ON ENERGY FLOW IN EARTHQUAKE RESPONSE

by

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Report CE 01-03

July, 2001
Los Angeles, California

www.usc.edu/dept/civil_eng/Earthquake_eng/
ABSTRACT

This report examines the flow of earthquake energy, from its source up to its destination in a soil-structure system, where the final part of this energy is converted into relative structure response. The basic seismological aspects of empirical scaling of seismic wave energy, $E_s$, are reviewed and it is shown how the radiated energy can be represented by functionals of strong ground motion. How this energy is attenuated with distance can be described via existing empirical scaling laws of strong ground motion. This attenuation is illustrated in a realistic setting of the three-dimensional geological structure in Los Angeles basin, for recordings of strong ground motion from the 1994 Northridge, California, earthquake. Next, two case studies, of a fourteen story reinforced concrete storage building in Hollywood and a seven story reinforced concrete hotel in Van Nuys, are presented. For these buildings, a quantitative correspondence between the total incident wave energy and the sum of all energies associated with the response of their soil-structure systems is demonstrated. Then, some elementary aspects of design, based on the power of the incident wave pulses, are discussed. It is shown how this power can be compared with the capacity of the structure to absorb the incident wave energy. Finally, the advantages of using the computed power of incident strong motion to design structures for linear or for partially destructive relative response are discussed.
ACKNOWLEDGEMENTS

This work was partially supported by grant CMS 0075070 from the National Science Foundation under the initiative Professional Opportunities for Women in Research and Education.
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1. INTRODUCTION

1.1 General Introduction

The modern era in Earthquake Engineering begins with the formulation of the concept of Response Spectrum by Biot [1932, 1933, 1934]. He presented the general theory, analyzed the first recorded accelerograms [Biot, 1941] and formulated the principles of response spectrum superposition [Biot, 1942]. Today, seventy years later, his ideas still govern the principles of earthquake resistant design.

The method of response spectrum superposition is an ideal tool for design of structures expected to vibrate without any damage during the largest possible levels of shaking. However, pragmatic considerations, analyses of uncertainties, and minimization of cost result in design of structures which may experience damage from rare and very strong earthquake shaking. Thus, during the last 30 years, many modifications and “corrections” have been introduced into the response spectrum method to reconcile its linear nature with its desired nonlinear use in design.

Well-designed structures are expected to have ductile behavior during the largest credible shaking, and large energy reserve to at least delay failure if it cannot be avoided. As the structure finally enters large nonlinear levels of response, it absorbs the excess of the input energy through ductile deformation of its components. Thus, it is logical to formulate earthquake resistant design procedures in terms of the energy driving this process. From the mechanics point of view, this brings nothing new, since the basic energy equations can be derived directly from the Newton’s second law. The advantage of using energy is that the duration of strong motion, the number of cycles to failure and dynamic instability, all can be quantified directly and explicitly. This, of course, requires scaling of the earthquake source and of the attenuation of strong motion to be described in terms of energy.

In 1934, Benioff proposed the use of seismic destructiveness to be measured by computing the area under the relative displacement response spectrum. It can be shown that this result can be related to the energy of strong motion [Arias, 1970; Trifunac and Brady, 1975a]. Benioff [1934] introduced his paper on earthquake destructiveness by stating that “The problem of designing structures to withstand destructive earthquakes is not in a very satisfactory condition. On the one hand engineers do not know what characteristics of the ground motion are responsible for destruction, and on the other hand seismologists have no measurements of seismic motion which are sufficiently adequate to serve for design, even if the destructive characteristics were known. Consequently, engineers have been forced to proceed on an empirical basis. From past
experience, chiefly in Japan, it has been found that buildings which are designed to withstand a constant horizontal acceleration of 0.1 gravity are, on the whole, fairly resistant to seismic damage. It is fortunate that such a simple formula works at all, in view of its inadequacy from the point of view of precise computation. We know that seismic motions do not exhibit constant accelerations; that instead they are made up of exceedingly variable oscillatory movements. A formula based upon constant acceleration may thus lead to large errors, especially when applied to new types of structures which have not been tested in actual earthquakes.”

In this report, we consider an alternative to the spectral method in earthquake resistant design by analyzing the flow of energy associated with strong motion. In Fig. 1.1, the principal stages of earthquake energy flow, from the earthquake source, along the propagation path, and to the final work leading to relative response of the structure, are outlined. The loss of energy at every stage is also outlined. These losses must be accounted for to properly quantify the remaining energy, which will excite the relative response of the structure.

1.2 Review of Past Studies on Energy Radiated by the Earthquake Source and on Energy of Structural Response

The seismological and earthquake engineering characterizations of the earthquake source begin by estimating its “size”. For centuries this was performed by means of earthquake intensity scales, which are not instrumental and are based on human description of the effects of earthquakes [Richter, 1958; Trifunac and Brady, 1975b]. In the early 1930’s, the first instrumental scale – the local earthquake magnitude $M_L$ was introduced in southern California [Richter, 1936; 1958]. Few years later, it was followed by the surface wave magnitude $M_s$ [Gutenberg and Richter, 1956a,b], and more recently by the moment magnitude $M_w = (\log_{10} M_0 - 16)/1.5$ and by the strong motion magnitude $M_{SM}$ [Trifunac, 1991]. The seismic energy associated with elastic waves radiated from the source, $E_s$, (Gutenberg and Richter, 1956a,b) has also been used to compare “sizes” of different earthquakes. The seismic energy, $E_s$, leaving the earthquake source is attenuated with increasing epicentral distance, $r$, through mechanisms of inelastic attenuation [Trifunac, 1994], scattering, and geometric spreading. In the near-field, for distances comparable to the source dimensions, different near-field terms attenuate like $r^{-4}$ and $r^{-2}$ [Haskell, 1969]. The body waves (P- and S-waves) attenuate like $r^{-1}$, while the surface waves attenuate like $r^{-1/2}$.

The seismic wave energy arriving towards the site is next attenuated by nonlinear response of shallow sediments and soil in the “free-field” [Joyner, 1975; Joyner and Chen, 1975; Trifunac and Todorovska, 1996, 1998], before it begins to excite the
Flow and Distribution of Seismic Energy

Fig. 1.1 An outline of principal stages in the flow of earthquake wave energy, from the earthquake source to a structure.

Potential energy in the rocks at the source prior to an earthquake

Elastic energy radiated in form of waves, $E_s$

Wave energy arriving towards the site

Wave energy incident upon a structural foundation

Response of a soil-foundation-structure systems

Energy available to excite the structure

Relative response of the structure

Energy losses:
- Heat
- Friction
- Crushing of rocks
- Work against gravity forces

Energy losses:
- Anelastic attenuation
- Scattering
- Geometric spreading

Energy losses:
- Non-linear response of shallow sediments and of soil in the "free-field"

Energy losses:
- Scattering and diffraction of incident waves from the building foundation

Energy losses:
- Non-linear response of soil due to soil-structure interaction
- Radiation damping

Energy losses:
- Hysteretic response of the structure
- Damping mechanism in the structure

Scaling parameters, or functionals based on measurements: $E_s, M, M_0$

Fourier and response spectra of "free-field" motion

Subject of this work
foundation. Once the foundation is excited by the incident waves, the response of the soil-structure-system is initiated. The incident wave energy is further reduced by nonlinear response of the soil during soil-structure interaction [Trifunac et al., 1999a; 2001a,b] and by radiation damping [Luco et al., 1985; Todorovska and Trifunac, 1991]. The seismic energy flow and distribution involving the last three stages in Fig. 1.1: (1) the response of soil-foundation-structure systems, (2) the energy available to excite the structure, and (3) the relative response of the structure, will be analyzed further in this work.

Engineering analyses of seismic energy flow and distribution among different aspects of the structural response have been carried out since the mid 1950’s. A review of this subject and examples describing the limit-state design of buildings can be found in the book by Akiyama [1985], and in collected papers edited by Fajfar and Krawinkler [1992]. In most engineering studies, the analysis begins by integrating the differential equation of dynamic equilibrium of a single degree of freedom system with respect to displacement, which results in

$$E_I = E_K + E_\zeta + E_E + E_H$$ (1.1)

where $E_I$ is the input energy, $E_K$ is the kinetic energy, $E_\zeta$ is the damping energy, $E_E$ is the elastic strain energy, and $E_H$ is the hysteretic energy.

An important omission in this approach is that the effects of soil-structure interaction are ignored, and because of that significant energy loss mechanisms (nonlinear response of the soil and radiation damping) are thus neglected (Fig. 1.1). Other simplifications and omissions in eqn (1.1) are that the dynamic instability and the effects of gravity on nonlinear response are usually ignored [Husid, 1967; Lee, 1979; Todorovska and Trifunac 1991, 1993].

1.3 Organization of this Report

Chapter 2 deals with the flow of earthquake energy, as follows. Section 2.1, reviews the basic seismological aspects of empirical scaling of seismic wave energy, $E_s$, and shows how the radiated energy can be represented by functionals of strong ground motion. Section 2.2 deals with the energy propagation and attenuation with distance, which is illustrated for the three-dimensional geological structure of Los Angeles basin during the 1994 Northridge, California earthquake. Section 3.3 focuses on representation of the seismic energy flow through the last three stages outlined in Fig. 1.1: the response of soil-foundation-structure systems, the energy available to excite the structure, and the relative response of the structure. For this purpose, a model is used (presented in Section 2.3),
which is verified in Chapters 3 and 4 against two case studies—a fourteen-story reinforced concrete building in Hollywood, and a seven-story reinforced concrete building in Van Nuys. For these buildings, a quantitative correspondence between the total incident wave energy and the sum of all energies associated with the response of the soil-structure system is demonstrated. In Chapter 5, some elementary aspects of design, based on the power of the incident wave pulses, are discussed. It is shown how this power can be compared with the capacity of the structure to absorb the incident wave energy, and the advantages of using the computed power of incident strong motion to design structures for linear or for partially destructive relative response are discussed. Chapter 6 presents a discussion and the conclusions of this study. Appendix A presents details of the modeling of the soil stiffness and damping coefficients, and Appendix B presents details about the force-deformation models for the soil considered. Appendix C presents results for the relative response and its energy and power for the Van Nuys hotel building during 12 earthquakes, and Appendix D presents a list of symbols used in each chapter.
2. FLOW OF EARTHQUAKE ENERGY

2.1 Energy at the Earthquake Source

During an earthquake, the potential energy in the rocks is converted into heat, into mechanical work moving the crustal blocks and crushing the material in the fault zone, and into energy $E_s$, associated with the emitted elastic waves (Fig. 1.1). At a given period, the energy in the elastic waves is of the form

$$\log_{10} E_s = C + 2M$$

(2.1)

where $C$ is a constant. Following numerous estimates of the energy radiated by different earthquakes, and the revisions of the empirical scaling equations based on eqn (2.1) using surface wave magnitude, $M_s$, this empirical relationship has evolved to become [Gutenberg and Richter, 1956b]

$$\log_{10} E_s = 4.8 + 1.5 M_s \text{ (Jouls)}$$

(2.2)

In terms of the local magnitude scale $M_L$, this is equivalent to [Richter, 1958]

$$\log_{10} E_s = 9.9 + 1.9 M_L - 0.024 M_L^2$$

(2.3)

For scaling in terms of earthquake intensity scales, we note here the work of Shebalin [1955], who studied 56 earthquakes and derived the following empirical relationship

$$0.9 \log_{10} E_s - I = 3.8 \log_{10} h - 3.3, \quad \text{for } h < 70 \text{ km}$$

(2.4)

where $E_s$ is energy in surface waves in MJouls ($10^{13}$ ergs), $h$ is hypocentral depth in kilometers, and $I$ is the maximum Modified Mercalli Intensity.

The amount of energy transmitted per unit time across unit area normal to the direction of wave propagation is $\rho \alpha v^2$ for plane P-waves and $\rho \beta v^2$ for plane S-waves, where $\rho$ is material density, $\alpha$ is velocity of P-waves, $\beta$ is velocity of S-waves, and $v$ is particle velocity. The shear wave energy transmitted through area $A$ during time interval $[0, T]$ is, for example

$$E_s = \rho A \beta \int_0^T v^2(t) dt$$

(2.5)

with analogous expression for the energy of P-waves. Using the Parceval’s theorem, eqn (2.5) becomes
\[ E_s = \frac{\rho A \beta}{2\pi} \int_0^\infty \left( \frac{F(\omega)}{\omega} \right)^2 d\omega \]  \hspace{1cm} (2.6)

where \( F(\omega) \) is the Fourier amplitude spectrum of ground acceleration (Fig. 2.1). For practical calculations using band-limited strong motion data, Trifunac [1995] defined the quantity \( en \) (proportional to energy)

\[ en = \int_{\pi/50}^{200\pi} \left| \frac{F(\omega)}{\omega} \right|^2 d\omega \]  \hspace{1cm} (2.7)

and showed how it can be used to develop empirical criteria for initiation of liquefaction.

