealed boundary case.
Fig. 3.8 (h) Rotation $\xi_{xy}/u_x$; (i) Rotation $\xi_{xx}/u_y$; (j) Stress $\tau_{zz}$; (k) Pore pressure $\sigma$ versus incident angle, to an incident P-wave for $\hat{h} = 0.3$, $\mu/K_f = 1$, scaled boundary case.
Fig. 3.9 (a) Amplitude coefficient $a_i$; (b) Amplitude coefficient $a_j$; (c) Amplitude coefficient $a_k$; (d) Displacement $u_i$; and (e) Displacement $u_j$. versus incident angle, to an incident P-wave for $\bar{\mu} = 0.3$, $\mu_c = 0.1$, scaled boundary case.
Fig. 3.9 (f) Surface strain $\gamma_x$; (g) Surface strain $\gamma_y$; (h) Rotation $\xi_{xy}/u_y$; (i) Rotation $\xi_{xy}/u_y$ versus incident angle, to an incident P-wave for $\hat{n} = 0.3$, $\mu/K_f = 0.1$, sealed boundary case.
Fig. 3.9 (j) Stress $\tau_{xx}$; (k) Pore pressure $\sigma$ versus incident angle, to an incident P-wave for $\mu = 0.3$, $\mu / K_r = 0.1$, sealed boundary case.

Fig. 3.10 (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude $b$ versus incident angle, to an incident P-wave for $\mu = 0.3$, $\mu / K_r = 0.01$, sealed boundary case.
Fig. 3.10 (d) Displacement $u_y$; (f) Surface strain $\gamma_y$; (g) Surface strain $\gamma_x$; (i) Incident angle, to an incident P-wave for $\bar{n} = 0.3, \mu = K_f = 0.01$, sealed boundary case.
Fig. 3.10 (b) Rotation $\tilde{\phi}$ / $u_y$; (i) Stress $t_{xy}$; (k) Pore pressure $\sigma$ versus incident angle, to an incident P-wave for $\bar{n} = 0.3$. $\mu / K = 0.01$, sealed boundary case.
Fig. 3.11 (a) Amplitude coefficient $a_x$; (b) Amplitude coefficient $a_y$; (c) Amplitude coefficient $a_z$; (d) Displacement $u_x$; and (e) Displacement $u_y$. versus incident angle, for an incident P-wave for $\theta = 0.3$, $V_r = 0.25$, $K_r = 0.01$, $\mu/K_r = 1$, $\mu/K_r = 10$, and $\mu/K_r = 0.1$.
Fig. 3.11 (f) Surface strain $\gamma_s$; (g) Surface strain $\gamma_r$; (h) Rotation $\xi_{sx}/u_x$; (i) Rotation $\xi_{xy}/u_y$ versus incident angle, to an incident P-wave for $\hat{n} = 0.3$, $v_s = 0.25$, sealed boundary case, with different $\mu/K_r$. 
Fig. 3.11 (j) Stress $\tau_{ex}$; (k) Pore pressure $\sigma$ versus incident angle, to an incident P-wave for $\bar{f} = 0.3$, $v_s = 0.25$, sealed boundary case, with different $\mu / K_r$. 
3.5 Comparison of open-boundary and sealed-boundary cases

Fig. 3.12 shows the displacements, surface strains, rotations, and stresses for elastic medium, for open-boundary and sealed boundary cases in porous medium for \( \nu_s = 0.25 \) and \( \mu / K_f = 0.1 \) (simulating soils). The peak values of displacement \( u_x \), surface strains \( \gamma_x \), \( \gamma_y \), rotation \( \xi_{xy} / u_y \) and stress \( \tau_{xx} \) for elastic medium are larger than those in porous medium.

Generally, the response for sealed-boundary case is larger than the response for open-boundary case. For sealed-boundary case, the surface strain \( \gamma_y \) and stress \( \tau_{xx} \) are no longer equal to zero for vertically incident P-wave. For soft solid cases, it is found that the peak amplitudes of \( \gamma_y \) and \( \tau_{xx} \) are induced by vertical incidence.

The peak value of \( u_y \) for sealed-boundary case exceeds the peak value, equal to two, for the elastic medium. In this case, the amplification of sealed-boundary case is larger than for the elastic medium in the vertical displacement component.

From Fig. 3.12(f) and the parts (i) in Fig. 3.2 through Fig. 3.11, it is found that the ratio of rotation to vertical displacement described by Eq. (3-12) is consistent with the case for elastic medium (Trifunac, 1982).

\[
\frac{\xi_{xy}}{u_y} = 2 \frac{k_{a1} \sin \theta_{a1}}{k_{\beta}}
\]  

(3-12)

From Eq. (3-12), it is seen that the ratio of rotation to vertical displacement is associated with the ratio of wave number \( k_{a1} \) to \( k_{\beta} \) and the incident angle for P-wave.
Fig. 3.12 Comparison of open boundary and sealed boundary cases of porous media, for \( \hat{n} = 0.3, \mu / K_c = 0.1, \) and elastic medium, \( v_r = 0.25. \)

(a) Displacement \( u_x \); (b) Displacement \( u_y \); (c) Surface strain \( \gamma_x \); (d) Surface strain \( \gamma_y \); versus incident angle, to an incident P-wave.
Fig. 3.12 Comparison of open boundary and sealed boundary cases of porous media, for $\hat{n} = 0.3$, $\mu / K_r = 0.1$, and elastic medium, $\nu_s = 0.25$.

(e) Rotation $\xi_{xy} / u_y$; (f) Rotation $\xi_{xy} / u_y$; (g) Stress $\tau_{xx}$ versus incident angle, to an incident P-wave.
3.6 Conclusions

The wave propagation amplitudes in fluid-saturated porous medium are influenced significantly by the stiffness and Poisson’s ratio of the solid-skeleton, and by the boundary drainage.

The solid stiffness dominates the amplitudes of elastic waves in porous medium. For solid-dominated case (large $\mu / K_f$ ratio), the porous medium behaves like elastic solid medium. The porous medium behaves like a fluid medium for fluid-dominated case (small $\mu / K_f$ ratio). Generally, the variations caused by the Poisson’s ratio are significant for the solid-dominated case, but are less significant with decreasing solid stiffness.

For open (drained) boundary, the peak amplitudes of displacements, surface strains, rotations, and stresses are always smaller than the amplitudes for an elastic medium. In this case, the fluid plays a passive role in the poroelastic system and reduces the dynamic response of the solid-skeleton.

For sealed (undrained) boundary, the first remarkable effect is that the peak values of normal displacement $u_y$ are no longer equal to two as in the elastic and the drained-boundary cases. The displacement responses for most $\mu / K_f$ ratios are amplified more than in the case of the elastic medium. Secondly, the surface strains $\gamma_y$ are no longer zero for vertical incidence, and the peak values of $\gamma_y$ exceed the values for elastic medium when the solid-skeleton is soft.

The pore pressures are not zero, and have the same amplitudes as the normal stresses $\tau_{yy}$ for the sealed boundary case. For soft and unconsolidated solids, it is found that the pore pressures exceed the stresses $\tau_{xx}$ (e.g. $\mu / K_f \leq 0.1$ and $\nu_s \geq 0.3$) although their amplitudes are small. In this case, if the solid-skeleton is formed with cohesionless granular particles, the initiation of liquefaction is possible because the pore fluid pressure may cause loss of effective stress between the particles near the ground surface.
4 Response of a fluid-saturated porous half-space to an incident plane SV-wave

4.1 The model

Consider a fluid-saturated porous half-space subjected to a plane SV-wave with incidence angle $\theta_\beta$ as shown in Fig. 4.1. We assume the wave has harmonic time dependence and, for brevity, we omit the $e^{-i\omega t}$ terms.