To account for all the radiated energy in the form of elastic waves, and to follow its distribution, starting with the earthquake source and ending with the structural response, first it is necessary to show that the above simple expressions valid for plane waves can be used approximately to account for all the elastic energy, \( E_s \), radiated by the source. As a first step, we next show that \( E_s \) computed from eqn (2.6), where \( F(\omega) \) is determined from strong motion data, is consistent with eqn (2.2), based on seismological estimates of surface wave magnitude \( M_s \). In Fig. 2.2 we plot \( \log_{10} (\rho_{source} \beta_{source} A_{source} en) \) versus \( \log_{10} E_s \), computed from eqn (2.2), for \( M = 4, 5, 6, 7 \) and 8. For this example, we take \( \beta_{source} = 3 \) km/s, \( \rho_{source} = 2\times10^3 \) kg/m\(^3\) and source area \( A_{source} = LW \), where \( L \) and \( W \) are the fault length and width [Trifunac, 1993, 1994] given by

\[ L = 0.01\times10^{0.5M} \text{ (km)} \]  \hspace{1cm} (2.8)

and

\[ W = 0.1\times10^{0.25M} \text{ (km)} \]  \hspace{1cm} (2.9)

It is seen that, except for \( M = 4 \) and \( M = 8 \), \( 2\rho \beta A \ en \sim E_s \) (the factor of 2 comes from the fact that we approximated the area through which \( E_s \) is radiated by \( 2A \), representing both sides of the fault plane). This implies that our empirical scaling equations for \( F(\omega) \) [Trifunac 1989; 1993; 1994], including their extrapolation to the fault surface, are satisfactory.

In the following example, we show how strong motion data can be used to compute magnitude of an earthquake. We present this example for the Northridge, California, earthquake of 17 January, 1994. Fig. 2.3 shows the horizontal projection of the fault plane (dashed lines). The gray shades in this figure outline the areas where the basement
Fig. 2.1 Fourier amplitude spectra of ground acceleration at a site on basement rock (s=2), and "rock" soil (s_t=0), for earthquake magnitudes $M = 4, 5, 6$ and 7, zero source depth and epicentral distance $R = 10$ km. The shaded region indicates the range where the empirical scaling laws apply [Trifunac 1989].

Fig. 2.2 Energy computed from recorded strong motion velocity, extrapolated to the fault surface ($\rho_{\text{source}} \beta_{\text{source}} A_{\text{source}} \int_0^T v^2 dt$) versus seismological (empirical) definition of $E_s$ in terms of earthquake magnitude.
Fig. 2.3 Location of twenty-five strong motion stations on basement rock which recorded the Northridge, California, earthquake of 17 January, 1994.
Fig. 2.4 Surface-wave magnitude, $M_s$, computed from $E_s$ (corrected for geometric spreading and inelastic attenuation), versus epicentral distance. All points shown are for stations located on basement rock (see Fig. 2.3).
rock is essentially at the surface. To minimize the effects of amplification by sediments and thus simulate conditions of typical seismological observatories, we consider only those sites of strong motion accelerographs that recorded this event on basement “rock”. This requirement leads to selection of 25 strong motion stations, shown in Fig. 2.3. At each of these stations, we compute \[ \int_0^T v^2(t) \, dt \] for the entire duration of all available recorded components, and assume that the result is a fair approximation of \( en \) in eqn (2.7). We combine contribution from three components of motion by square root of the sum of the squares. To correct for average inelastic attenuation and geometric spreading, we multiply all the recorded velocities by \( \Delta \exp \left( \frac{\omega \Delta}{2Q\beta} \right) \) where \( \Delta \) is hypocentral distance and \( Q \) is the quality factor. We assume \( \omega \sim 6.28 \text{ rad/s}, \beta \sim 1 \text{ km/s} \) and \( Q = 500 \). Finally, we multiply the integrals of velocities squared by \( \rho_{\text{source}} \beta_{\text{source}} A_{\text{source}} \) and assume that \( \rho_{\text{source}} = 1.6 \times 10^3 \text{ kg/m}^3 \), \( \beta_{\text{source}} = 1 \text{ km/s} \) and \( A_{\text{source}} = 100 \text{ km}^2 \). From the computed \( E_s \) (using eqn (2.5)), we evaluate \( M_s \) using equation (2.2). The results are shown in Fig. 2.4. The data indicates average estimate equal to \( M_s = 6.7 \), and standard deviation of 0.20. This is in good agreement with other magnitude estimates for the Northridge earthquake (local magnitude \( M_L = 6.4 \) and moment magnitude \( M_w = 6.7 \)).

Other examples of using wave energy in interpretation of recorded strong motion accelerations can be found in Trifunac [1972 a,b].

### 2.2 Energy Propagation and Attenuation

The nature of growth of the integral in eqn (2.5) versus time is shown in Fig. 2.5. It increases rapidly, at first, and then tends asymptotically towards its final value. The time during which this integral grows rapidly is called “strong motion duration” and in this example corresponds to realization of 90 percent of strong motion energy at this site. Duration of strong motion increases with magnitude, epicentral distance, depth and dimensions of sedimentary layers through which the waves propagate (see the review article by Trifunac and Novikova [1994]).

Figures 2.6 and 2.7 show the spatial distribution of the durations (in seconds) of strong motion during the 1994 Northridge earthquake. The duration of acceleration shows the duration of high frequency motion (Fig. 2.6) while the duration of velocity represents the duration of intermediate frequencies (Fig. 2.7). Both groups of figures show how the duration increases with epicentral distance, and how its complexity is influenced by the irregular deep sediments of the Los Angeles basin.
Fig. 2.5 Strong motion velocity (top), “wave energy” ($\int_0^t v^2(\tau)d\tau$) and its (90 percent) duration (center), and maximum incident power ($\frac{\partial}{\partial t} \int_0^t v^2d\tau$) (bottom).
Fig. 2.6a Contours of the duration of horizontal acceleration (seconds).
Fig. 2.6b Contours of the duration of vertical acceleration (seconds).
Fig. 2.7a Contours of the duration of horizontal velocity (seconds).
Fig. 2.7b  Contours of the duration of vertical velocity (seconds).
$\log_{10} \int_0^t a_h^2(\tau) d\tau \text{ (cm}^2/\text{s}^3)$

Fig. 2.8a  Contours of $\log_{10} \int_0^t a_h^2(\tau) d\tau$ cm$^2$/s$^3$. 
Fig. 2.8b  Contours of $\log_{10} \int_0^t a^2_c(\tau) d\tau$ (cm$^2$/s).
Fig. 2.9a Contours of $\log_{10} \int_0^T \nu_{h}^2(\tau) d\tau$ (cm$^2$/s).
\[ \log_{10} \int_0^t v_v^2(\tau) d\tau \text{ cm}^2/\text{s} \]

Fig. 2.9b Contours of \( \log_{10} \int_0^t v_v^2(\tau) d\tau \text{ cm}^2/\text{s} \).
Fig. 2.10a Contours of $\log_{10}$ (average “power” of $a_h(t)$) (cm$^2$/s$^4$).
Fig. 2.10b Contours of $\log_{10}$ (average "power" of $a_r(t)$) (cm$^2$/s$^2$).
Fig. 2.11a Contours of $\log_{10}$ (average “power” of $v_h(t)$) (cm$^2$/s$^2$).
Fig. 2.11b  Contours of \( \log_{10} \) (average “power” of \( v(t) \)) \((\text{cm}^2/\text{s}^2)\).
Fig. 2.12a Contours of $\log_{10}$ (maximum “power” of $a_h(t)$) (cm$^2$/s$^4$).
Fig. 2.12b Contours of $\log_{10}$ (maximum "power" of $a_v(t)$) (cm$^2$/s$^4$).
Fig. 2.13a Contours of $\log_{10}$ (maximum “power” of $v_h(t)$) (cm$^2$/s$^2$).
Fig. 2.13b Contours of $\log_{10}$ (maximum "power" of $v(t)$ (cm$^2$/s$^2$).
Figures 2.8 and 2.9 show contours of the logarithms of the integrals $IA = \int a^2 dt$ (in cm$^2$/s$^3$) and of $IV = \int v^2 dt$ (in cm$^2$/s). It is seen that, for horizontal strong motion, $10^{3.2} < IA < 10^6$ cm$^2$/s$^3$ and $10^{3.2} < IV < 10^6$ cm$^2$/s. In San Fernando Valley and in the Los Angeles – Santa Monica areas, where the Northridge earthquake damaged many structures [Trifunac and Todorovska, 1997a,b; 1999; 2001], $IV$ is greater than $10^5$ cm$^2$/s.

Figures 2.10 and 2.11 present contours of the logarithms of average power for $IA$ and $IV$, while Figures 2.12 and 2.13 present contours of the logarithms of maximum power for $IA$ and $IV$.

2.3 Energy during Soil-Foundation-Structure System Response

2.3.1 Model

To describe the energy flow through a soil-foundation-structure system, we use the model shown in Fig. 2.14. In this model, the building is represented by an equivalent single-degree-of-freedom oscillator founded on a “rigid” embedded rectangular foundation. The top of the foundation is at the same level as the surface of the soil. The soil has shear modulus $G$, shear wave velocity $\beta$, Poisson’s ratio $\nu$, and mass density $\rho_s$. The oscillator has only one degree-of-freedom with respect to the foundation, $\theta_{rel}$. The mass of the oscillator is $m_b$. It has height $H$ and radius of gyration $r_b$. The oscillator is connected to the foundation at point $O$ through a rotational spring and a viscous damper. The spring has stiffness $K_b$, and the viscous damper has damping constant $C_b$. The stiffness is chosen such that the natural period of the oscillator, $T_1$, is equal to the period of the fundamental mode of the building. Assuming that the equivalent SDOF oscillator has same mass per unit length as the real building, then, $H$ and $r_b$ are related to $H_{sb}$ and $W_{sb}$, the height and width of the real building [Todorovska and Trifunac, 1993], as

$$H = \frac{H_{sb}}{\sqrt{3}}, \quad r_b = \frac{W_{sb}}{\sqrt{3}} \quad (2.10)$$

The rectangular foundation has width $W_{sb}$, depth $D$, mass $m_f$, and mass moment of inertia $I_f$. To simplify the analysis, we assume that the stiffness of the soil in the vertical direction is infinite. The foundation has two degrees-of-freedom with respect to its center of gravity (point $CG$): horizontal translation, $u$, and rotation, $\phi$. The foundation is surrounded by springs and dashpots, which model the reactive forces caused by deformation developed in the soil [Richart et al., 1970]. In Fig. 2.14, $k_h$ and $c_h$ are the stiffness and damping constants of horizontal springs and dashpots around the foundation representing the horizontal reactive forces on the vertical faces of the foundation; $k_s$ and
Fig. 2.14 The model of a soil-foundation-building system in undeformed (left) and deformed (right) configurations.
$c_s$ are the stiffness and damping constants of horizontal springs and dashpots at the base of the foundation representing the shear forces acting on the interface; and $K_r$ and $C_r$ are the rotational stiffness and damping constant representing the resisting moments in the half-space. Evaluation of the stiffness ($k_h$, $k_s$ and $K_r$) and damping ($c_h$, $c_s$ and $C_r$) constants is discussed in Appendix A. This soil-foundation-structure system is subjected to horizontal and vertical excitations ($u_g$ and $v_g$).

In our modeling, we assume the foundation is rigid, as commonly done, to reduce the number of degrees-of-freedom of the model [Duncan, 1952; Trifunac and Todorovska, 2001]. Such models give an approximation of the system response for long wavelengths relative to the foundation dimensions [Lee, 1979]. For short wavelengths, this assumption can result in nonconservative estimates of the relative deformations in the structure [Trifunac, 1997; Trifunac and Todorovska, 1997b] and, in general, is expected to result in excessive estimates of scattering of the incident wave energy and in excessive radiation damping [Todorovska and Trifunac 1990a,b,c; 1991; 1992; 1993]. The extent to which this simplifying assumption is valid depends on the stiffness of the foundation system relative to that of the soil, and also on the overall rigidity of the structure [Iguchi and Luco, 1982; Liou and Huang, 1994; Hayir et al., 2001; Todorovska et al., 2001a,b,c; Trifunac et al.,1999a].

Ivanović et al. [2000] and Trifunac et al. [2001a,b,c] suggested that the soil behavior is nonlinear during most earthquakes. To investigate this, the oscillator can be assumed to deform in the linear strain range, and the nonlinear force-deformation relation of the soil can be analyzed. To simulate nonlinear behavior of the soil, we assume the soil on the side of the foundation is represented by a hysteretic slip model (see Fig. 2.15a). The slip model emphasizes the pinching effects of soil with large stresses and the gap generated by soil compression. For the soil at the base of the foundation, the bilinear and softening characteristics are represented in Fig. 2.15b. Detailed description of these slip and bilinear hysteretic systems is discussed in Appendix B.

### 2.3.2 Equations of Motions

The equations of motion for the system are derived and solved including the nonlinear geometry and soil behavior, coupling of the vertical acceleration with the rocking and horizontal translation, and the effects of gravity forces ($m_b g$ and $m_f g$). We do not have to employ the small deformation assumption, and arbitrary material nonlinearity is allowed. The response of the building and of the soil can enter inelastic range during strong ground shaking. In that case, however, the analysis of soil-structure interaction (SSI) will be quite complicated. Consequently, at first, the building will be assumed to be linear, and the soil will be assumed to exhibit inelastic force-deformation relation.
Fig. 2.15 Force-deformation relations used to represent the nonlinear behavior of the soil.

Fig. 2.16 Free-body diagrams for the deformed model in Fig. 2.14.
Referring to Fig. 2.16, the total displacements due to horizontal, vertical and rocking motion of the center of mass of the foundation (point $CG$) are

\[ u_{CG} = u_g + u \]  
\[ v_{CG} = v_g \]  
\[ \phi_{CG} = \phi \]

and those of point $O$ are

\[ u_O = u_{CG} + \frac{D}{2} \sin \phi \]  
\[ v_O = v_{CG} + \frac{D}{2} (1 - \cos \phi) \]  
\[ \phi_O = \phi \]

Similarly, the total displacements of the center of mass of the building are

\[ u_b = u_{CG} + \frac{D}{2} \sin \phi + H \sin(\phi + \theta_{rel}) \]  
\[ v_b = v_{CG} + \frac{D}{2} (1 - \cos \phi) + H[1 - \cos(\phi + \theta_{rel})] \]  
\[ \phi_b = \phi + \theta_{rel} \]

The equations of motion for the system then can be derived from the equilibrium of forces and moments. From the equations of dynamic equilibrium of forces in the horizontal and vertical directions, and all moments acting on the oscillator about point $O$, the interactive forces and the moment between the oscillator and foundation are

\[ \Sigma F_x = 0 \Rightarrow f_{x,b} = m_b \ddot{u}_b \]  
\[ \Sigma F_z = 0 \Rightarrow f_{z,b} = -m_b (\ddot{v}_b - g) \]  
\[ \Sigma M_o = 0 \Rightarrow M_{o,b} = -m_b \ddot{u}_o H \cos(\phi + \theta_{rel}) - m_b (\ddot{v}_o - g) H \sin(\phi + \theta_{rel}) - (m_b \dot{v}_b^2 + m_b H^2) (\ddot{\phi} + \dot{\theta}_{rel}). \]
Since
\[ M_{O,b} = K_b \theta^{rel} + C_b \dot{\theta}^{rel} \] (2.23)
the substitution of equation (2.23) into (2.22) gives
\[ I_O (\ddot{\phi} + \ddot{\theta}^{rel}) + m_b \ddot{u}_O H \cos(\phi + \theta^{rel}) + m_b (\ddot{v}_O - g) H \sin(\phi + \theta^{rel}) + K_b \theta^{rel} + C_b \dot{\theta}^{rel} = 0 \] (2.24)
where \[ I_O = m_b r_o^2 \left[ 1 + \left( \frac{r_o}{H} \right)^2 \right] \]

From the equations of dynamic equilibrium of all forces and moments acting on the foundation about point \( CG \), it follows that
\[ \Sigma F_x = 0 \Rightarrow \sum_{i=1}^{m} f_{h,i} + \sum_{j=1}^{n} f_{s,j} = -m_j \ddot{u}_{CG} - f_{x,b} \] (2.25)
\[ \Sigma F_z = 0 \Rightarrow f_{z,f} = -m_j (\ddot{v}_{CG} - g) + f_{z,b} \] (2.26)
\[ \Sigma M_{CG} = 0 \Rightarrow M_{B,f} = (f_{z,b} + f_{z,f}) \frac{D}{2} \sin \phi - f_{x,b} \frac{D}{2} \cos \phi + M_{O,b} - I_f \ddot{\phi} \]
\[ - \sum_{i=1}^{m} (f_{h,i} \cos \phi) d_i + \sum_{i=1}^{m} (f_{h,i} \sin \phi) \frac{W_{ib}}{2} - \sum_{j=1}^{n} (f_{s,j} \sin \phi) l_i + \sum_{j=1}^{n} (f_{s,j} \cos \phi) \frac{D}{2} \] (2.27)
and
\[ M_{B,f} = K_f \phi + C_f \dot{\phi} \] (2.28)

In equations (2.25), (2.26) and (2.27), \( f_{h,i} \) and \( f_{s,j} \), represent the spring and damping forces of the soil, and can be expressed as
\[ f_{h,i} = f_{h,i}^o + f_{h,i}^s = c_{h,i} \ddot{u}_i + k_{h,i} u_i \] (2.29)
\[ f_{s,j} = f_{s,j}^o + f_{s,j}^s = c_{s,j} \ddot{u}_j + k_{s,j} u_j \] (2.30)
in which
\[ u_i = u + d_i \sin \phi - \frac{W_{ib}}{2} (1 - \cos \phi), \quad \dot{u}_i = \dot{u} + d_i \cos \phi \ddot{\phi} - \frac{W_{ib}}{2} \sin \phi \dot{\phi} \] (2.31)
\[ u_j = u - \frac{D}{2} \sin \phi + l_j (1 - \cos \phi) , \quad \dot{u}_j = \dot{u} - \frac{D}{2} \cos \phi \dot{\phi} + l_j \sin \phi \dot{\phi} \] (2.32)

\( d_i \) is the distance from the \( x \)-axis to the point where \( f_{h,i} \) is acting, and \( l_j \) is the distance from the \( z \)-axis to the point where \( f_{s,j} \) is acting. It should be noted that the stiffness and damping constants are associated with yielding of soil. Hence, \( c_{h,i} \), \( k_{h,i} \), \( c_{s,j} \) and \( k_{s,j} \) are time dependent.

Next, from equations (2.11), (2.12), (2.17) and (2.18) it follows

\[ \ddot{u}_{CG} = \ddot{u}_g + \ddot{u} \] (2.33)

\[ \ddot{v}_{CG} = \ddot{v}_g \] (2.34)

\[ \ddot{u}_b = \ddot{u}_{CG} + \frac{D}{2} (-\sin \phi \ \dddot{\phi}^2 + \cos \phi \ \dddot{\phi}) + H[-\sin(\phi + \theta^{rel})(\ddot{\phi} + \dot{\theta}^{rel})^2 \right. \]

\[ \left. + \cos(\phi + \theta)(\dddot{\phi} + \dddot{\theta}^{rel}) \right] \] (2.35)

and

\[ \ddot{v}_b = \ddot{v}_{CG} - \frac{D}{2} (-\cos \phi \ \dddot{\phi}^2 - \sin \phi \ \dddot{\phi}) - H[-\cos(\phi + \theta^{rel})(\ddot{\phi} + \dot{\theta}^{rel})^2 \right. \]

\[ \left. - \sin(\phi + \theta)(\dddot{\phi} + \dddot{\theta}^{rel}) \right]. \] (2.36)

Then, in eqns (2.20) through (2.22), \( f_{x,b} \), \( f_{z,b} \) and \( M_{o,b} \) can be expressed in terms of \( \theta^{rel}, u \) and \( \phi \)

\[ f_{x,b} = m_b (\ddot{u}_g + \ddot{u}) + m_b \frac{D}{2} (-\sin \phi \ \dddot{\phi}^2 + \cos \phi \ \dddot{\phi}) \]

\[ + m_b H[-\sin(\phi + \theta^{rel})(\ddot{\phi} + \dot{\theta}^{rel})^2 + \cos(\phi + \theta)(\dddot{\phi} + \dddot{\theta}^{rel})] \] (2.37)

\[ f_{z,b} = -m_b (\ddot{v}_g - g) + m_b \frac{D}{2} (-\cos \phi \ \dddot{\phi}^2 - \sin \phi \ \dddot{\phi}) \]

\[ + m_b H[-\cos(\phi + \theta^{rel})(\ddot{\phi} + \dot{\theta}^{rel})^2 - \sin(\phi + \theta)(\dddot{\phi} + \dddot{\theta}^{rel})] \] (2.38)

and

\[ M_{o,b} = -m_b (\ddot{u}_g + \ddot{u}) H \cos(\phi + \theta^{rel}) - m_b \frac{D}{2} (-\sin \phi \ \dddot{\phi}^2 + \cos \phi \ \dddot{\phi}) H \cos(\phi + \theta^{rel}) \]

\[ - m_b (\ddot{v}_g - g) H \sin(\phi + \theta^{rel}) + m_b \frac{D}{2} (-\cos \phi \ \dddot{\phi}^2 - \sin \phi \ \dddot{\phi}) H \sin(\phi + \theta^{rel}) \]

\[ - I_o (\dddot{\phi} + \dddot{\theta}^{rel}) \] (2.39)
so that, eqn (2.24) becomes

\[
I_o(\phi + \theta^{rel}) + m_b(\ddot{u}_g + \ddot{u})H\cos(\phi + \theta^{rel}) + m_b \frac{D}{2}(-\sin\phi \quad \phi^2 + \cos\phi \quad \phi)H\cos(\phi + \theta^{rel}) \\
+ m_b(\ddot{v}_g - g)H\sin(\phi + \theta^{rel}) - m_b \frac{D}{2}(-\cos\phi \quad \phi^2 - \sin\phi \quad \phi)H\sin(\phi + \theta^{rel}) + K_b\theta^{rel} + C_b\dot{\theta}^{rel} = 0
\]  

(2.40)

Now we substitute eqn (2.37) into (2.25), (2.38) into (2.26), and obtain

\[
m_f(\ddot{u}_g + \ddot{u}) + m_b(\ddot{u}_g + \ddot{u}) + m_b \frac{D}{2}(-\sin\phi \quad \phi^2 + \cos\phi \quad \phi) \\
+ m_bH[-\sin(\phi + \theta^{rel})\dot{\phi} + \theta^{rel})^2 + \cos(\phi + \theta)(\dot{\phi} + \dot{\theta}^{rel})] + \sum_{i=1}^{m} f_{h,i} + \sum_{j=1}^{n} f_{s,j} = 0
\]  

(2.41)

and

\[
f_{z,f} = -m_f(\ddot{v}_g - g) - m_b(\ddot{v}_g - g) + m_b \frac{D}{2}(-\cos\phi \quad \phi^2 - \sin\phi \quad \phi) \\
+ m_bH[-\cos(\phi + \theta^{rel})(\dot{\phi} + \dot{\theta}^{rel})^2 - \sin(\phi + \theta)(\dot{\phi} + \dot{\theta}^{rel})]
\]  

(2.42)

Similarly, we substitute eqns (2.28), (2.37) through (2.39) and (2.42) into (2.27) to get

\[
K_\phi + C_\phi \dot{\phi} + m_b(\ddot{v}_g - g)D\sin\phi - m_b \frac{D^2}{2}(-\cos\phi \quad \phi^2 - \sin\phi \quad \phi)\sin\phi \\
- m_bHD[-\cos(\phi + \theta^{rel})(\dot{\phi} + \dot{\theta}^{rel})^2 - \sin(\phi + \theta^{rel})(\dot{\phi} + \dot{\theta}^{rel})^2] \sin\phi \\
+ m_f(\ddot{v}_g - g) \frac{D}{2} \sin\phi + m_b(\ddot{u}_g + \ddot{u}) \frac{D}{2} \cos\phi + m_b \frac{D^2}{4}(-\sin\phi \quad \phi^2 + \cos\phi \quad \phi) \cos\phi \\
+ m_bH \frac{D}{2}[-\sin(\phi + \theta^{rel})(\dot{\phi} + \dot{\theta}^{rel})^2 + \cos(\phi + \theta^{rel})(\dot{\phi} + \dot{\theta}^{rel})] \cos\phi \\
+ m_b(\ddot{u}_g + \ddot{u})H \cos(\phi + \theta^{rel}) + m_bH \frac{D}{2}(-\sin\phi \quad \phi^2 + \cos\phi \quad \phi) \sin(\phi + \theta^{rel}) \\
+ m_b(\ddot{v}_g - g)H \sin(\phi + \theta^{rel}) - m_bH \frac{D}{2}(-\cos\phi \quad \phi^2 - \sin\phi \quad \phi) \sin(\phi + \theta^{rel}) \\
+ I_o(\ddot{\phi} + \ddot{\theta}^{rel}) + I_f \ddot{\phi} + \sum_{i=1}^{m} f_{h,i}(d_i \cos\phi - \frac{W_{max}}{2} \sin\phi) + \sum_{j=1}^{n} f_{s,j}(l_j \sin\phi - \frac{D}{2} \cos\phi) = 0
\]  

(2.43)

The general equations of motion for the nonlinear soil-foundation-oscillator system are eqns (2.40), (2.41) and (2.43). These equations are in terms of three principal unknown displacements. The unknown displacements then can be solved numerically in time domain, for example by the Runge-Kutta method.
The response of the foundation will contribute towards dissipation of the earthquake energy. Because of this, the overall displacement of the equivalent oscillator representing the building will be decreased. Also, the dynamic stability of the system may be important. Therefore it is necessary to consider the system in which the displacements of the foundation are included explicitly in the dynamic equilibrium equations of the system. By assuming the displacements are small, the general equations of motion can be simplified, without losing the coupling terms, the gravity force effects and the vertical acceleration effects.

2.3.3 Dissipation of Energy

When a system is subjected to earthquake shaking, the incident wave energy propagates into it. During strong ground motion, part of this incident energy is dissipated by scattering from the foundation and by the deformation of the soil, and the rest is transmitted into the building.