![Fluid-saturated porous half-space subjected to an incident SV-wave.](image)

The incident SV-wave potential is

$$\psi^i = b_0 e^{ik_x (x \sin \theta_\beta + y \cos \theta_\beta)}$$

and the reflected wave potentials are

$$\phi^r_1 = a_1 e^{ik_{\kappa 1} (x \sin \theta_{\kappa 1} - y \cos \theta_{\kappa 1})}$$

$$\phi^r_2 = a_2 e^{ik_{\kappa 2} (x \sin \theta_{\kappa 2} - y \cos \theta_{\kappa 2})}$$

$$\psi^r = b e^{ik_x (x \sin \theta_\beta - y \cos \theta_\beta)}$$

The apparent velocity along the half-space surface is

$$V_0 = \frac{V_\beta}{\sin \theta_\beta} = \frac{V_{\alpha 1}}{\sin \theta_{\alpha 1}} = \frac{V_{\alpha 2}}{\sin \theta_{\alpha 2}}$$
From Table 2.2, it is seen that the fast P-wave is always faster than S-wave. If the incident angle is beyond the critical angle $\theta_{cr}$, the reflected angle of the fast P-waves becomes complex, because $\sin \theta_{a1} = (V_{a1}/V_\beta) \sin \theta_\beta > 1$. The critical angle for the fast P-wave (the first critical angle) is determined from

$$\theta_{cr} = \sin^{-1} (V_\beta/V_{a1})$$

(4-3)

It is seen that the critical angle depends on the ratio of P-wave and S-wave velocities. A critical angle for the slow P-wave (the second critical angle) does not usually exist, because $V_{a2}/V_\beta > 1$ only in a soft, unconsolidated solid-skeleton (see Table 2.2, $\mu/K_f \leq 0.1$ and $\nu = 0.4$). We rewrite Eq.(4-1) as follows.

$$\psi^r = b_0 e^{i k_0 + \nu \beta y}$$

(4-4a)

$$\phi_1^r = a_1 e^{i k_0 x - \nu a_1 y}$$

(4-4b)

$$\phi_2^r = a_2 e^{i k_0 x - \nu a_2 y}$$

(4-4c)

$$\psi^r = b e^{i k_0 x - \nu \beta y},$$

(4-4d)

where $k_0 = k_\beta \sin \theta_\beta = k_{a1} \sin \theta_{a1} = k_{a2} \sin \theta_{a2}$ is the apparent wave number, and

$$\nu_\beta = i k_0 \cot \theta_\beta = i \sqrt{k_\beta^2 - k_0^2}$$

(4-5a)

$$\nu_{a1} = i k_0 \cot \theta_{a1} = \begin{cases} 
  i \sqrt{k_{a1}^2 - k_0^2} & \text{when } V_0 \geq V_{a1} \quad (\theta_\beta \leq \theta_{cr1}) \\
  -\sqrt{k_0^2 - k_{a1}^2} & \text{when } V_0 < V_{a1} \quad (\theta_{cr1} < \theta_\beta)
\end{cases}$$

(4-5b)

$$\nu_{a2} = i k_0 \cot \theta_{a2} = \begin{cases} 
  i \sqrt{k_{a2}^2 - k_0^2} & \text{when } V_0 \geq V_{a2} \quad (\theta_\beta \leq \theta_{cr2}) \\
  -\sqrt{k_0^2 - k_{a2}^2} & \text{when } V_0 < V_{a2} \quad (\theta_{cr2} < \theta_\beta)
\end{cases}$$

(4-5c)

The value of $\nu_\beta$ is imaginary for $0 \leq \theta_\beta \leq 90^\circ$, $k_0 \leq k_\beta$. The value of $\nu_{a1}$ is imaginary when $\theta_\beta \leq \theta_{cr1}$, and the potential remains as a harmonic wave. The value of $\nu_{a1}$ becomes a negative real number when $\theta_{cr1} < \theta_\beta$, and the potential describes a surface wave decaying exponentially with depth ($y \leq 0$). Similar conditions also hold for $\nu_{a2}$.

The boundary conditions will be used to determine the unknown amplitude coefficients. To investigate the ground responses for drained and undrained boundaries, both open and sealed boundaries will be considered in the following.
4.1.1 Open-boundary case

This case has been studied by Deresiewicz (1960). Applying the conditions for open-boundary in Table 2.4, we obtain the boundary conditions in Cartesian coordinates as

\[
\begin{align*}
\begin{cases}
\tau_{yy} \\
\tau_{xy} \\
\sigma_y
\end{cases}
= \begin{cases}
0 \\
0 \\
0
\end{cases}
\quad y=0
\end{align*}
\tag{4-6}
\]

Substituting Eq.(4-4) into Eq.(4-6), the equations can be simplified as in Eq.(4-7), where the notation used is summarized in Appendix A.

\[
\begin{pmatrix}
G_{11,1}^* & G_{11,2}^* & -G_{12}^* \\
-G_{21,1}^* & G_{21,2}^* & G_{22}^* \\
G_{61,1}^* & G_{61,2}^* & 0
\end{pmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
b
\end{bmatrix}
= -b_0
\begin{bmatrix}
G_{12}^* \\
G_{22}^* \\
0
\end{bmatrix}
\tag{4-7}
\]

The three amplitude coefficients are determined as

\[
\begin{bmatrix}
a_1 / b_0 \\
a_2 / b_0 \\
b / b_0
\end{bmatrix}
= \frac{1}{\det[G^*]}
\begin{bmatrix}
c_1 \\
c_2 \\
c_3
\end{bmatrix}
\tag{4-8}
\]

where

\[
c_1 = 4ik_0k_{a2}S_{a2}(2k_0^2 - k_\beta^2)v_\beta
\tag{4-9a}
\]

\[
c_2 = -4ik_0k_{a1}S_{a1}(2k_0^2 - k_\beta^2)v_\beta
\tag{4-9b}
\]

\[
c_3 = 4k_0^2k_{a1}^2S_{a1}v_{a2}v_\beta - 4k_0^2k_{a2}^2S_{a2}v_{a1}v_\beta
\tag{4-9c}
\]

\[
+ (2k_0^2 - k_\beta^2)[2k_0^2(k_{a1}^2S_1 - k_{a2}^2S_2) + k_{a1}^2k_{a2}^2(M_1S_2 - M_2S_1)]
\]

\[
\det[G^*] = 4k_0^2k_{a1}^2S_{a1}v_{a2}v_\beta - 4k_0^2k_{a2}^2S_{a2}v_{a1}v_\beta
\tag{4-9d}
\]

\[
- (2k_0^2 - k_\beta^2)[2k_0^2(k_{a1}^2S_1 - k_{a2}^2S_2) + k_{a1}^2k_{a2}^2(M_1S_2 - M_2S_1)]
\]

with \(M_j, S_j (j = 1, 2)\) defined in Appendix A.

In Eq.(4-8) and Eq.(4-9), the amplitude coefficients are real for incident angle \(\theta_\beta < \theta_{\text{cr}}\). From Eq.(4-9a) and Eq.(4-9b), it is seen that no P-waves are reflected when \(2k_0^2 - k_\beta^2 = 0\), in which case, the incident angle \(\theta_\beta\) equals 45°. This result is consistent with the elastic solid media (Achenbach, 1973). From Eq.(4-9c) and Eq.(4-9d), the ratio
of $|b/b_0| = 1$ exits in the case of incident SV-wave beyond the critical angle of the slow P-wave (both $\nu_{\alpha 1}$ and $\nu_{\alpha 2}$ are real) in a soft, unconsolidated solid-skeleton ($V_{\alpha 2}/V_\beta > 1$).