Referring to the free-body diagram in Fig. 2.16, all forces and moments move through the corresponding displacements and thus do work. To evaluate this work, we integrate all six equilibrium equations of the system, eqns (2.20), (2.21), (2.24), (2.25), (2.26) and (2.27), with respect to their corresponding displacements, as follows.

\[
\int_{u_b} \text{eqn (2.20)} du_b = \int (m_b \ddot{u}_b - f_{s,b}) \, du_b = 0
\]  
\( (2.44) \)

\[
\int_{v_b} \text{eqn (2.21)} dv_b = \int [m_b(\ddot{v}_b - g) + f_{z,b}] \, dv_b = 0
\]  
\( (2.45) \)

\[
\int_{\phi_b} \text{eqn (2.24)} d\phi_b = \int [m_b \ddot{\phi}_b H \cos \phi_b + m_b(\ddot{V}_o - g)H \sin \phi_b + I_o \dddot{\phi}_b + K_o \theta^{rel}_o + C_b \dot{\theta}^{rel}_b] \, d\phi_b = 0
\]  
\( (2.46) \)

\[
\int_{u_{CG}} \text{eqn (2.25)} du_{CG} = \int (m_j \ddot{u}_{CG} + f_{x,b} + \sum_{i=1}^{m} f_{h,i} + \sum_{j=1}^{n} f_{s,j}) \, du_{CG} = 0
\]  
\( (2.47) \)

\[
\int_{v_{CG}} \text{eqn (2.26)} dv_{CG} = \int [m_j(\ddot{v}_{CG} - g) - f_{z,b} + f_{z,f}] \, dv_{CG} = 0
\]  
\( (2.48) \)

and
\[ \int_{\phi_{CG}} \text{d}\phi_{CG} = \int (K_r \phi + C \phi - f_{z,b} \frac{D}{2} \sin \phi - f_{z,f} \frac{D}{2} \sin \phi + f_{s,b} \frac{D}{2} \cos \phi - K_b \theta_{rel}^r - C_b \dot{\theta}_{rel}^r + I_f \ddot{\phi} + \sum_{i=1}^{m} f_{h_i} \cos \phi d_i - \sum_{i=1}^{m} f_{h_i} \sin \phi \frac{W_{ib}}{2} + \sum_{j=1}^{n} f_{s,j} \sin \phi l_j - \sum_{j=1}^{n} f_{s,j} \cos \phi \frac{D}{2} \dot{\phi}_{CG} \text{d}\phi_{CG} = 0 \] (2.49)

The total work done in the system is then computed by superposition of eqns (2.44) through (2.49).

These contributions to the system energy can be evaluated by rewriting the integrals with respect to time. Thus

eqn (2.44) \( \Rightarrow \int (m_j \ddot{u}_b - f_{x,b}) u_b \text{d}t = 0 \) (2.50)

eqn (2.45) \( \Rightarrow \int [m_b (\ddot{u}_b - g) + f_{z,b}] \dot{v}_b \text{d}t = 0 \) (2.51)

eqn (2.46) \( \Rightarrow \int [m_j \ddot{u}_o H \cos \phi_b + m_b (\ddot{u}_o - g) H \sin \phi_b + I_o \dddot{\phi}_b + K_b \theta_{rel}^r + C_b \dot{\theta}_{rel}^r] \dot{\phi}_b \text{d}t = 0 \) (2.52)

eqn (2.47) \( \Rightarrow \int (m_j \ddot{u}_{CG} + f_{s,b} + \sum_{i=1}^{m} f_{h_i} + \sum_{j=1}^{n} f_{s,j}) u_{CG} \text{d}t = 0 \) (2.53)

eqn (2.48) \( \Rightarrow \int [m_j (\ddot{v}_{CG} - g) - f_{x,b} + f_{z,f}] \dot{v}_{CG} \text{d}t = 0 \) (2.54)

and

eqn (2.49) \( \Rightarrow \int (K_r \phi + C \phi - f_{z,b} \frac{D}{2} \sin \phi - f_{z,f} \frac{D}{2} \sin \phi + f_{s,b} \frac{D}{2} \cos \phi - K_b \theta_{rel}^r - C_b \dot{\theta}_{rel}^r + I_f \ddot{\phi} + \sum_{i=1}^{m} f_{h_i} \cos \phi d_i - \sum_{i=1}^{m} f_{h_i} \sin \phi \frac{W_{ib}}{2} + \sum_{j=1}^{n} f_{s,j} \sin \phi l_j - \sum_{j=1}^{n} f_{s,j} \cos \phi \frac{D}{2} \dot{\phi}_{CG} \text{d}\phi_{CG} = 0 \) (2.55)

Recalling eqns (2.11) through (2.19), the above equations can be expressed in terms of \( \theta_{rel}^r, u \) and \( \phi \).
To simplify these energy formulae, we keep only the first order terms of the Taylor series expansions of the angles $\theta^\text{rel}$ and $\phi$ (and their linear combination), and eliminate the products of small angles and of their derivatives. Then, the above six equations give

\[
\int \left\{ m_b \left( \ddot{u} + \frac{D}{2} \dddot{\phi} + H \left( \dddot{\phi} + \dddot{\theta}^\text{rel} \right) \right) - f_{x,b} \right\} \dot{u}_b \, dt = -\int m_b \dddot{u}_b \ddot{u}_b \, dt \tag{2.56}
\]

\[
\int (-m_b g + f_{z,b}) \dot{v}_b \, dt = -\int m_b \dddot{v}_b \ddot{v}_b \, dt \tag{2.57}
\]

\[
\int \left[ m_b (\dddot{u}_o + \frac{D}{2} \dddot{\phi}) H - m_b g H \phi_b + I_{o\phi} \dddot{\phi}_b + K_b \theta^\text{rel} + C_b \dot{\theta}^\text{rel} \right] \dot{\phi}_b \, dt = -\int \left( m_b \dddot{u}_g \, H + m_b \dddot{v}_g \, H \phi_b \right) \dot{\phi}_b \, dt \tag{2.58}
\]

\[
\int (m_j \dddot{u}_b + f_{x,b}) \dot{u}_CG \, dt + \int \left\{ \sum_{i=1}^{m} f_{h,i} + \sum_{j=1}^{n} f_{s,j} \right\} \dot{u} \, dt = -\int m_j \dddot{u}_g \ddot{u}_CG \, dt \tag{2.59}
\]

\[
\int (-m_j g - f_{z,b} + f_{z,f}) \dot{v}_CG \, dt = -\int m_j \dddot{v}_g \ddot{v}_CG \, dt \tag{2.60}
\]

and

\[
\int (K_b \phi + C_b \dot{\phi} - f_{z,b} \frac{D}{2} \phi - f_{z,f} \frac{D}{2} \phi + f_{x,b} \frac{D}{2} - K_b \theta^\text{rel} - C_b \dot{\theta}^\text{rel} + I_{f\phi} + \sum_{i=1}^{m} f_{h,i} d_i - \sum_{i=1}^{m} f_{h,i} \phi \frac{W_{ub}}{2} + \sum_{j=1}^{n} f_{s,j} \phi l_i - \sum_{j=1}^{n} f_{s,j} \frac{D}{2} \phi CG \, dt = 0 \tag{2.61}
\]

Next, we group the energy terms, according to their physical nature, into the following categories:

\[
E_K(t) = \text{kinetic energy}
\]

\[
E_p(t) = \text{potential energy of gravity forces}
\]

\[
E_D^{\text{bldg}}(t) = \text{damping energy dissipated in the building}
\]

\[
E_E^{\text{bldg}}(t) = \text{recoverable elastic strain energy in the building}
\]

\[
E_D^{\text{soil}}(t) = \text{energy dissipated by dashpots of the soil}
\]
\[ E_s^{\text{soil}}(t) = \text{elastic strain energy in the soil} \]

\[ E_Y^{\text{soil}}(t) = \text{irrecoverable hysteretic energy in the soil} \]

\[ E_{S,Y}^{\text{soil}}(t) = E_s^{\text{soil}}(t) + E_Y^{\text{soil}}(t) \]

\[ E_I(t) = \text{total earthquake input energy.} \]

First, based on eqns (2.56) through (2.61), the earthquake input energy is the sum

\[ E_I(t) = -\int [m_b \dddot{u}_b + m_b \dddot{v}_b + (m_b \dddot{u}_b + m_b \dddot{v}_b H + m_f \dddot{v}_b H \phi_b) \phi_b + m_f \dddot{u}_b \phi_{CG} + m_f \dddot{v}_b \phi_{CG}] \, dt \quad (2.62) \]

The kinetic energy associated with absolute motion of masses is

\[ E_K(t) = \int \left\{ m_b \dddot{u}_b + \frac{D}{2} \dddot{\phi} + H \dddot{\phi}_b \right\} \dddot{\phi}_b + [m_b \dddot{u}_b + \frac{D}{2} \dddot{\phi} + I_c \dddot{\phi}_b] \dddot{\phi}_b + m_f \dddot{u}_b \phi_{CG} + I_f \dddot{\phi}_b \phi_{CG} \, dt \quad (2.63) \]

It can be seen that \( E_K(t) \) is equal to the integrals of the inertial forces with respect to their absolute velocities. The potential energy associated with the gravity forces is

\[ E_P(t) = -\int (m_b \dddot{v}_b + m_b g \dddot{\phi}_b + m_f g \dddot{\phi}_{CG}) \, dt \quad (2.64) \]

The energy dissipated by viscous damping in the building can be calculated from

\[ E_D^{\text{bldg}}(t) = \int C_b (\dddot{\phi}^{rel})^2 \, dt \quad (2.65) \]

The recoverable strain energy of the building (for linear response) is

\[ E_s^{\text{bldg}}(t) = \frac{1}{2} K_b (\theta^{rel})^2 \quad (2.66) \]

For this illustration the building is assumed to deform in linear manner only, the irrecoverable hysteretic energy in the building will be zero. Then, the energy dissipated by the dashpots in the soil is
The energy dissipated by the yielding and the recoverable strain energy of the soil can be obtained from

\[
E^\text{soil}_D(t) = \int \left( \sum_{i=1}^{m} f^D_{h,i} + \sum_{j=1}^{n} f^D_{s,j} \right) \ddot{u} \, dt + \int C_i \dot{\phi}^2 \, dt
+ \int \left\{ \sum_{i=1}^{m} f^D_{h,i} \left( d_i - \frac{W_{sb}}{2} \phi \right) + \sum_{j=1}^{n} f^D_{s,j} \left( l_j \phi - \frac{D}{2} \right) \right\} \dot{\phi} \, dt
\]  
(2.67)

The energy dissipated by the yielding and the recoverable strain energy of the soil can be obtained from

\[
E^\text{soil}_S(t) = \int \left( \sum_{i=1}^{m} f^S_{h,i} + \sum_{j=1}^{n} f^S_{s,j} \right) \ddot{u} \, dt + \int K_i \dot{\phi} \, dt
+ \int \left[ \sum_{i=1}^{m} f^S_{h,i} \left( d_i - \frac{W_{sb}}{2} \phi \right) + \sum_{j=1}^{n} f^S_{s,j} \left( l_j \phi - \frac{D}{2} \right) \right] \dot{\phi} \, dt
\]  
(2.68)

in which

\[
E^\text{soil}_S(t) = \sum_{i=1}^{m} \left( \frac{f^S_{h,i}}{2k_{h,i}} + \frac{f^S_{s,j}}{2k_{s,j}} \right) + \frac{K_i \phi^2}{2}
\]  
(2.69)

where \(k_{h,i}\) and \(k_{s,j}\) are the initial stiffnesses of the inelastic soil.

Based on these energy “components”, the statement of energy balance of the system is then expressed as

\[
E_k(t) + E_P(t) + E^\text{bldg}_D(t) + E^\text{bldg}_S(t) + E^\text{soil}_D(t) + E^\text{soil}_S(t) = E_f(t)
\]  
(2.70)

The foregoing analysis of the nonlinear system models the energy components of the simple nonlinear SSI system in Fig. 2.14, rather than that of the fixed-base system [Akiyama, 1985; 1988; 1997; Uang and Bertero, 1988].
3. STUDIES OF THE HOLLYWOOD STORAGE BUILDING

The Hollywood Storage Building (HSB, Fig. 3.1) is the first structure in California equipped with permanent strong motion accelerographs, in 1933. It is also the first building in California where strong motion was recorded (October, 1933), and also the first building for which it could be shown that both theoretical analysis and observation of soil-structure interaction are consistent [Duke et al., 1970]. This building served as a testing ground for intuitive, theoretical and quantitative [Duke et al., 1970] studies of soil-structure interaction. The data recorded in and near this building was also used in several other related studies, reviewed by Trifunac et al. [2001c].

3.1 Description of the building

The Hollywood Storage Building is 217 ½ feet east to west by 51 feet south to north (1 foot=30.48 cm). The roof low point is 148 feet 9 inches above the first floor, and the typical story height is 10 feet 6 inches (1 inch=2.54 cm). A basement, nine feet below the first floor, underlies the six westerly bays of the structure. A one-story structure of timber truss roof construction abuts the west wall and the westerly portion of the north wall.

The fifth to twelfth floor framing plan, illustrated in Figs 3.2 through 3.4, is representative of the third to thirteenth floors inclusively. The framing for the east ten bays of each floor is a flat slab with drop panels and column capitals, while the west three bays at the freight elevators are of joist and beam construction. In the transverse direction, the columns and flat slabs comprise three bay moment frames. The sizes for any given column type vary within any given story.

The fourteenth floor is typically concrete joist and beam construction, with girders at selected lines, and with clear span over two bays in the transverse direction. Where spanning two bays, the girders are concrete encased structural steel. The roof construction is typically of concrete joists framing to concrete encased structural steel girders with clear span in all three bays in the transverse direction. Some girders along selected lines have clear span for only two bays. Elevator penthouses and machine rooms over the west three bays of the structure are typically of concrete wall and concrete joist and beam construction.

The second floor features side mezzanines created by voiding the central portion of the framing for the east five bays of the structure. The mezzanines to either side of the void are of concrete joist and beam construction, while the remaining framing is as described for the third to thirteenth stories. The first floor is typically a five-inch concrete slab on
Fig. 3.1a A sketch of the Hollywood Storage Building in the early 1950’s. The location of the strong motion accelerographs is indicated, one in the basement, one on the roof and one at a “free-field” site, 112 feet west of the south-west corner of the building (after Duke et al., 1970).
Fig. 3.1b  A sketch of the Hollywood Storage Building showing the sensor locations and their orientation for the strong motion instrumentation since 1976 (twelve accelerometers), maintained by California Division of Mines and Geology.
Fig. 3.2 Hollywood Storage Building, 5\textsuperscript{th} – 12\textsuperscript{th} floor plans.
Fig. 3.3   Hollywood Storage Building – transverse section.
Fig. 3.4 Hollywood Storage Building – selected structural details.
grade, except for the portion over the basement, which is of concrete joist and beam framing.

The wall lines are set back in varying amounts to create ledges for the local support of cast stone ornamentation at the third, thirteenth, and roof levels. The longitudinal wall lines are at the building lines in the first and second stories. From the third to twelfth stories, the longitudinal wall lines are set back four inches. Above the thirteenth floor, the longitudinal walls are set back approximately by additional five inches, and also set back diagonally across each corner of the east bay.

The west wall is an unadorned eight-inch concrete shear wall, pierced by two tiers of window openings. A loading door opening is located in the north bay, at the platform height of 3 feet 1 inch above the finish at first floor.

The north and south walls below the third floor are unadorned concrete with some large window openings in selected bays. The west five bays of the first story of the north wall have loading door openings at platform height. Above the third floor, each of the north and south walls are comprised of five shear piers separated by vertically continuous window openings. The east wall is essentially a three bay moment frame with columns above the third floor separated by vertically continuous window openings similar to those of the longitudinal walls. Below the third story, while remaining a three bay moment frame, the window treatment differs, and the wall line sets back approximately five inches from the building line to receive an elaborate cast stone facade.

All concrete was noted on the design drawings to be Grade #1, proportioned 1:2½:3½ cement, fine aggregate and coarse aggregate, respectively. Such a mix is considered to have a nominal 28-day compressive strength of 2500 psi.

The footings are constructed of driven steel shell pilings filled with Grade #1 concrete, and driven to a penetration giving an allowable loading value of 40 tons. Although a 16-foot pile length was selected as the basis for bid, in practice the piles were driven to penetration from 10 to 30 feet.

3.2 Instrumentation and Earthquake Records

The basement accelerograph (Coast and Geodetic Survey (C&GS) Standard; for location see Fig. 3.1a) was installed in June 1933. This recorder was removed in April of 1975, by the United States Geological Survey (USGS), because the California Division of Mines and Geology (CDMG) was scheduled to take over the strong motion observation at this site. In April 1976, CDMG installed a central Recording System (CRA-I) shown in Fig. 3.1b. The instrument in the parking lot (see Fig. 3.1a) was installed in December 1939. It
Fig. 3.5 Earthquakes in Southern California large enough to trigger the strong motion accelerographs in Hollywood Storage Building. The solid stars indicate events for which strong motion was recorded and is available in processed form (see Table 3.1). The open stars indicate events for which strong motion may have been recorded, but so far is not reported and strong motion data is not available in digital form.
Table 3.1  Apparent frequencies of the response during seven earthquakes, at the beginning and end of shaking, maximum and minimum values during shaking ($f_{\text{beg}}, f_{\text{end}}, f_{\text{max}}, f_{\text{min}}$), the largest difference ($\Delta f_{\text{max}} = f_{\text{max}} - f_{\text{min}}$), peak ground velocity ($v_{\text{G, max}}$), and peak instantaneous difference between the velocity recorded at the roof and at ground level ($\theta_{\text{max}}$).

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Date</th>
<th>$M_L$</th>
<th>$R$ (km)</th>
<th>$f_{\text{beg}}$</th>
<th>$f_{\text{end}}$</th>
<th>$f_{\text{max}}$</th>
<th>$f_{\text{min}}$</th>
<th>($\Delta f_{\text{max}}$)</th>
<th>$v_{\text{G, max}}$ (cm/s)</th>
<th>$\theta_{\text{max}}$ ($\times10^3$ rad/s)</th>
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<tbody>
<tr>
<td>Southern California</td>
<td>10/02/1933</td>
<td>5.4</td>
<td>38</td>
<td>0.599</td>
<td>0.671</td>
<td>0.671</td>
<td>0.599</td>
<td>0.073</td>
<td>2.47</td>
<td>0.79630</td>
</tr>
<tr>
<td>Kern County</td>
<td>07/21/1952</td>
<td>7.7</td>
<td>120</td>
<td>0.533</td>
<td>0.599</td>
<td>0.711</td>
<td>0.533</td>
<td>0.178</td>
<td>6.72</td>
<td>3.39614</td>
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<tr>
<td>Borrego Mountain</td>
<td>04/08/1968</td>
<td>6.4</td>
<td>225</td>
<td>0.780</td>
<td>0.453</td>
<td>0.780</td>
<td>0.453</td>
<td>0.327</td>
<td>2.14</td>
<td>2.12623</td>
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<tr>
<td>Lytle Creek</td>
<td>09/12/1970</td>
<td>5.4</td>
<td>74</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>0.67</td>
<td>--</td>
</tr>
<tr>
<td>San Fernando</td>
<td>02/09/1971</td>
<td>6.4</td>
<td>38</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>16.96</td>
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<tr>
<td>Whittier Narrows</td>
<td>10/01/1987</td>
<td>5.9</td>
<td>24</td>
<td>0.420</td>
<td>0.472</td>
<td>0.540</td>
<td>0.420</td>
<td>0.120</td>
<td>9.23</td>
<td>4.26965</td>
</tr>
<tr>
<td>Landers</td>
<td>06/28/1992</td>
<td>7.5</td>
<td>171</td>
<td>0.468</td>
<td>0.468</td>
<td>0.468</td>
<td>0.468</td>
<td>0.000</td>
<td>5.93</td>
<td>4.42882</td>
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<tr>
<td>Big Bear</td>
<td>06/28/1992</td>
<td>6.5</td>
<td>135</td>
<td>0.438</td>
<td>0.475</td>
<td>0.550</td>
<td>0.438</td>
<td>0.112</td>
<td>3.68</td>
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<td>Northridge</td>
<td>01/17/1994</td>
<td>6.4</td>
<td>23</td>
<td>0.462</td>
<td>0.438</td>
<td>0.462</td>
<td>0.360</td>
<td>0.102</td>
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<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Apparent EW frequency (Hz)</th>
<th>EW velocities</th>
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<td>1.710</td>
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<td>1.838</td>
<td>1.710</td>
<td>0.128</td>
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<td>1.61330</td>
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<td>Kern County</td>
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<td>1.595</td>
<td>1.950</td>
<td>1.350</td>
<td>0.600</td>
<td>8.85</td>
<td>3.52799</td>
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<td>Borrego Mountain</td>
<td>1.950</td>
<td>1.710</td>
<td>1.930</td>
<td>1.483</td>
<td>0.447</td>
<td>3.03</td>
<td>0.75532</td>
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<td></td>
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<tr>
<td>Lytle Creek</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
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<td></td>
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</tr>
<tr>
<td>San Fernando</td>
<td>--</td>
<td>--</td>
<td>--</td>
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<td>--</td>
<td>--</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Whittier Narrows</td>
<td>1.720</td>
<td>1.545</td>
<td>1.720</td>
<td>1.305</td>
<td>0.415</td>
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<td>3.00368</td>
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<tr>
<td>Landers</td>
<td>1.745</td>
<td>1.512</td>
<td>1.745</td>
<td>1.332</td>
<td>0.413</td>
<td>7.93</td>
<td>2.10864</td>
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<tr>
<td>Big Bear</td>
<td>1.750</td>
<td>1.572</td>
<td>1.750</td>
<td>1.572</td>
<td>0.178</td>
<td>4.03</td>
<td>1.20905</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Northridge</td>
<td>1.378</td>
<td>1.250</td>
<td>1.378</td>
<td>1.060</td>
<td>0.318</td>
<td>18.88</td>
<td>8.81063</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*--: record is not available at the roof.
1: see Hudson [1976]; Trifunac and Lee [1978].
*: digitized by Trifunac from xerox copies of CDMG reports.
Fig. 3.6 Directions of wave arrival from nine earthquakes recorded in Hollywood Storage Building.
was housed in 6×9 foot galvanized metal shed, 112 feet west of the west wall of the building. In April 1973, this instrument was removed. In April 1975 CDMG constructed a new free field instrument shelter, about 80 feet west of the south west corner of the building, and installed an RFT-250 accelerograph there.

Numerous earthquakes triggered the instruments both in the building and in the parking lot. However, so far the data has been digitized and processed only for nine earthquakes (Fig. 3.5). Table 3.1 summarizes the dates, magnitudes and epicentral distances for all the available digitized data. The accelerograms of Southern California, 1933, Kern County, 1952, Borrego Mountain, 1968, Lytle Creek, 1970, and San Fernando, 1971, earthquakes were digitized for the uniform strong motion data base (Hudson, 1976; Trifunac and Lee 1978), the data for Whittier-Narrows, 1987, Landers, 1992, and Big Bear, 1992, earthquakes were digitized by Trifunac from Xerox copies of CDMG reports OSMS 87-05, OSMS 92-09, and OSMS 92-10. The data for Northridge, 1994 earthquake was supplied by CDMG.

Fig. 3.5 shows the epicenters of these nine events (shown by solid stars). Many other earthquakes in the area, shown by open stars in the same Figure, could have triggered the instruments in the Hollywood storage building, but these data have not been announced and made available in digitized form. Figure 3.6 shows the plan view of the directions of wave arrival for the above nine events.

### 3.3 Time And Amplitude Dependent Response

Figures 3.7, 3.8 and 3.9 show transfer functions of the EW and NS, (roof/basement) responses recorded near the west end or center of the building. The three figures show only the frequencies up to 5 Hz. It is clear from these figures that with increasing amplitudes of the incident waves, the system becomes softer. For EW motions the system frequency is near 1.9 Hz (close to 2.0 Hz as reported by Carder, 1936; 1964) for excitation from Borrego Mountain Earthquake of 1968. As Fig. 3.7 shows, the EW system frequency decreases for larger amplitudes of motion, and is near 1.25 Hz for excitation from the Northridge earthquake. For NS translational response, the system frequency is near 0.6 to 0.7 Hz for small ground motions (Southern California earthquake of 1933) and falls to ~0.45 Hz for the largest recorded motions (Northridge earthquake of 1994). During ambient and forced vibration tests, this frequency was 0.83 Hz (Carder, 1964). The fundamental frequency in torsion was reported by Carder (1964) to be in the range 1.57–1.67 Hz. Figure 3.9 shows that it was as low as 1.1 Hz for excitation by the waves arriving from east (Fig. 3.6) during Landers earthquake of 1992.
Fig. 3.7 Amplitudes of the transfer-functions between EW translation at the roof center and at the basement center during seven earthquakes. The vertical lines show the system frequencies determined from small amplitude full-scale tests in 1938 [Carder, 1964], and from modal minimization method using recorded response to earthquake excitation [Papageorgiou and Lin, 1991].
Roof Motion Relative to Basement Motion
N-S Comp. (center)

Hollywood Storage Bldg.
- Northridge
- Whittier Narrows
- Big Bear
- Landers

Fig. 3.8  Same as Fig. 3.7 but for NS motions.
Fig. 3.9 Same as Fig. 3.7 but for NS motions recorded along the western end of the building.
The peaks in the transfer function in Figs 3.7 through 3.9 are “broad”, because the system changes during the excitation. These changes versus time are shown in Figs 3.10 through 3.18 (center) for nine earthquakes. Parts a and b of these figures correspond respectively to the EW and NS responses. These changes are summarized in Table 3.1 and in Fig. 3.19. Figure 3.19 also compares the variations in the system frequencies during strong motion with the values from low amplitude testing (horizontal solid lines) [Carder, 1936; 1964] and system identification studies (horizontal dashed lines) [Papageorgiou and Lin, 1991]. The following describes in more detail Figs 3.10 through 3.18.

The top parts of Figs 3.10 through 3.18 show time histories of band-pass filtered relative displacements at the roof, computed from recorded data (dashed lines) and predicted using the model described in Section 2.4.1 and shown in Fig. 2.14. Figures 3.13 and 3.14 show only the model response because during the 1970 Lytle Creek and 1971 San Fernando earthquakes the accelerographs at roof malfunctioned. The band-pass filters were chosen to include the system frequency, and were determined after analyzing the instantaneous transfer-function between the relative horizontal response on the roof and the horizontal motion at the base.

The instantaneous system frequencies, $f_p$, shown in the central parts of Figs 3.10 through 3.18 were determined by two methods: (1) zero-crossing analysis (shown by open and full circles), and (2) moving window Fourier analysis (shown by the solid and dashed lines). Both methods were applied to the filtered data shown in the top parts of these figures. The solid points and solid line correspond to the predicted model response, and the open circles and dashed lines correspond to the recorded response. The zero-crossing analysis consisted of determining the half periods for all approximately symmetric peaks in the relative response, shown in the top parts of these figures, assuming that the filtered relative displacements locally can be approximated by a sine wave. The time windows for the moving window Fourier analysis depended on the sampling rate of the available processed strong motion data. For all except one earthquake, corrected (for the baseline and for the instrument response) data were available equally spaced at $\Delta t = 0.01$ s. The exception was the 1994 Northridge earthquake for which data were available with $\Delta t = 0.02$ s. For the data with $\Delta t = 0.01$ s, 4 s windows were used (0.5 s ramp up, 3 s flat, and 0.5 s ramp down), and a sliding interval of 2 s. For the data with $\Delta t = 0.02$ s, 8 s windows were used (1 s ramp up, 6 s flat, and 1 s ramp down), and a sliding interval of 4 s.