### 4.1.2 Sealed-boundary case

The following boundary conditions for sealed-boundary case are obtained from Table 2.4.

$$\begin{bmatrix} \tau_{yy} + \sigma \\ \tau_{xy} \\ u_y - U_y \end{bmatrix} \bigg|_{y=0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{4-10}$$

Substituting Eq.(4-4) into Eq.(4-10), the equations can be simplified to Eq.(4-11), where the notation used is summarized in Appendix A.

$$\begin{bmatrix} G_{11,1}^* + G_{61,1}^* \\ -G_{21,1}^* \\ -(1 - f_1)G_{41,1}^* \end{bmatrix} + \begin{bmatrix} G_{11,2}^* + G_{61,2}^* - G_{12}^* \\ -G_{21,2}^* \\ (1 - f_2)G_{41,2}^* - (1 - f_3)G_{42}^* \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ b \end{bmatrix} = -b_0 \begin{bmatrix} G_{12}^* \\ G_{22}^* \\ (1 - f_3)G_{42}^* \end{bmatrix} \tag{4-11}$$

The three amplitude coefficients are determined from

$$\begin{bmatrix} a_1/b_0 \\ a_2/b_0 \\ b/b_0 \end{bmatrix} = \frac{1}{\det[G^*]} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \tag{4-12}$$

where

$$c_1 = -4ik_0[2(2 - f_2 - f_3)k_0^2 - (1 - f_3)k_\beta^2]v_{a2}v_\beta$$ \hspace{1cm} (4-13a)

$$c_2 = 4ik_0[2(2 - f_1 - f_3)k_0^2 - (1 - f_1)k_\beta^2]v_{a1}v_\beta$$ \hspace{1cm} (4-13b)

$$c_3 = -[2(2 - f_1 - f_3)k_0^2 - (1 - f_1)k_\beta^2][2k_0^2 - (M_2 + S_2)k_{a2}^2]v_{a1}$$
$$+ [2(2 - f_2 - f_3)k_0^2 - (1 - f_3)k_\beta^2][2k_0^2 - (M_1 + S_1)k_{a1}^2]v_{a2}$$
$$+ 4(f_1 - f_2)k_0^2v_{a1}v_{a2}v_\beta$$ \hspace{1cm} (4-13c)

$$\det[G^*] = [2(2 - f_1 - f_3)k_0^2 - (1 - f_1)k_\beta^2][2k_0^2 - (M_2 + S_2)k_{a2}^2]v_{a1}$$
$$- [2(2 - f_2 - f_3)k_0^2 - (1 - f_3)k_\beta^2][2k_0^2 - (M_1 + S_1)k_{a1}^2]v_{a2}$$
$$+ 4(f_1 - f_2)k_0^2v_{a1}v_{a2}v_\beta \tag{4-13d}$$

In Eq.(4-12) and Eq.(4-13), the amplitude coefficients are real for incident angle $\theta_\beta < \theta_{cr1}$. From Eq.(4-13a) and Eq.(4-13b), it is seen that no P-waves are reflected when
2(2 – f_j – f_3)k_0^2 – (1 – f_j)k_p^2 = 0. From Eq.(4-13c) and Eq.(4-13d), the ratio of \(|b/b_o| = 1\) exits in the case of incident SV-wave beyond the critical angle for the slow P-wave (both \(\nu_{\alpha 1}\) and \(\nu_{\alpha 2}\) are real numbers) in a soft, unconsolidated solid-skeleton (\(V_{\alpha 2}/V_\beta > 1\)).

### 4.1.3 Surface Response

The following responses of the solid-skeleton at the half-space surface are calculated.

#### Displacements

Once the amplitude coefficients are determined, the displacement responses of the fluid-saturated porous medium at the ground surface can be obtained as

\[
\begin{bmatrix}
u_x \\
u_y
\end{bmatrix}_{y=0} =
\begin{bmatrix}
G_{32}^* & G_{31,1}^* & G_{31,2}^* & -G_{32}^* \\
G_{42}^* & -G_{41,1}^* & -G_{41,2}^* & G_{42}^*
\end{bmatrix}
\begin{bmatrix}
b_0 \\
a_1 \\
a_2 \\
b
\end{bmatrix} e^{ik_0 x}. 
\tag{4-14}
\]

#### Surface strains

The surface strains are defined by the following derivatives of displacements

\[
\begin{bmatrix}
\gamma_x \\
\gamma_y
\end{bmatrix}_{y=0} =
\begin{bmatrix}
\partial u_x / \partial x \\
\partial u_y / \partial y
\end{bmatrix} \quad \tag{4-15}
\]

Substituting Eq. (4-4) into Eq.(4-15), we obtain

\[
\begin{bmatrix}
\gamma_x \\
\gamma_y
\end{bmatrix}_{y=0} =
\begin{bmatrix}
G_{72}^* & G_{71,1}^* & G_{71,2}^* & -G_{72}^* \\
G_{82}^* & G_{81,1}^* & G_{81,2}^* & -G_{82}^*
\end{bmatrix}
\begin{bmatrix}
b_0 \\
a_1 \\
a_2 \\
b
\end{bmatrix} e^{ik_0 x}. 
\tag{4-16}
\]

#### Surface rotation

The surface rocking for an incident SV-wave is

\[
\psi_{xy} = \frac{1}{2} \begin{bmatrix}
\partial u_y \\
\partial x \end{bmatrix} - \begin{bmatrix}
\partial u_x \\
\partial y
\end{bmatrix} \quad \tag{4-17}
\]

\[
= \frac{1}{2} (b_0 + b)k_1^2 e^{ik_0 x}
\]
and the normalized rotation is (Trifunac, 1982; Lee and Trifunac, 1987)

\[ \xi_{xy} = \frac{\psi_{xy} (\lambda_\beta / \pi)}{ik_\beta e^{ik_0 x}} = -(b_0 + b)e^{i\xi} \]  

(4-18)

where \( \lambda_\beta \) is the wavelength of SV-wave, and \( ik_\beta e^{ik_0 x} \) is the displacement induced by incident SV-wave.

It is seen that the rotation is phase-shifted relative to the incident motion by \( \pi/2 \) for \( \theta_\beta < \theta_{cr1} \).

**Stresses**

The stresses at the ground surface can be obtained as

\[
\begin{pmatrix}
\tau_{yy} \\
\tau_{sy} \\
\tau_{sx} \\
\sigma
\end{pmatrix}
\big|_{y=0} = \mu
\begin{pmatrix}
G_{12}^* & G_{11,1}^* & G_{11,2}^* & -G_{12}^* \\
G_{22}^* & -G_{21,1}^* & -G_{21,2}^* & G_{22}^* \\
G_{52}^* & G_{51,1}^* & G_{51,2}^* & -G_{52}^* \\
0 & G_{61,1}^* & G_{61,2}^* & 0
\end{pmatrix}
\begin{pmatrix}
b_0 \\
a_1 e^{ik_0 x} \\
a_2 \\
b
\end{pmatrix}
\]  

(4-19)

**4.2 An example of surface response analysis**

A fluid-saturated porous half-space subjected to an incident SV-wave with unit amplitude is investigated for surface response. We assume porous medium with 30% porosity consisting of spherical solid particles (\( \tau_r = 0.5, \rho_s/\rho_f = 2.7 \)). The relations among material constants in Table 2.1 are used to calculate the responses for various Poisson’s ratios of the solid-skeleton and \( \mu/K_f \) ratios.

The surface response for the open boundary and sealed boundary cases are discussed in sections 4.3 and 4.4, respectively. Fig. 4.2 through Fig. 4.5 and Fig. 4.7 through Fig. 4.10 show the amplitude coefficients, displacements, surface strains, rotations and stresses versus incident angle for various solid materials with different Poisson’s ratios for the open-boundary cases and sealed-boundary cases, respectively. Fig. 4.6 and Fig. 4.11 show the effects of variable solid stiffness, with the same Poisson’s ratio, \( \nu = 0.25 \), for the open-boundary case and sealed-boundary case, respectively. Fig.
4.12 illustrates the effects of open-boundary and sealed-boundary for porous material with $\mu / K_f$ ratio = 0.1 and $\nu_s = 0.25$.

It is noted that the amplitude coefficients become complex numbers beyond the critical angle. The displacements are normalized by a factor $k_\beta$, which gives the displacement intensity of the incident SV-wave. The surface strains are normalized by a factor $k_\beta^2$, which is the strain intensity of incident SV-wave. The stresses are normalized by a factor $k_\beta^2$, which is the stress intensity induced by incident SV-wave (Pao and Mao, 1971). All the above complex amplitudes are represented in the plots by their absolute values.

4.3 Results and discussion for open-boundary case

4.3.1 Amplitude coefficients

From parts (a), (b) and (c) in Fig. 4.2 through Fig. 4.5, it is seen that no P-waves are reflected at incident angle of 45°. The peak amplitude coefficients of the slow P-waves are much smaller ($10^{-2}$) than those of the fast P-waves for $\mu / K_f = 10$ and increase to 0.49 for $\mu / K_f = 0.01$. The coefficient variations with respect to Poisson’s ratio are significant for the solid-dominated case (large $\mu / K_f$ ratio), and diminish with decreasing solid stiffness. The amplitude spikes at the first critical angle also diminish with decreasing solid stiffness. The peak displacements $u_x$, are between 1.55 and 1.69. Fig. 4.6(d) and (e) show how the amplitudes of $u_x$ decrease with decreasing solid stiffness.