The bottom parts of Figs. 3.10 through 3.18 show a comparison of the predicted total system energy with the input wave energy. To properly compare these results, we band-pass filtered the model results and the input wave energy (the processed velocity
Fig. 3.10a  Top: comparison of recorded (dashed lines) and predicted (solid line) EW relative displacement response at the roof during the 1933 Southern California earthquake. Center: time dependent changes of the system frequency $f_P$ computed from recorded (dashed lines and open circles) and simulated (continuous line and solid dots) responses. Bottom: contributions to the system energy: $E_{\text{soil}}$, $E_{\text{soil}}$, $E_{\text{bldg}}$ and $E_K$ and $E_p$ and their sum $E_I$. The input wave energy $\alpha A \beta \int_0^T v^2(\tau) d\tau$ is shown by a dashed line.
Fig. 3.10b  Same as Fig. 3.10a but for the N-S response.
Fig. 3.11a  Same as Fig. 3.10a but for the 1952 Kern County earthquake.
Fig. 3.11b  Same as Fig. 3.10b but for the 1952 Kern County earthquake.
Fig. 3.12a Same as Fig. 3.10a but for the 1968 Borrego Mountain earthquake.
Fig. 3.12b  Same as Fig. 3.10b but for the 1968 Borrego Mountain earthquake.
Fig. 3.13a Same as Fig. 3.10a but for the 1970 Lytle Creek earthquake.
Fig. 3.13b  Same as Fig. 3.10b but for the 1970 Lytle Creek earthquake.
Fig. 3.14a  Same as Fig. 3.10a but for the 1971 San Fernando earthquake.
Fig. 3.14b  Same as Fig. 3.10b but for the 1971 San Fernando earthquake.
Fig. 3.15a Same as Fig. 3.10a but for the 1987 Whittier-Narrows earthquake.
Fig. 3.15b  Same as Fig. 3.10b but for the 1987 Whittier-Narrows earthquake.
Fig. 3.16a Same as Fig. 3.10a but for the 1992 Landers earthquake.
Fig. 3.16b Same as Fig. 3.10b but for the 1992 Landers earthquake.
Fig. 3.17a  Same as Fig. 3.10a but for the 1992 Big Bear earthquake.
Fig. 3.17b  Same as Fig. 3.10b but for the 1992 Big Bear earthquake.
Fig. 3.18a  Same as Fig. 3.10a but for the 1994 Northridge earthquake.
Fig. 3.18b Same as Fig. 3.10b but for the 1994 Northridge earthquake.
Fig. 3.19  A summary of the time dependent changes of the EW (top) and NS (bottom) system frequencies of Hollywood Storage Building during seven earthquakes between 1933 and 1994. The horizontal lines show the system frequencies determined from ambient vibration and forced vibration tests (light solid lines), [Carder 1936, 1964], and those identified by Papageorgiou and Lin [1991] (dashed lines). For each earthquake, the horizontal ticks represent pre and post earthquake estimates of the system frequencies.
was filtered before integration). The pass-bands are summarized in Table 3.2. The results concerning energy are discussed in more detail in Section 3.4.

Figures 3.20a and b compare the “rocking angles” (the displacement at the roof less the displacement at ground level, all divided by the building height) versus the instantaneous apparent frequency computed for most half-period segments of the response during seven earthquakes. It is seen, that the apparent system frequency depends on the amplitude of the excitation, and for small amplitudes approaches the frequencies measured by Carder [1936; 1964] by full-scale ambient and forced vibration tests.

3.4 Energies of Response

The bottom parts of Figs 3.10 through 3.18 show time dependent growth of different energies in the computed model response. The top solid line represents the total system energy computed by summing all the other response energies. The dashed line represents

\[ a_0 \int_0^t v^2(\tau) d\tau \]

with \( a_0 \) determined by least squares fit of the “total” energy in terms of

\[ \int_0^t v^2(\tau) d\tau . \]

As we already explained, the integral \( a_0 \int_0^t v^2(\tau) d\tau \) represents the cumulative energy arriving to the site in form of seismic waves. The “total” energy represents the sum of all response energies of soil-structure system, and in this work of the model shown in Fig. 2.14.

Figure 3.21 shows the plot of “total” energies, \( E_I \), in kNm versus \( a_0 \int_0^t v^2(\tau) d\tau \), for the EW (solid circles) and NS (solid triangles) responses for all nine earthquakes. For 1970 Lytle Creek and 1971 San Fernando earthquakes, the \( E_I \) energies are based on assumed model parameters only, since without recorded roof motion it is not possible to find the best estimates of the systems parameters. The least squares fit through the data gives \( a_0 = 1.7 \times 10^4 \) kg/s. All data is above \( 10^4 \int_0^t v^2(\tau) d\tau \) and below \( 4 \times 10^4 \int_0^t v^2(\tau) d\tau \).

For vertically incident plane shear waves, and neglecting wave scattering from the foundation, the coefficient \( a_0 \) should equal \( \rho A \beta \), where \( \rho \) is density, \( \beta \) is the shear wave velocity in the soil surrounding the foundation, and \( A \) is the area of the plan of the building foundation. Assuming \( \rho = 1.2 \times 10^3 \) kg/m³, \( \beta = 300 \) m/s and \( A = 1028 \) m², we obtain \( \rho A \beta = 3.6 \times 10^8 \) kg/s. Since \( v(t) \) in \( \int_0^t v^2 d\tau \) has been calculated in units of cm/s, while the response energies are presented in kNm, the above estimate of \( \rho A \beta \) must be divided by \( 10^4 \) giving “\( \rho A \beta \)” \( = 3.6 \times 10^4 \) kg/s. It is seen that \( a_0 \) is smaller by a factor of about 2. It is clear that the incident seismic waves are not vertically arriving plane S-
Table 3.2  Pass-bands for filtering the model results and the ground velocity before integration, for Hollywood Storage Building.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>N-S Comp.</th>
<th>E-W Comp.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pass-band for model analysis (Hz)</td>
<td>Pass-band for $\int v^2(t) , dt$ (Hz)</td>
</tr>
<tr>
<td>Southern California</td>
<td>0.350 - 0.450 to 0.800 - 0.950</td>
<td>0.400 - 0.475 to 0.750 - 0.850</td>
</tr>
<tr>
<td>Kern County</td>
<td>0.200 - 0.350 to 0.850 - 1.000</td>
<td>0.350 - 0.425 to 0.725 - 0.825</td>
</tr>
<tr>
<td>Borrego Mountain</td>
<td>0.400 - 0.550 to 1.000 - 1.150</td>
<td>0.325 - 0.400 to 1.000 - 1.100</td>
</tr>
<tr>
<td>Lytle Creek</td>
<td>0.250 - 0.325 to 0.850 - 0.950</td>
<td>0.400 - 0.475 to 0.750 - 0.850</td>
</tr>
<tr>
<td>San Fernando</td>
<td>0.175 - 0.250 to 0.750 - 0.850</td>
<td>0.250 - 0.325 to 0.575 - 0.675</td>
</tr>
<tr>
<td>Whittier Narrows</td>
<td>0.250 - 0.325 to 0.850 - 0.950</td>
<td>0.250 - 0.325 to 0.750 - 0.850</td>
</tr>
<tr>
<td>Landers</td>
<td>0.275 - 0.350 to 0.750 - 0.850</td>
<td>0.275 - 0.350 to 0.675 - 0.775</td>
</tr>
<tr>
<td>Big Bear</td>
<td>0.275 - 0.350 to 0.800 - 0.900</td>
<td>0.300 - 0.375 to 0.625 - 0.725</td>
</tr>
<tr>
<td>Northridge</td>
<td>0.125 - 0.225 to 0.750 - 0.850</td>
<td>0.250 - 0.325 to 0.575 - 0.675</td>
</tr>
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</table>
Fig. 3.20a  Dependence of the apparent system frequency of Hollywood Storage Building on the amplitude of the EW response ("rocking angle"). The solid vertical lines show estimates of the system frequencies determined from small amplitude (ambient vibration and forced vibration) tests by Carder [1936, 1964].
Fig. 3.20b  Same as Fig. 3.20a but for the NS response.
\( \bar{a}_0 = 17114.74 \)
\( \bar{a}_1 = 13813.20 \), \( \bar{e}_i = 691.60 \)

Hollywood Storage Bldg.

\[ y = \bar{a}_1 x + \bar{e}_i \]

\( \rho \beta A = 3.6 \times 10^4 \text{ kg/s}^4 \)

\( \rho \beta A = 1.7 \times 10^4 \text{ kg/s}^4 \)

\( \sum v d\tau = e_n = \int_0^t v^2(\tau)d\tau \) (cm²/s)

Fig. 3.21 Total computed response energy \( E_i \) (kN·m) versus input wave “energy” \( e_n = \int_0^t v^2(\tau)d\tau \) for nine earthquakes.
Table 3.3 Percentages of the energy components of the predicted response at the end of shaking, for Hollywood Storage Building.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Code</th>
<th>$E_I$ (kN·m)</th>
<th>$E_I^*$ (kN·m)</th>
<th>$E_D^{\text{blend}}/E_I$ (%)</th>
<th>$E_S^{\text{blend}}/E_I$ (%)</th>
<th>$E_D^{\text{total}}/E_I$ (%)</th>
<th>$E_S^{\text{total}}/E_I$ (%)</th>
<th>$(E_K + E_P)_{\text{max}}/E_I$ (%)</th>
<th>$E_I$ (kN·m)</th>
<th>$E_I^*$ (kN·m)</th>
<th>$E_D^{\text{blend}}/E_I$ (%)</th>
<th>$E_S^{\text{blend}}/E_I$ (%)</th>
<th>$E_D^{\text{total}}/E_I$ (%)</th>
<th>$E_S^{\text{total}}/E_I$ (%)</th>
<th>$(E_K + E_P)_{\text{max}}/E_I$ (%)</th>
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<tbody>
<tr>
<td>Southern California</td>
<td>SC</td>
<td>10.55</td>
<td>3.30</td>
<td>88.14</td>
<td>0.11</td>
<td>6.71</td>
<td>4.69</td>
<td>23.17</td>
<td>36.76</td>
<td>26.57</td>
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<td>0.00</td>
<td>26.42</td>
<td>9.34</td>
<td>86.19</td>
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<tr>
<td>Kern County</td>
<td>KC</td>
<td>366.5</td>
<td>278.6</td>
<td>33.46</td>
<td>0.53</td>
<td>13.19</td>
<td>51.87</td>
<td>48.74</td>
<td>196.0</td>
<td>34.89</td>
<td>59.92</td>
<td>0.07</td>
<td>32.13</td>
<td>7.88</td>
<td>27.13</td>
</tr>
<tr>
<td>Borrego Mountain</td>
<td>BM</td>
<td>85.14</td>
<td>53.43</td>
<td>50.13</td>
<td>0.02</td>
<td>5.08</td>
<td>43.94</td>
<td>24.95</td>
<td>8.90</td>
<td>5.92</td>
<td>63.33</td>
<td>0.29</td>
<td>24.21</td>
<td>12.08</td>
<td>31.81</td>
</tr>
<tr>
<td>Lytle Creek</td>
<td>LC</td>
<td>0.59</td>
<td>1.33</td>
<td>61.58</td>
<td>0.51</td>
<td>14.72</td>
<td>5.42</td>
<td>79.60</td>
<td>1.43</td>
<td>0.53</td>
<td>73.24</td>
<td>0.12</td>
<td>22.26</td>
<td>3.99</td>
<td>82.40</td>
</tr>
<tr>
<td>San Fernando</td>
<td>SF</td>
<td>365.7</td>
<td>591.8</td>
<td>43.87</td>
<td>0.38</td>
<td>17.12</td>
<td>38.60</td>
<td>26.58</td>
<td>1668</td>
<td>1238</td>
<td>13.19</td>
<td>0.00</td>
<td>71.02</td>
<td>15.78</td>
<td>38.48</td>
</tr>
<tr>
<td>Whittier Narrows</td>
<td>WT</td>
<td>176.2</td>
<td>236.5</td>
<td>35.34</td>
<td>1.89</td>
<td>12.71</td>
<td>49.22</td>
<td>64.44</td>
<td>178.8</td>
<td>50.33</td>
<td>54.07</td>
<td>0.04</td>
<td>31.79</td>
<td>14.05</td>
<td>70.23</td>
</tr>
<tr>
<td>Landers</td>
<td>LN</td>
<td>551.8</td>
<td>319.0</td>
<td>24.08</td>
<td>0.16</td>
<td>20.81</td>
<td>54.71</td>
<td>16.49</td>
<td>212.2</td>
<td>91.41</td>
<td>53.30</td>
<td>0.06</td>
<td>31.26</td>
<td>15.38</td>
<td>43.69</td>
</tr>
<tr>
<td>Big Bear</td>
<td>BB</td>
<td>160.2</td>
<td>137.0</td>
<td>24.39</td>
<td>0.22</td>
<td>11.85</td>
<td>60.25</td>
<td>21.54</td>
<td>108.9</td>
<td>77.78</td>
<td>29.64</td>
<td>0.04</td>
<td>53.71</td>
<td>16.50</td>
<td>49.79</td>
</tr>
<tr>
<td>Northridge</td>
<td>NR</td>
<td>842.8</td>
<td>1004</td>
<td>29.71</td>
<td>0.11</td>
<td>24.55</td>
<td>44.25</td>
<td>51.28</td>
<td>2036</td>
<td>925.4</td>
<td>19.10</td>
<td>0.00</td>
<td>68.25</td>
<td>12.65</td>
<td>64.90</td>
</tr>
</tbody>
</table>

$E_I^*$: total energy at the instant when $(E_K + E_P)$ is maximum.
Fig. 3.22 Percentages of various response energies versus peak ground velocity, $v_{G,\text{max}}$. 
Fig. 3.23  Percentages of the various response energies versus peak “rocking” velocity, $\dot{\theta}_{\text{max}}$.
waves, but represent a complex sequence of body and surface waves whose angles of approach vary vertically (different phase velocities, see for example Fig. 4.23) and horizontally as shown in Fig. 3.6. Furthermore “coefficients” $a_0$ and $a_1$ depend on the soil-structure interaction, which in this example is further complicated by the presence of piles. We conclude that considering the complexities of the energy transfer from the soil into the building [Trifunac et al., 1999a; Trifunac et al., 2001b], the agreement of the computed $E_I$ versus $\int_0^t v^2 \, dt$ is satisfactory to warrant further and more detailed studies of this energy transfer mechanism.

In Table 3.3, we summarize the distribution of $E_I$ among $E_D^{\text{bldg}}$, $E_S^{\text{bldg}}$, $E_D^{\text{soil}}$ and $E_S^{\text{soil}} + E_Y$ at the end of the excitation and $(E_K + E_P)_{\text{max}}$ for all nine earthquakes. In Figs 3.22 and 3.23, we show the trend of those energies versus $v_{G,\text{max}}$ in the basement and versus $\dot{\theta}_{\text{max}} = |v_{\text{roof}} - v_{\text{base}}|_{\text{max}} / H$. As could be expected, the participation (percentage) of $E_D^{\text{bldg}}$ decreases while for $E_D^{\text{soil}}$ and $E_S^{\text{soil}} + E_Y$ increase, with both $v_{G,\text{max}}$ and $\dot{\theta}_{\text{max}}$. 
4. STUDIES OF THE VAN NUYS HOTEL

As a second case study to illustrate our model for energy flow and partitioning in a soil-structure system we use the Van Nuys seven story hotel (VN7SH), studied by many investigators following Northridge, 1994, California earthquake (e.g. see De la Llera et al. [2001], and Trifunac et al. [2001a,b]). This building has become a useful general-purpose test case for many comparative studies. It experienced damaging motions in 1971 and 1994, there are multiple recordings in the building of weak, intermediate and strong earthquake response [Trifunac et al., 1999b], and it was tested using ambient vibration method [Trifunac et al., 1999a; Ivanović et al., 1999; 2000]. Because of its simplicity and nearly symmetric properties this building has also served as a model test for response analyses using wave propagation methods [Todorovska et al., 2001a,b; Trifunac et al., 2001d].

4.1 Background

4.1.1 Description of the Building

The Van Nuys 7-story hotel (VN7SH) was designed in 1965, constructed in 1966 [Blume et al., 1973], served as a hotel until 1994, and reopened again as a hotel in 1999. Its plan dimensions are 62 by 150 feet (1 foot=30.48 cm). Figure 4.1 (a, b, c) shows a typical floor layout, the foundation layout and a side view of the structure. The typical framing consists of columns spaced at 20-foot centers in the transverse direction and 19-foot centers in the longitudinal direction. Spandrel beams surround the perimeter of the structure. The lateral forces in each direction are resisted by the interior column-slab frames and exterior column spandrel beam frames. The added stiffness associated with the spandrel beams, creates exterior frames that are roughly twice as stiff as the interior frames. With the exception of some light framing members supporting the stairway and elevator openings, the structure is essentially symmetric. Except for two small areas at the ground floor, covered by one-story canopies, the plan configurations of the floors, including the roof, are the same. The floor system is a reinforced concrete flat slab, 10 inches thick at the second floor, 8.5 inches thick at the third to seventh floors and 8 inches thick at the roof (1 inch=2.54 cm). The north side of the building, along column line D (Fig. 4.1a), has four bays of brick masonry walls located between the ground and the second floor at the east end of the structure. Nominal ½-inch expansion joints, separate the walls from the underside of the second floor spandrel beams. Although none of the wall elements described are designed as a part of the lateral force-resisting system, they do contribute in varying degrees to the stiffness of the structure [Islam, 1996].
Fig. 4.1  a) Typical floor plan.  b) Foundation plan.  c) Typical transverse section.  d) Log of typical soil boring.
Fig. 4.2  Southern California earthquakes for which recorded data in the VN7SH building has been digitized (solid stars).  The open stars show earthquakes that may have been recorded in VN7SH, but so far the data is not available in digitized form.
Table 4.1  Apparent frequencies of the response during twelve earthquakes, at the beginning and end of shaking, maximum and minimum values during shaking ($f_{beg}$, $f_{end}$, $f_{max}$, $f_{min}$), the largest difference (($Δf_{max} = f_{max} - f_{min}$), peak ground velocity ($v_{G,max}$), and peak instantaneous difference between the velocity recorded at the roof and at ground level ($\dot{θ}_{max}$), for VN7SH.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Date</th>
<th>$M_L$</th>
<th>$R$ (km)</th>
<th>$f_{beg}$</th>
<th>$f_{end}$</th>
<th>$f_{max}$</th>
<th>$f_{min}$</th>
<th>($Δf_{max}$)</th>
<th>$v_{G,max}$ (cm/s)</th>
<th>$\dot{θ}_{max}$ (×10^3 rad/s)</th>
<th>$f_{beg}$</th>
<th>$f_{end}$</th>
<th>$f_{max}$</th>
<th>$f_{min}$</th>
<th>($Δf_{max}$)</th>
<th>$v_{G,max}$ (cm/s)</th>
<th>$\dot{θ}_{max}$ (×10^3 rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Fernando</td>
<td>02/09/1971</td>
<td>6.6</td>
<td>10*</td>
<td>0.683</td>
<td>0.545</td>
<td>0.683</td>
<td>0.545</td>
<td>0.138</td>
<td>29.28</td>
<td>17.5245</td>
<td>0.815</td>
<td>0.600</td>
<td>0.815</td>
<td>0.600</td>
<td>0.215</td>
<td>23.72</td>
<td>9.8280</td>
</tr>
<tr>
<td>Whittier Narrows</td>
<td>10/01/1987</td>
<td>5.9</td>
<td>41</td>
<td>0.850</td>
<td>0.787</td>
<td>0.850</td>
<td>0.787</td>
<td>0.063</td>
<td>8.14</td>
<td>2.8390</td>
<td>--</td>
<td>--</td>
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<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Whittier 12th Aft.</td>
<td>10/04/1987</td>
<td>5.3</td>
<td>41</td>
<td>0.880</td>
<td>1.020</td>
<td>1.020</td>
<td>0.880</td>
<td>0.140</td>
<td>1.33</td>
<td>0.7675</td>
<td>0.775</td>
<td>0.950</td>
<td>0.950</td>
<td>0.775</td>
<td>0.175</td>
<td>2.18</td>
<td>1.7240</td>
</tr>
<tr>
<td>Pasadena</td>
<td>12/03/1988</td>
<td>4.9</td>
<td>32</td>
<td>1.075</td>
<td>0.930</td>
<td>1.075</td>
<td>0.930</td>
<td>0.145</td>
<td>1.46</td>
<td>0.4040</td>
<td>0.980</td>
<td>1.070</td>
<td>1.070</td>
<td>0.980</td>
<td>0.090</td>
<td>0.94</td>
<td>0.6495</td>
</tr>
<tr>
<td>Malibu</td>
<td>01/19/1989</td>
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<td>35</td>
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<td>0.970</td>
<td>1.180</td>
<td>0.970</td>
<td>0.210</td>
<td>0.93</td>
<td>1.0210</td>
<td>1.110</td>
<td>1.030</td>
<td>1.110</td>
<td>1.030</td>
<td>0.070</td>
<td>0.96</td>
<td>0.3785</td>
</tr>
<tr>
<td>Montebello</td>
<td>06/12/1989</td>
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<td>34</td>
<td>1.360</td>
<td>1.520</td>
<td>1.520</td>
<td>1.360</td>
<td>0.160</td>
<td>0.45</td>
<td>0.2115</td>
<td>1.250</td>
<td>1.360</td>
<td>1.360</td>
<td>1.115</td>
<td>0.245</td>
<td>0.85</td>
<td>0.1055</td>
</tr>
<tr>
<td>Sierra Madre</td>
<td>06/28/1991</td>
<td>5.8</td>
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<td>0.790</td>
<td>0.960</td>
<td>0.790</td>
<td>0.170</td>
<td>4.40</td>
<td>3.8190</td>
<td>1.070</td>
<td>0.915</td>
<td>1.070</td>
<td>0.915</td>
<td>0.155</td>
<td>2.78</td>
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<tr>
<td>Landers</td>
<td>06/28/1992</td>
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<td>0.695</td>
<td>0.910</td>
<td>0.695</td>
<td>0.215</td>
<td>10.42</td>
<td>13.8710</td>
<td>0.910</td>
<td>0.705</td>
<td>0.910</td>
<td>0.705</td>
<td>0.205</td>
<td>10.64</td>
<td>5.3720</td>
</tr>
<tr>
<td>Big Bear</td>
<td>06/28/1992</td>
<td>6.5</td>
<td>149</td>
<td>0.650</td>
<td>0.860</td>
<td>0.860</td>
<td>0.650</td>
<td>0.210</td>
<td>3.87</td>
<td>2.9480</td>
<td>0.765</td>
<td>0.765</td>
<td>0.915</td>
<td>0.765</td>
<td>0.150</td>
<td>3.58</td>
<td>3.1885</td>
</tr>
<tr>
<td>Northridge</td>
<td>01/17/1994</td>
<td>6.4</td>
<td>4*</td>
<td>0.720</td>
<td>0.475</td>
<td>0.720</td>
<td>0.457</td>
<td>0.245</td>
<td>35.32</td>
<td>20.9745</td>
<td>0.765</td>
<td>0.435</td>
<td>0.765</td>
<td>0.435</td>
<td>0.330</td>
<td>50.93</td>
<td>14.3420</td>
</tr>
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<td>03/10/1994</td>
<td>5.2</td>
<td>1</td>
<td>0.600</td>
<td>0.520</td>
<td>0.600</td>
<td>0.520</td>
<td>0.080</td>
<td>7.61</td>
<td>0.7340</td>
<td>0.780</td>
<td>0.700</td>
<td>0.780</td>
<td>0.700</td>
<td>0.080</td>
<td>4.83</td>
<td>0.1455</td>
</tr>
<tr>
<td>Northridge Mar. Aft.</td>
<td>03/10/1994</td>
<td>5.2</td>
<td>1</td>
<td>0.713</td>
<td>0.675</td>
<td>0.713</td>
<td>0.495</td>
<td>0.218</td>
<td>2.58</td>
<td>0.3500</td>
<td>0.770</td>
<td>0.680</td>
<td>0.770</td>
<td>0.590</td>
<td>0.180</td>
<td>4.21</td>
<td>0.2340</td>
</tr>
<tr>
<td>Northridge Dec. Aft.</td>
<td>12/06/1994</td>
<td>4.3</td>
<td>11</td>
<td>0.662</td>
<td>0.772</td>
<td>0.772</td>
<td>0.662</td>
<td>0.110</td>
<td>2.67</td>
<td>0.7865</td>
<td>0.687</td>
<td>0.770</td>
<td>0.770</td>
<td>0.687</td>
<td>0.083</td>
<td>2.41</td>
<td>0.2675</td>
</tr>
</tbody>
</table>

---: record is not available.
*: horizontal projection of the closet distance to fault surface.
Fig. 4.3  General setting of VN7SH building in central San Fernando Valley. Horizontal projections of fault planes of S. Fernando, 1971, and Northridge, 1994, earthquakes are outlined by dashed lines. Directions and epicentral distances to seven other earthquakes are shown. Epicenters of two aftershocks of Northridge 1994 earthquake are shown by solid stars.

Fig. 4.4  a) Location of three AR-240 accelerographs which recorded San Fernando 1971 earthquake. b) Location and orientation of 13 sensitivity vectors of CR-1 recording system. Channels 14, 15 and 16 belong to SMA-1 accelerograph.
The foundation system (Fig. 4.1b) consists of 38-inch-deep pile caps, supported by groups of two to four poured-in-place 24-inch-diameter reinforced concrete friction piles. These are centered under the main building columns. All pile caps are connected by a grid of the beams. Each pile is roughly 40 feet long and has design capacity of over 100 kips vertical load and up to 20 kips lateral load. The structure is constructed of regular weight reinforced concrete [Blume et al., 1973]. The site lies on recent alluvium, with 300 m/s average shear-wave velocity in the top 30 m. A typical boring log (Fig. 4.1d) shows the underlying soil to be primarily fine sandy silts and silty fine sands.