4.3.2 Displacements

From parts (d) and (e) in Fig. 4.2 through Fig. 4.5, it is seen that the effects of variations in Poisson’s ratio are significant for displacement amplitudes in the solid-dominated case. These effects diminish with decreasing solid stiffness. The displacement $u_x$ is zero at incident angle of 45°. The spikes of the displacement $u_x$ at the first critical angle have a maximum value 5.34 for $\mu / K_f = 10$ and $\nu_s = 0.1$. These spikes diminish with decreasing of $\mu / K_f$ ratio. The peak displacements $u_y$, are between 1.55 and 1.69. Fig. 4.6(d) and (e) show how the amplitudes of $u_x$ decrease with decreasing solid stiffness.
4.3.3 Surface strains

From parts (f) and (g) in Fig. 4.2 through Fig. 4.5, it is seen that the surface strains are zero for incident angle of 45°. The surface strain $\gamma_x$ variations for different Poisson’s ratios are significant for the solid-dominated case, and reduce with decreasing of solid stiffness. The variations of surface strain $\gamma_y$ for different Poisson’s ratios are significant for all conditions. The spikes of surface strains $\gamma_x$ at the first critical angle have a maximum value 3.37 for $\mu/K_f = 10$ and $\nu_s = 0.1$. These spikes diminish with decreasing of $\mu/K_f$ ratio. The peak surface strain $\gamma_y$ has the maximum value of 0.84 for $\mu/K_f = 10$ and $\nu_s = 0.4$, and decreases with decreasing $\mu/K_f$ ratio. For $\mu/K_f = 0.01$, the peak of $\gamma_y$ reduces to 0.6. Fig. 4.6(f) and (g) show how the surface strains decrease with decreasing of solid stiffness and are compared to the amplitudes for the elastic medium.

4.3.4 Rotations

The parts (h) and (i) in Fig. 4.2 through Fig. 4.5 show that the normalized rotation and its phase with respect to the angle of incident motion. It is seen that the dependence of rotation on Poisson’s ratio is significant for the solid-dominated case, and reduces with decreasing solid stiffness. The peak normalized rotation $\xi_{xy}$ always equals 2 at incident angle of 45°, for all the cases. The phase equals to $\pi/2$ when $\theta_{\beta} < \theta_{cr1}$. From parts (j) and (k) in Fig. 4.2 through Fig. 4.5, it is seen that the amplitudes $\xi_{xy}/u_x$ blow up at incident angle of 45°, due to $u_x = 0$, and the ratio $\xi_{xy}/u_y$ equals $2\sin \theta_{\beta}$ for all conditions. Fig. 4.6(h) and (i) show how the rotations vary with variations in solid stiffness, and relative to the rotations in the elastic medium.

4.3.5 Stresses

From parts (l) in Fig. 4.2 through Fig. 4.5, it is seen that the dependence of stress on Poisson’s ratios is always significant. The spikes of stress $\tau_{xx}$ at the first critical angle have maximum value of 7.5 as $\mu/K_f = 10$ and $\nu_s = 0.1$, and diminish with decreasing $\mu/K_f$ ratio. For $\mu/K_f = 0.01$, the peak $\tau_{xx}$ reduces to 3. Fig. 4.6(j) shows how the amplitudes of $\tau_{xx}$ vary with the solid stiffness, and relative to the amplitudes in an elastic medium.
Fig. 4.2 (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude coefficient $b$; (d) Displacement $u_x$; (e) Displacement $u_y$ versus incident angle, to an incident SV-wave for $h = 0.3$, $\mu / K_f = 10$, open boundary case.
Fig. 4.2 (a) Surface strain $\gamma_s$: (b) Normalized Rotation $\beta_s$, (c) Phase of $\gamma_s$ versus incident angle, to an incident SV-wave for $\bar{a} = 0.3$, $\mu_s = 0$, open boundary case.
Fig. 4.2  (j) Rotaion $\xi_{x}/u_x$;  (k) Rotaion $\xi_{x}/u_y$;  (l) Stress $\tau_{xx}$ versus incident angle, to an incident SV-wave for $\hat{n} = 0.3$, $\mu / K_v = 10$, open boundary case.
Fig. 4.3 (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude coefficient $b$; (d) Displacement $u_x$; (e) Displacement $u_y$ versus incident angle, to an incident SV-wave for $\bar{n} = 0.3, \mu / K_r = 1$, open boundary case.
Fig. 4.3 (f) Surface strain $\gamma_y$; (g) Surface strain $\gamma_x$; (h) Normalized Rotation $\gamma_7$; (i) Phase of $\gamma_7$ vs. incident angle, to an incident SV-wave for $m = 0.3$, $\mu / \mu_i = 1$, open boundary case.
Fig. 4.3 (j) Rotation $\xi_{xy}/u_x$; (k) Rotation $\xi_{xy}/u_y$; (l) Stress $\tau_{xx}$ versus incident angle, to an incident SV-wave for $\hat{n} = 0.3, \mu / K_T = 1$, open boundary case.
Fig. 4.4 (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude coefficient $b$; (d) Displacement $u_x$; (e) Displacement $u_y$ versus incident angle, to an incident SV-wave for $\hat{n} = 0.3$, $\mu / K_v = 0.1$, open boundary case.
Fig. 4.4: (f) Surface strain $y_{f}$; (g) Normalized Rotation $R_{n}$; (h) Phase of $\xi_{p}$; (i) Phase of incident angle, to an incident SV-wave for $\tilde{a} = 0.3$, $\mu / \lambda = 0.1$, open boundary case.
Fig. 4.4 (j) Rotation $\xi_{xy}/u_y$; (k) Rotation $\xi_{xy}/u_x$; (l) Stress $\tau_{xx}$ versus incident angle, to an incident SV-wave for $\hat{a} = 0.3$, $\mu/K_r = 0.1$, open boundary case.
Fig. 4.5 (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude coefficient $b_1$; (d) Displacement $u_1$; (e) Displacement $u_2$. Versus incident angle, for an incident SV-wave for $\bar{n} = 0.3$, $\mu = 0.01$, open boundary case.
Fig. 4.5: (f) Surface strain $\gamma_z$; (g) Surface strain $\gamma_r$. (h) Normalized Rotation $\theta_n$. (i) Phase of $E_{zy}$ versus incident angle, to an incident SV-wave for $\eta = 0.3$, $\mu / K_e = 0.01$, open boundary case.
Fig. 4.5 (j) Rotation $\xi_{xy} / u_x$; (k) Rotaion $\xi_{xy} / u_y$; (l) Stress $\tau_{xx}$ versus incident angle, to an incident SV-wave for $\hat{n} = 0.3$, $\mu / K_r = 0.01$, open boundary case.
Fig. 4.6  (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude coefficient $b$; (d) Displacement $u_x$; (e) Displacement $u_y$ versus incident angle, to an incident SV-wave for $\tilde{n} = 0.3$, $\nu_t = 0.25$, open boundary case, with different $\mu / K_f$. 
Fig. 4.6  (g) Surface strain $\gamma_s$; (h) Normalized Rotation $\theta_0$; (i) Phase of $F_{x0}$ versus incident angle, to an incident SV-wave for $\tilde{a}=0.3, \nu=0.25$, open boundary case, with different $\mu/K_s$.
Fig. 4.6  (j) Rotation $\xi_{xy}/u_y$; (k) Rotation $\xi_{xy}/u_y$; (l) Stress $\tau_{xx}$ versus incident angle, to an incident SV-wave for $\hat{n} = 0.3$, $v_s = 0.25$, open boundary case, with different $\mu/K_r$. 
4.4 Results and discussion for sealed-boundary case

4.4.1 Amplitude coefficients

From parts (a), (b) and (c) in Fig. 4.7 through Fig. 4.10, it is seen that the coefficient variations with respect to Poisson’s ratio are always significant for any set of conditions. The incident angles leading to no reflections of P-waves are no longer 45°, because of the solid-fluid interaction. The amplitude spikes at the first critical angle diminish with decreasing solid stiffness. The peak amplitude coefficients of the slow P-waves are much smaller ($10^{-2}$) than those of the fast P-waves for $\mu / K_f = 10$ and increase with decreasing $\mu / K_f$ ratio. The effects of the second critical angle become significant for soft, unconsolidated solid-skeleton ($\mu / K_f \leq 0.1$ and $\nu_s = 0.4$). The spike of amplitude $a_2$ at the second critical angle equals 2.99 as $\mu / K_f = 0.01$ and $\nu_s = 0.4$. Fig. 4.11(a), (b) and (c) show how the amplitude coefficients vary with decreasing solid stiffness, and relative to the amplitudes for an elastic medium.

4.4.2 Displacements

From parts (d) and (e) in Fig. 4.7 through Fig. 4.10, it is seen that the displacement variations caused by Poisson’s ratios are significant for the solid-dominated case. These effects diminish with decreasing solid stiffness except for $\mu / K_f \leq 0.1$ and $\nu_s = 0.4$. The zero-amplitude displacement $u_x$ near incident angle of 45° can be found in the solid-dominated case, but not in the fluid-dominated case. The amplitude spikes at the first critical angle also diminish with decreasing solid stiffness. The effects of the second critical angle become significant for soft, unconsolidated solid-skeleton ($\mu / K_f \leq 0.1$ and $\nu_s = 0.4$). The spike of displacement $u_x$, at the second critical angle, approaches 1.1 as $\mu / K_f = 0.01$ and $\nu_s = 0.4$. The peak displacements $u_y$, are between 1.66 and 1.69. Fig. 4.11(d) and (e) show how the amplitudes of $u_x$ vary with respect to the solid stiffness. It is found that the peak value of $u_y$ for the sealed-boundary case is larger than in the case for an elastic medium.
4.4.3 Surface strains

From parts (f) and (g) in Fig. 4.7 through Fig. 4.10, it is seen that the variations in surface strain with respect to Poisson’s ratio are significant for the solid-dominated case, and reduce with decreasing solid stiffness except for $\mu/K_f \leq 0.1$ and $\nu_s = 0.4$. The zero-amplitude strain $\gamma$ near incident angle of $45^\circ$ can be found only for $\mu/K_f = 10$. The amplitude spikes at the first critical angle also diminish with decreasing solid stiffness. The effects of the second critical angle become significant for soft, unconsolidated solid-skeleton ($\mu/K_f \leq 0.1$ and $\nu_s = 0.4$). The spikes at the second critical angle approach 0.862 for $\gamma$, component, and 0.964 for $\gamma$ component as $\mu/K_f = 0.01$ and $\nu_s = 0.4$. Fig. 4.11(f) and (g) show how the amplitudes of surface strains vary with the solid stiffness. It is found that the peak value of $\gamma$ for the sealed-boundary case is larger than that for an elastic medium when the solid is soft.

4.4.4 Rotations

The parts (h) and (i) in Fig. 4.7 through Fig. 4.10 show the normalized rotation and its phase versus incident angle. It is seen that the variations in rotation in terms of Poisson’s ratios are significant for the solid-dominated case. Unlike the open-boundary case, the peak normalized rotation $\xi_{xy}$ no longer equals 2. The phase equals $\pi/2$ when $\theta_\beta < \theta_{cr1}$. The effects of the second critical angle become significant for soft, unconsolidated solid-skeleton ($\mu/K_f \leq 0.1$ and $\nu_s = 0.4$). From parts (j) and (k) in Fig. 4.7 through Fig. 4.10, it is seen that the amplitudes $\xi_{xy}/u_x$ are large around $45^\circ$ since $u_x$ is small, and the ratio of $\xi_{xy}/u_y$ equals to $2\sin\theta_\beta$ for all conditions. Fig. 4.11(h) and (i) show how the rotations vary respect to solid stiffness and relative to the amplitudes for the elastic medium.

4.4.5 Stresses

From parts (l) and (m) in Fig. 4.7 through Fig. 4.10, it is seen that the stress dependence on Poisson’s ratios is always significant. The spikes at the first critical angle have maximum value of 10.4 for stress $\tau_{xx}$ and 0.311 for pore pressure $\sigma$ when $\mu/K_f = 10$ and $\nu_s = 0.1$. These effects diminish with decreasing $\mu/K_f$ ratio. The effects of the second critical angle become significant for soft, unconsolidated solid-skeleton ($\mu/K_f \leq 0.1$ and
$\nu_s = 0.4)$. The spikes at the second critical angle approach 2.57 for $\tau_{xx}$ and 2.77 for $\sigma$ as $\mu/K_f = 0.01$ and $\nu_s = 0.4$. Fig. 4.11(j) and (k) show how the stress $\tau_{xx}$ and pore fluid pressure vary with respect to the solid stiffness, and relative to the case of an elastic medium.
Fig. 4.7 (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude coefficient $b$; (d) Displacement $u_x$; (e) Displacement $u_y$ versus incident angle, to an incident SV-wave for $\tilde{n} = 0.3$, $\mu / K_r = 10$, sealed boundary case.
Fig. 4.7  (f) Surface strain $\gamma_x$; (g) Surface strain $\gamma_y$; (h) Normalized Rotation $\xi_{xy}$; (i) Phase of $\xi_{xy}$ versus incident angle, to an incident SV-wave for $\bar{\eta} = 0.3$, $\mu / K_r = 10$, sealed boundary case.
Fig. 4.7  (j) Rotaion $\xi_{xy} / u_y$ ; (i) Rotaion $\xi_{xy} / u_y$ ; (l) Stress $\tau_{xx}$ ; (m) Pore Pressure $\sigma$ versus incident angle, to an incident SV-wave for $\hat{n} = 0.3$, $\mu / K_r = 10$, sealed boundary case.
Fig. 4.8  (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude coefficient $b$; (d) Displacement $u_x$; (e) Displacement $u_y$ versus incident angle, to an incident SV-wave for $\bar{n} = 0.3$, $\mu / K_r = 1$, sealed boundary case.
Fig. 4.8: (f) Surface strain $\gamma_x$; (g) Surface strain $\gamma_y$; (h) Normalized Rotation $\xi_x$; (i) Phase of $\xi_x$ versus incident angle, to an incident SV-wave for $k = 0.3$, $\mu/K_r = 1$, scaled boundary case.
Fig. 4.9 (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude coefficient $b$; (d) Displacement $u_x$; (e) Displacement $u_y$ versus incident angle, to an incident SV-wave for $\hat{\eta} = 0.3$, $\mu / K_r = 0.1$, sealed boundary case.
Fig. 4.9 (f) Surface strain $\gamma_L$; (g) Normalized Rotation $\xi_L$; (i) Phase of $\xi_L$ versus incident angle, to an incident SV-wave for $n = 0.3$, $\mu/K_r = 0.1$, scaled boundary case.
Fig. 4.9
(i) Rotation $\frac{\xi}{n}$; (k) Rotation $\frac{\xi}{n}$; (j) Stress $\sigma$; (l) Stress $\sigma$; (m) Pore Pressure $\sigma$ versus incident angle, to an incident SV-wave for $\bar{n} = 0.3$.
Fig. 4.10 (a) Amplitude coefficient $a$; (b) Amplitude coefficient $b$; (c) Amplitude coefficient $c$; (d) Displacement $u_x$; (e) Displacement $u_y$ versus incident angle, to an incident SV-wave for $n = 0.3$, $\mu = 0.01$, sealed boundary case.
Fig. 4.10 (f) Surface strain $\gamma$: (g) Surface strain $\gamma$: (h) Normalized Rotation $\xi$; (i) Phase of $\xi_{xy}$ versus incident angle, to an incident SV-wave for $h = 0.3, \mu' / K_r = 0.01$, scaled boundary case.
Fig. 4.10 (j) Rotation $\xi_{xy}/u_x$; (k) Rotation $\xi_{xy}/u_y$; (l) Stress $\tau_{xy}$; (m) Pore Pressure $\sigma$ versus incident angle, to an incident SV-wave for $\hat{n} = 0.3$, $\mu/K_r = 0.01$, sealed boundary case.
Fig. 4.11 (a) Amplitude coefficient $a_1$; (b) Amplitude coefficient $a_2$; (c) Amplitude coefficient $b$; (d) Displacement $u_1$; (e) Displacement $u_y$ versus incident angle, to an incident SV-wave for $\hat{n} = 0.3, \nu_r = 0.25$, sealed boundary case, with different $\mu / K_r$. 
Fig. 4.11 (f) Surface strain $\gamma_s$; (g) Surface strain $\gamma_s$; (h) Normalized Rotation $\xi_{xy}$; (i) Phase of $\xi_{xy}$ versus incident angle, to an incident SV-wave for $\hat{n} = 0.3$, $\nu_s = 0.25$, sealed boundary case, with different $\mu / K_r$. 
Fig. 4.11 (j) Rotaion $\xi_{xy}/u_x$; (k) Rotaion $\xi_{xy}/u_y$; (l) Stress $\tau_{xx}$; (m) Pore pressure $\sigma$ versus incident angle, to an incident SV-wave for $\mu = 0.3$, $\nu_s = 0.25$, sealed boundary case, with different $\mu/K_f$. 
4.5 Comparison of open-boundary and sealed-boundary cases

While comparing the open-boundary with the sealed-boundary case in Fig. 4.2 through Fig. 4.11, it is seen that the effects of slow P-wave are barely noticeable for the open-boundary case. In contrast, the effects of the second critical angle for slow P-wave become significant for the sealed-boundary case in a soft, unconsolidated solid-skeleton ($\mu / K_f \leq 0.1$ and $\nu_s = 0.4$).

Fig. 4.12 shows the displacements, surface strains, rotations, and stresses for an elastic medium, for open-boundary and for sealed boundary cases in porous medium for $\nu_s = 0.25$ and $\mu / K_f = 0.1$ (simulating soils). It is seen that the amplitudes at the critical angle for elastic medium are significant compared to the poroelastic medium.

Generally, the peak amplitudes for the sealed-boundary case are larger than the ones for the open-boundary case, except the normalized rotation $\xi_{xy}$. It is seen that the peak values of $u_y$ and $\gamma_y$ for the sealed-boundary case exceed the peak values for the elastic medium. In such cases, the amplification of vertical motion of the sealed-boundary is larger than that for a classical elastic medium.

It is found that the ratio between the rotation and the vertical displacement can be described by Eq. (4-20), consistent with the results for elastic medium (Trifunac, 1982).

$$\left| \frac{\xi_{xy}}{u_y} \right| = 2 \sin \theta_\beta$$  \hspace{1cm} (4-20)

From Eq. (4-20), the ratio of rotation to the vertical displacement is only associated with the incident angle of SV-wave.
Fig. 4.12 Comparison of sealed boundary and sealed boundary cases of porous media, for \( n = 0.3 \), \( \mu / K_r = 0.1 \), and elastic medium, \( v = 0.25 \).

(a) Displacement \( u \); (b) Displacement \( u' \); (c) Surface strain \( \gamma \); (d) Surface strain \( \gamma' \), versus incident angle, to an incident SV-wave.
Fig. 4.12 Comparison of sealed boundary and sealed boundary cases of porous media, for $\tilde{n} = 0.3$, $\mu / K_r = 0.1$, and elastic medium, $\nu_s = 0.25$.

(e) Normalized Rotation $\tilde{\xi}_{xy}$; (f) Rotation $\xi_{xy} / u_s$; (g) Rotation $\xi_{xy} / u_y$; and (h) Stress $\tau_{xx}$ versus incident angle, to an incident SV-wave.
4.6 Conclusions

The surface response of a fluid-saturated porous half-space is influenced significantly by the stiffness and Poisson’s ratio of the solid-skeleton, and by the boundary drainage.

The solid stiffness governs the amplitudes of elastic waves in a porous medium. For the solid-dominated case (large $\mu/K_f$ ratio), the porous medium behaves similarly as an elastic medium. The porous medium behaves like a fluid medium for the fluid-dominated case (small $\mu/K_f$ ratio). Generally, the variations caused by Poisson’s ratio are significant for the solid-dominated case, but reduce with decreasing solid stiffness.

It is interesting to note that the ratio between normalized rotation and the vertical displacement is only associated with the incident angle of SV-wave. From Eq.(3-12) and Eq.(4-20), we found that this ratio is equal to the ratio of the apparent wave number to the SV-wave number as in

$$\frac{\xi_{xy}}{u_y} = 2\frac{k_i \sin \theta_i}{k_\beta} = \frac{2k_i}{k_\beta},$$

(4-21)

where $k_i$ is the wave number of incident wave and $\theta_i$ is the angle of incidence.

For open (drained) boundary, the peak amplitudes of displacements, surface strains, rotations, and stresses are smaller than the amplitudes for an elastic medium. In this case, the fluid plays a passive role in the poroelastic system and reduces the dynamic response of the solid-skeleton. It is also seen that the effects of the first critical angle diminish with decreasing solid stiffness.

For sealed (undrained) boundary, the effects of the first critical angle diminish with decreasing solid stiffness. In contrast, the response at the second critical angle for the undrained boundary becomes significant for soft, unconsolidated solid-skeleton ($\mu/K_f \leq 0.1$ and $\nu_s = 0.4$). The peak values of the displacement response $u_y$ for the sealed boundary case are larger than those for an elastic medium, except for the case of $\mu/K_f = 10$. The peak values of $\gamma_y$ also exceed those for the elastic medium, when the solid-skeleton is soft.

The pore pressures are not zero for the sealed boundary case, and increase with decreasing solid stiffness. If the solid-skeleton is formed with cohesionless granular
particles, the initiation of liquefaction becomes possible because the pore fluid pressure may cause loss of effective stresses between the particles near the ground surface.
5 Case study and general conclusions

5.1 Case study: strong motion records at Port Island, during the Kobe Earthquake on January 17, 1995

During the Kobe earthquake on January 17, 1995 ($M_w = 6.9$), strong ground motions were recorded by a downhole array in the water-saturated soils at a reclaimed island, Port Island. Port Island is located on the southwest side of Kobe city, and is approximately 20 km from the epicenter (Fig. 5.1). The downhole array consisted of four three-component accelerometers located on the surface and at depths of 16, 32, and 83 m below the ground surface. Each accelerometer recorded two horizontal motions (east-west, and north-south directions), and one vertical motion. The soil profile (with water table at 2.4 m below the surface) and the distribution of recorded peak accelerations with depth at the site are shown in Fig. 5.2. Fig. 5.3 shows all the accelerograms of the downhole array recorded during the 1995 Kobe earthquake.

From 5.2(b), it is seen that the peak acceleration of the vertical component is 1.5 to 2 times larger than of the two horizontal components. The recorded accelerations at different depths also suggest that the surface motions in the vertical direction are amplified, while the horizontal motions are reduced, as shown in Fig. 5.3.

Many researchers (Sato et al., 1996; Aguirre and Irikura, 1997; Kokusho and Matsumoto, 1999; Yang et al., 2000) concluded that the reduction of horizontal motions was associated with soil nonlinearity and liquefaction in the surface reclaimed layers. Yang and Sato (2000, 2001) suggested that the amplification of vertical motions was caused by incomplete saturation of near-surface soils in the layered half-space subjected to a vertical P-wave incidence.

In place of those analyses, we apply the model studied in this report to make a rough preliminary explanation of the same recording. Obviously, more detailed analysis based on wave propagation in this medium will have to be carried out to explain the observations with some certainty. As seen in Fig. 5.2 (a), the soil profile mainly consists
Fig. 5.1 Map of the Osaka Bay region, showing the geographical locations of the earthquake epicenter and of the reclaimed island – Port Island (EERC, 1995)
of sands and the water table (at 2.4 m depth) is close to the ground surface. Therefore, we assume that water-saturated half-space is an applicable approximate representation for this site.

We assume the medium has 30% porosity and consists of soil particles with specific gravity of 2.7. The S wave velocities at this site have been found to be in the range from 170 m/s to 350 m/s (Yang and Sato, 2000). By using Table 2.3, it is seen that the case with $\mu/K_f = 0.1$ is the closest for this site. From the geographical distances between the earthquake hypocenter (depth $\approx 10$ km) and the strong motion station (epicentral distance $\approx 20$ km), it can be assumed that the angle of incidence is around 60°.

Both P and SV wave incidences are considered in the present example. Fig. 5.4 shows the normalized displacement amplitudes versus incident angle for the chosen porous medium ($\hat{n} = 0.3$, $\nu_s = 0.25$, $\mu/K_f = 0.1$) for both P and SV wave incidences. It is seen that near 60° angle of incidence, the displacement amplification is less than 0.5 for the horizontal component, and is around 1 to 1.2 for the vertical component, for both

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**Fig. 5.2** (a) The soil profile, and (b) distribution of recorded peak accelerations with depth in three components at the downhole strong motion array, Port Island (Yang and Sato, 2001).
Fig. 5.3 Vertical array acceleration records at Port Island during the 1995 Kobe earthquake (After Yang et al., 2000).
open and sealed boundary cases. Thus, this also might explain why the recorded accelerograms in the horizontal directions are de-amplified, and are amplified in the vertical component as in Fig. 5.3.

This example illustrates de-amplification of earthquake motions in the horizontal direction for fluid-saturated porous media by Biot’s theory. Of course, further studies are needed to analyze this particular site amplification in detail. For example, a layered porous half-space can be employed to study the amplification effects for soils with different water-saturation conditions. We will report on such studies in our future work.

Fig. 5.4 Displacement amplitudes versus incident angle for a water-saturated porous half-space (\( \hat{n} = 0.3 \), \( \nu_s = 0.25 \), \( \mu / K_f = 0.1 \)) subjected to both P and SV wave incidences. For P-wave incidence: (a) horizontal displacement; (b) vertical displacement, and for SV-wave incidence: (c) horizontal displacement; (d) vertical displacement.
5.2 Conclusions

It is hoped that the foregoing analysis will help illustrate the complex nature of the departures from the “classical” theory of reflection of plane elastic waves, from the plane half space boundary. This classical theory has been the first step in numerous engineering interpretations of observed amplitudes of strong earthquake ground motion, and its analysis for predicting the effects on engineering structures. Whether in the construction of artificial strong motion (translations: Trifunac 1971; Wong and Trifunac 1978; rotations: Lee and Trifunac 1985; 1987; curvograms: Trifunac 1990; or strains: Lee, 1990; Trifunac and Lee 1996), or in interpretation of highly concentrated areas of damage (Kawase and Aki, 1990), it is seen that the response of porous water saturated sands, with water table essentially at the ground surface can be significantly different form that of elastic homogeneous half-space. Obviously, further studies of water saturated porous media subjected to incident seismic waves are needed if we wish to improve our ability to predict and to interpret the true nature of destructive strong earthquake shaking in such materials.
Acknowledgements

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References


Appendix A – Summary of potential-displacement-stress relations

The following expressions are based on the Cartesian coordinate system shown in Figure A-1.

A.1 Table of Notations

<table>
<thead>
<tr>
<th></th>
<th>Solid</th>
<th>Fluid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>( \mathbf{u} = (u_x, \ u_y, \ u_z) )</td>
<td>( \mathbf{U} = (U_x, \ U_y, \ U_z) )</td>
</tr>
<tr>
<td>Strain</td>
<td>( \begin{bmatrix} \gamma_{xx} &amp; \gamma_{xy} &amp; \gamma_{xz} \ \gamma_{yx} &amp; \gamma_{yy} &amp; \gamma_{yz} \ \gamma_{zx} &amp; \gamma_{zy} &amp; \gamma_{zz} \end{bmatrix} )</td>
<td>( \varepsilon = \text{div}(\mathbf{U}) )</td>
</tr>
<tr>
<td>Stress</td>
<td>( \begin{bmatrix} \tau_{xx} &amp; \tau_{xy} &amp; \tau_{xz} \ \tau_{yx} &amp; \tau_{yy} &amp; \tau_{yz} \ \tau_{zx} &amp; \tau_{zy} &amp; \tau_{zz} \end{bmatrix} )</td>
<td>( \begin{bmatrix} \sigma &amp; 0 &amp; 0 \ 0 &amp; \sigma &amp; 0 \ 0 &amp; 0 &amp; \sigma \end{bmatrix} )</td>
</tr>
</tbody>
</table>

where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \).

A.2 Stress-Strain Relations

\[
\begin{bmatrix}
\tau_{xx} \\
\tau_{yy} \\
\tau_{zz} \\
\sigma \\
\tau_{xy} \\
\tau_{yz} \\
\tau_{zx}
\end{bmatrix}
= \begin{bmatrix}
P & \lambda & \lambda & Q & 0 & 0 & 0 \\
\lambda & P & \lambda & Q & 0 & 0 & 0 \\
\lambda & \lambda & P & Q & 0 & 0 & 0 \\
Q & Q & Q & R & 0 & 0 & 0 \\
0 & 0 & 0 & 2\mu & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2\mu & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2\mu
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\varepsilon_{xy} \\
\varepsilon_{yz} \\
\varepsilon_{zx}
\end{bmatrix}
\]

For In-plane waves:

\[
\begin{align*}
\mathbf{u}_x &= \sum_{j=1}^{2} \left( \frac{\partial \phi_j}{\partial x} \right) + \left( \frac{\partial \psi}{\partial y} \right) \\
\mathbf{u}_y &= \sum_{j=1}^{2} \left( \frac{\partial \phi_j}{\partial y} \right) - \left( \frac{\partial \psi}{\partial x} \right) \\
\tau_{yy} &= \sum_{j=1}^{2} \left( \lambda + f_j Q \right) \nabla^2 \phi_j + 2\mu \left( \frac{\partial^2 \phi_j}{\partial y^2} \right) - 2\mu \left( \frac{\partial^2 \psi}{\partial x \partial y} \right)
\end{align*}
\]

Figure A-1. Cartesian coordinate system.
\[ \tau_{xy} = \sum_{j=1}^{2} \left( 2 \mu \frac{\partial^2 \phi_j}{\partial x \partial y} + \mu \left( -\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) \right) \]

\[ \tau_{xx} = \sum_{j=1}^{2} \left( (\lambda + f_j Q) \nabla^2 \phi_j + 2 \mu \frac{\partial^2 \phi_j}{\partial x^2} \right) + 2 \mu \left( \frac{\partial^2 \psi}{\partial x \partial y} \right) \]

\[ \sigma = Q \nabla^2 \phi + R \nabla^2 \Phi = \sum_{j=1}^{2} (Q + f_j R) \nabla^2 \phi_j, \quad (j = 1, 2) \]

\[ \gamma_x = \frac{\partial u_x}{\partial x} = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \psi}{\partial x \partial y} \]

\[ \gamma_y = \frac{\partial u_y}{\partial y} = \frac{\partial^2 \phi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x \partial y} \]

\[ \psi_{xy} = \frac{1}{2} \left( \frac{\partial u_x}{\partial x} - \frac{\partial u_y}{\partial y} \right) = -\frac{1}{2} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) \]

Let: \[ \phi_j = e^{ik_{\alpha_j} (x \sin \theta_{\alpha_j} + y \cos \theta_{\alpha_j})} \]

\[ \psi = e^{ik_{\beta} (x \sin \theta_{\beta} + y \cos \theta_{\beta})} \]

| \( \tau_{yy} \) due to | \( \phi_j : \mu G_{31,j} e^{ik_{\alpha_{j}} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) | \( G_{31,j} = -k_{\alpha_{j}}^2 (M_{j} - 2 \sin^2 \theta_{\alpha_{j}}) \)
| \( G_{12} = k_{\beta}^2 \sin 2\theta_{\beta} \) |
| \( \psi : \pm \mu G_{12} e^{ik_{\alpha_{j}} (x \sin \theta_{\beta} + y \cos \theta_{\beta})} \) |
| \( \tau_{xy} \) due to | \( \phi_j : \pm \mu G_{31,j} e^{ik_{\alpha_{j}} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) | \( G_{21,j} = -k_{\alpha_{j}}^2 \sin \theta_{\alpha_{j}} \)
| \( G_{22} = -k_{\beta}^2 \cos 2\theta_{\beta} \) |
| \( \psi : \mu G_{32} e^{ik_{\beta} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) |
| \( u_x \) due to | \( \phi_j : G_{31,j} e^{ik_{\alpha_{j}} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) | \( G_{31,j} = ik_{\alpha_{j}} \sin \theta_{\alpha_{j}} \)
| \( G_{32} = ik_{\beta} \cos \theta_{\beta} \) |
| \( \psi : \pm G_{32} e^{ik_{\beta} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) |
| \( u_y \) due to | \( \phi_j : \pm G_{31,j} e^{ik_{\alpha_{j}} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) | \( G_{41,j} = ik_{\alpha_{j}} \cos \theta_{\alpha_{j}} \)
| \( G_{42} = -ik_{\beta} \sin \theta_{\beta} \) |
| \( \psi : G_{32} e^{ik_{\beta} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) |
| \( \tau_{xx} \) due to | \( \phi_j : \pm \mu G_{51,j} e^{ik_{\alpha_{j}} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) | \( G_{51,j} = -k_{\alpha_{j}}^2 (M_{j} - 2 \cos^2 \theta_{\alpha_{j}}) \)
| \( G_{52} = -k_{\beta}^2 \sin 2\theta_{\beta} \) |
| \( \psi : \pm \mu G_{52} e^{ik_{\beta} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) |
| \( \sigma \) due to | \( \phi_j : \mu G_{61,j} e^{ik_{\alpha_{j}} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) | \( G_{61,j} = -k_{\alpha_{j}}^2 S_{j} \)
| \( \psi = e^{ik_{\beta} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) |
| \( \gamma_x \) due to | \( \phi_j : G_{71,j} e^{ik_{\alpha_{j}} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) | \( G_{71,j} = -k_{\alpha_{j}}^2 \sin^2 \theta_{\alpha_{j}} \)
| \( G_{72} = -k_{\beta}^2 \sin \theta_{\beta} \cos \theta_{\beta} \) |
| \( \psi : \pm G_{72} e^{ik_{\beta} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) |
| \( \gamma_y \) due to | \( \phi_j : G_{81,j} e^{ik_{\alpha_{j}} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) | \( G_{81,j} = -k_{\alpha_{j}}^2 \cos^2 \theta_{\alpha_{j}} \)
| \( G_{82} = k_{\beta}^2 \sin \theta_{\beta} \cos \theta_{\beta} \) |
| \( \psi : \pm G_{82} e^{ik_{\beta} (x \sin \theta_{\alpha_{j}} + y \cos \theta_{\alpha_{j}})} \) |
Let: \( \phi_j = e^{ik_0x_0y_0} \) \( \psi = e^{ik_0x_0y_1} \)

<table>
<thead>
<tr>
<th>( \tau_{yy} ) due to</th>
<th>( \phi_j : \mu G_{11}^* e^{ik_0x_0y_0} )</th>
<th>( \psi : \mu G_{12}^* e^{ik_0x_0y_0} )</th>
<th>( G_{11,j} = 2k_0^2 - M_j k_0^2 )</th>
<th>( G_{12}^* = -2ik_0v_\beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_{xy} ) due to</td>
<td>( \phi_j : \pm \mu G_{21}^* e^{ik_0x_0y_0} )</td>
<td>( \psi : \mu G_{22}^* e^{ik_0x_0y_0} )</td>
<td>( G_{21,j} = 2ik_0v_aj )</td>
<td>( G_{22}^* = 2k_0^2 - k_\beta^2 )</td>
</tr>
<tr>
<td>( u_x ) due to</td>
<td>( \phi_j : G_{31}^* e^{ik_0x_0y_0} )</td>
<td>( \psi : \pm G_{32}^* e^{ik_0x_0y_0} )</td>
<td>( G_{31,j} = ik_0 )</td>
<td>( G_{32}^* = v_\beta )</td>
</tr>
<tr>
<td>( u_y ) due to</td>
<td>( \phi_j : \pm G_{41}^* e^{ik_0x_0y_0} )</td>
<td>( \psi : G_{42}^* e^{ik_0x_0y_0} )</td>
<td>( G_{41,j} = v_aj )</td>
<td>( G_{42}^* = -ik_0 )</td>
</tr>
<tr>
<td>( \tau_{xx} ) due to</td>
<td>( \phi_j : \mu G_{51}^* e^{ik_0x_0y_0} )</td>
<td>( \psi : \pm \mu G_{52}^* e^{ik_0x_0y_0} )</td>
<td>( G_{51,j} = -2k_0^2 - (M_j - 2)k_0^2 )</td>
<td>( G_{52}^* = 2ik_0v_\beta )</td>
</tr>
<tr>
<td>( \sigma ) due to</td>
<td>( \phi_j : \mu G_{61}^* e^{ik_0x_0y_0} )</td>
<td>( \psi : \pm \mu G_{62}^* e^{ik_0x_0y_0} )</td>
<td>( G_{61,j} = -S_j k_0^2 )</td>
<td></td>
</tr>
<tr>
<td>( \gamma_x ) due to</td>
<td>( \phi_j : G_{71}^* e^{ik_0x_0y_0} )</td>
<td>( \psi : \pm G_{72}^* e^{ik_0x_0y_0} )</td>
<td>( G_{71,j} = -k_0^2 )</td>
<td>( G_{72}^* = ik_0v_\beta )</td>
</tr>
<tr>
<td>( \gamma_y ) due to</td>
<td>( \phi_j : G_{81}^* e^{ik_0x_0y_0} )</td>
<td>( \psi : \pm G_{82}^* e^{ik_0x_0y_0} )</td>
<td>( G_{81,j} = k_0^2 - k_0^2 )</td>
<td>( G_{82}^* = -ik_0v_\beta )</td>
</tr>
</tbody>
</table>

where \( M_j = (P + f_jQ) / \mu \), \( S_j = (Q + f_jR) / \mu \), \( j = 1, 2 \)
Appendix B – Summary of Notations

\[ A = PR - Q^2 \]
\[ \hat{b} = \hat{n}^2 \hat{\mu} \quad \text{dissipative coefficient} \]
\[ B = \rho_{11}R + \rho_{22}P - 2\rho_{12}Q \]
\[ C = \rho_{11}\rho_{22} - \rho_{12}^2 \]
\[ f_j = \text{wave potential factors between solid and fluid (} j = 1, 2, 3 \text{)} \]
\[ \hat{k} = \text{permeability} \]
\[ k_0 = \text{apparent wave number along half-space surface} \]
\[ k_i = \text{wave number of incident wave} \]
\[ k_{u,j} = \text{P-wave numbers} \]
\[ k_p = \text{S-wave number} \]
\[ K_f = \text{bulk modulus of fluid} \]
\[ K_s = \text{bulk modulus of solid-skeleton} \]
\[ \hat{n} = \text{porosity} \]
\[ p = \text{hydraulic pressure on cubic unit of aggregate} \]
\[ P = \lambda + 2\mu \]
\[ Q = \text{a measure of the coupling between the volume change of solid and liquid.} \]
\[ R = \text{a measure of the pressure exerted on the fluid to remain at a constant volume.} \]
\[ V_{u,j} = \text{P-wave velocities} \]
\[ V_p = \text{S-wave velocity} \]
\[ \delta = \text{compressibility of the solid-fluid system (unjacketed compressibility)} \]
\[ \gamma = \text{compressibility of the pore fluid (fluid infiltrating coefficient)} \]
\[ \kappa_f = \text{fluid compressibility} \]
\[ \kappa_s = \text{compressibility of the solid-skeleton (jacketed compressibility)} \]
\[ \lambda = \text{Lamé constant for the solid-fluid system} \]
\[ \lambda_s = \text{Lamé constant for the solid-skeleton} \]
\[ \lambda_\beta = \text{wavelength of S-wave} \]
\[ \mu = \text{shear modulus for the solid-fluid system} \]
\[ \mu_s = \text{shear modulus for the solid-skeleton} \]
\[ \hat{\mu} = \text{absolute viscosity of the fluid} \]
\[ \nu_s = \text{Poisson’s ratio for the solid-skeleton} \]
\[ \rho = \text{unit mass of the solid-fluid aggregate} \]
\[ \rho_s = \text{solid mass in unit cube (unit mass of solid-skeleton)} \]
\[ \rho_f = \text{fluid mass in unit cube} \]
\[ \rho_{11} = \text{effective solid mass for the solid-fluid system} \]
\[ \rho_{12} = \text{mass coupling coefficients between fluid and solid} \]
\( \rho_{22} \) = effective fluid mass for the solid-fluid system
\( \rho_s \) = density of solid material
\( \rho_j \) = density of fluid
\( \tau, \rho_j \) = the induced mass due to the oscillation of solid particle in fluid
\( \tau_a \) = toruosity parameter
\( \theta_i \) = incidence angle
\( \omega \) = circular frequency
\( \xi_{xy} \) = normalized rotation