4.1.2 Earthquake Recordings

The epicenters of 12 of the recorded earthquakes are shown in Fig. 4.2 and are listed in Table 4.1. Columns 1, 2, 3, 4 show the earthquake name, date, magnitude and epicentral distance, and columns 5 through 8 show the peak velocity recorded at ground level, $v_{G,\text{max}}$, and an estimate of peak rocking angular velocity $\dot{\theta}_{\text{max}}$. Closest to the building were the sources of the 1971 San Fernando and of the 1994 Northridge earthquakes, and two of its aftershocks. Figure 4.3 shows the building location, the major freeways, the fault planes of these two earthquakes, the epicenters of the two aftershocks of the latter (two stars), and arrows toward the epicenters of the more distant recorded earthquakes. During the 1971 San Fernando earthquake, the first strong motion waves started to arrive from N22°E, having originated at depth 9 to 13 km below the epicenter. With rupture propagating up towards south at about 2 km/s, the last direct waves were arriving from N62°E, 9-10 s later [Trifunac, 1974]. The 1987 Whittier-Narrows, 1992 Landers, and 1992 Big Bear earthquakes occurred at epicentral distances $R = 41, 186$ and 149 km and caused strong motion arrivals from E27°S, East, and E15°S. During the 1994 Northridge earthquake, the first motions started to arrive from the west, with the last arrivals coming from N42°W, about 7-10 s later [Wald et al., 1996].

The 1971 event was recorded by three self-contained tri-axial AR-240 accelerographs (Fig. 4.4a), and the other events were recorded by a CR-1 recording system (the sensor locations for this recording system are shown in Fig. 4.4b). The records of the San Fernando earthquake were digitized manually, at a sampling rate of minimum 50 points per second [Trifunac and Lee 1973; 1978, Trifunac et al., 1973], of the Whittier-Narrows, Landers, Big Bear and Northridge earthquakes were digitized by the California Division of Mines and Geology (e.g. Shakal et al. [1994]), and of the other earthquakes were digitized by the authors using the LeAuto software [Lee and Trifunac, 1990] from Xerox copies of the original recordings (supplied by CDMG; [Graizer, 1997]).
Fig. 4.5 Schematic representation of damage from the 1994 Northridge earthquake: (top) frame D (North view), and (bottom) frame A (South view). The sensor locations for channels 1-8 and 13 (oriented towards North), are also shown (see also Fig. 4.4).
4.1.3 Earthquake Damage

The February 9, 1971, San Fernando earthquake caused minor structural damage. Epoxy was used to repair the spalled concrete of the second floor beam column joints on the north side and east end of the building. The nonstructural damage, however, was extensive and about 80 percent of all repair cost was used to fix the drywall partitions, bathroom tiles and plumbing fixtures [Blume et al., 1973].

The building was damaged again by the Northridge earthquake of 17 January 1994 and its aftershocks. The structural damage was extensive in the exterior north (D) and south (A) frames, designed to take most of the lateral load in the longitudinal direction. Severe shear cracks occurred in the middle columns of frame A, near the contact with the spandrel beam of the fifth floor (Fig. 4.5). Those cracks significantly decreased the axial, moment and shear capacity of the columns. The shear cracks in the north (D) frame on the third and fourth floors, and the damage of columns D2, D3 and D4 on the first floor caused minor to moderate changes in the capacity of these structural elements. No major damage of the interior longitudinal (B and C) frames was noticed. There was no visible damage in the slabs and around the foundation. The nonstructural damage was significant. Almost every guestroom suffered considerable damage. Severe cracks were noticed in the masonry brick walls, and in the exterior cement plaster [Trifunac et al., 1999b].

4.1.4 Ambient Vibration Tests

Two detailed ambient vibration experiments were conducted by the authors and co-workers following the 1994 Northridge earthquake, one on February 4−5, 1994, and the other one on April 19−20, 1994, following the $M = 5.2$ aftershock of March 20, 1994 [Ivanović et al., 1999; 2000]. From the measurements in the transverse (NS) direction, the following apparent modes and frequencies were identified: first translational (1.4 Hz), first torsional (1.6 Hz), second translational (3.9 Hz), and second torsional (4.9 Hz). The frequencies for the respective modes determined from the measurements in the longitudinal (EW) direction were 1.0, 3.5, 5.7 and 8.1 Hz.

4.1.5 Time and Amplitude Dependent Response

Trifunac et al. [2001a,b] describe the apparent changes of the system period of this building during 12 earthquakes. In Figs 4.6 through 4.18, we show these changes together with the evolution in time of the different energies of response. Parts “a” of these figures correspond to the EW response, and parts “b” correspond to the NS response. For the Whittier-Narrows earthquake of 1987, there are no results for the EW response,
Fig. 4.6a  Top: comparison of recorded (dashed lines) and predicted (solid line) EW relative displacement response at the roof during the 1971 San Fernando earthquake. Center: time dependent changes of the system frequency $f_P$ computed from recorded (dashed lines and open circles) and simulated (continuous line and solid dots) responses. Bottom: contributions to the system energy: $E_{S+Y}^{\text{soil}}$, $E_{D}^{\text{soil}}$, $E_{D}^{\text{bldg}}$, $E_K$, and $E_p$ and their sum $E_I$. The input wave energy $\alpha A \beta \int_0^t v^2(\tau) d\tau$ is shown by a dashed line.
Fig. 4.6b  Same as Fig. 4.6a but for the N-S response.
Fig. 4.7 Same as Fig. 4.6b but for the 1987 Whittier-Narrow earthquake.
Fig. 4.8a  Same as Fig. 4.6a but for the Whittier-Narrows aftershock of October 4, 1987.
Fig. 4.8b  Same as Fig. 4.6b, but for for the Whittier-Narrows aftershock of October 4, 1987.
Fig. 4.9a  Same as Fig. 4.6a but for the 1988 Pasadena earthquake.
Fig. 4.9b  Same as Fig. 4.6b, but for the 1988 Pasadena earthquake.
Fig. 4.10a  Same as Fig. 4.6a but for the 1989 Malibu Earthquake.
Fig. 4.10b  Same as Fig. 4.6b but for the 1989 Malibu Earthquake.
Fig. 4.11a Same as Fig. 4.6a but for the 1989 Montebello earthquake.
Fig. 4.11b  Same as Fig. 4.6b but for the 1989 Montebello earthquake.
Fig. 4.12a  Same as Fig. 4.6a but for the 1991 Sierra Madre earthquake.
Fig. 4.12b  Same as Fig. 4.6b but for the 1991 Sierra Madre earthquake.
Fig. 4.13a Same as Fig. 4.6a but for the 1992 Landers earthquake.
Fig. 4.13b  Same as Fig. 4.6b but for the 1992 Landers earthquake.
Fig. 4.14a  Same as Fig. 4.6a but for the 1992 Big Bear earthquake.
Fig. 4.14b  Same as Fig. 4.6b but for the 1992 Big Bear earthquake.
Fig. 4.15a  Same as Fig. 4.6a but for the 1994 Northridge earthquake.
Fig. 4.15b  Same as Fig. 4.6b, but for the 1994 Northridge earthquake.
Fig. 4.16a  Same as Fig. 4.6ba but for the first segment of the Northridge aftershock of March 20, 1994.
Fig. 4.16b Same as Fig. 4.6b, but for the first segment of the Northridge aftershock of March 20, 1994.
Fig. 4.17a Same as Fig. 4.6a, but for the second segment of the Northridge aftershock of March 20, 1994.
Fig. 4.17b  Same as Fig. 4.6b, but for the second segment of the Northridge aftershock of March 20, 1994.
Fig. 4.18a  Same as Fig. 4.6a but for the Northridge aftershock of December 6, 1994.
Fig. 4.18b  Same as Fig. 4.6b but for the Northridge aftershock of December 6, 1994.
Table 4.2  Pass-bands for filtering the model results and the ground velocity before integration for VN7SH.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Pass-band for model analysis (Hz)</th>
<th>Pass-band for $\int v^2(t) , dt$ (Hz)</th>
<th>Pass-band for model analysis (Hz)</th>
<th>Pass-band for $\int v^2(t) , dt$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Fernando</td>
<td>0.350 - 0.425 to 0.875 - 0.950</td>
<td>0.350 - 0.425 to 0.875 - 0.975</td>
<td>0.500 - 0.600 to 1.125 - 1.225</td>
<td>0.450 - 0.550 to 1.250 - 1.350</td>
</tr>
<tr>
<td>Whittier Narrows</td>
<td>0.350 - 0.425 to 0.975 - 1.075</td>
<td>0.350 - 0.425 to 0.850 - 0.950</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Whittier 12th Aft.</td>
<td>0.375 - 0.450 to 1.375 - 1.475</td>
<td>0.425 - 0.500 to 1.300 - 1.400</td>
<td>0.450 - 0.525 to 1.200 - 1.300</td>
<td>0.575 - 0.650 to 1.125 - 1.225</td>
</tr>
<tr>
<td>Pasadena</td>
<td>0.475 - 0.575 to 1.450 - 1.550</td>
<td>0.825 - 0.925 to 1.675 - 1.775</td>
<td>0.400 - 0.500 to 1.300 - 1.400</td>
<td>0.600 - 0.700 to 1.250 - 1.350</td>
</tr>
<tr>
<td>Malibu</td>
<td>0.650 - 0.725 to 1.475 - 1.575</td>
<td>0.675 - 0.775 to 1.300 - 1.400</td>
<td>0.625 - 0.725 to 1.450 - 1.550</td>
<td>0.775 - 0.875 to 1.225 - 1.325</td>
</tr>
<tr>
<td>Montebello</td>
<td>1.000 - 1.100 to 1.775 - 1.875</td>
<td>1.100 - 1.200 to 1.600 - 1.700</td>
<td>0.675 - 0.750 to 1.750 - 1.850</td>
<td>0.900 - 1.000 to 1.525 - 1.625</td>
</tr>
<tr>
<td>Sierra Madre</td>
<td>0.400 - 0.500 to 1.200 - 1.300</td>
<td>0.575 - 0.650 to 1.100 - 1.200</td>
<td>0.525 - 0.600 to 1.300 - 1.400</td>
<td>0.675 - 0.750 to 1.175 - 1.275</td>
</tr>
<tr>
<td>Landers</td>
<td>0.300 - 0.375 to 1.200 - 1.300</td>
<td>0.375 - 0.450 to 1.025 - 1.125</td>
<td>0.450 - 0.550 to 1.250 - 1.350</td>
<td>0.575 - 0.650 to 1.050 - 1.150</td>
</tr>
<tr>
<td>Big Bear</td>
<td>0.300 - 0.375 to 1.150 - 1.250</td>
<td>0.350 - 0.425 to 1.175 - 1.275</td>
<td>0.275 - 0.450 to 1.250 - 1.350</td>
<td>0.475 - 0.550 to 1.100 - 1.200</td>
</tr>
<tr>
<td>Northridge</td>
<td>0.200 - 0.275 to 0.850 - 0.925</td>
<td>0.225 - 0.300 to 0.800 - 0.875</td>
<td>0.200 - 0.275 to 0.750 - 0.825</td>
<td>0.175 - 0.250 to 0.750 - 0.825</td>
</tr>
<tr>
<td>Northridge Mar. Aft.</td>
<td>0.250 - 0.325 to 0.750 - 0.850</td>
<td>0.300 - 0.375 to 0.700 - 0.775</td>
<td>0.300 - 0.375 to 0.950 - 1.050</td>
<td>0.400 - 0.475 to 0.825 - 0.900</td>
</tr>
<tr>
<td>Northridge Mar. Aft.</td>
<td>0.350 - 0.425 to 0.975 - 1.075</td>
<td>0.350 - 0.425 to 0.850 - 0.950</td>
<td>0.275 - 0.350 to 0.925 - 1.000</td>
<td>0.275 - 0.350 to 0.850 - 0.925</td>
</tr>
<tr>
<td>Northridge Dec. Aft.</td>
<td>0.300 - 0.375 to 0.950 - 1.050</td>
<td>0.350 - 0.425 to 0.950 - 1.050</td>
<td>0.375 - 0.450 to 0.900 - 1.000</td>
<td>0.400 - 0.475 to 0.800 - 0.900</td>
</tr>
</tbody>
</table>

--: data at the ground floor is not available.
Fig. 4.19  Schematic summary of the changes of the predominant system frequency for NS (top) and EW (bottom) responses, derived from moving window and zero-crossing analyses.
Fig. 4.20 Maximum change of predominant system frequency during strong shaking, $\Delta f_{\text{max}}$, versus the difference between the peak velocity at the roof and the ground floor of VN7SH, $(\Delta v)_{\text{max}}$, and the peak ground velocity of strong motion, $v_{G,\text{max}}$. 
Fig. 4.21a Peaks of the relative EW rocking $\theta(t)$ versus instantaneous frequency $f_p$ for the twelve earthquakes in Table 4.1.
Fig. 4.21b Peaks of the relative NS rocking $\theta(t)$ versus instantaneous frequency $f_p$ for the twelve earthquakes in Table 4.1.
because Channel No. 16 malfunctioned and the relative response could not be evaluated.

The top parts of Figs 4.6 through 4.18 show comparison of the recorded relative response (dashed lines) and the one predicted (solid line) by simulation using the recorded motion and response of the model in Fig. 2.14. These results were band-pass filtered and the bands are summarized in Table 4.2. The central parts of Figs 4.6 through 4.18 show the time dependent changes of the systems frequency \( f_p \). The dashed line shows the changes in \( f_p \) evaluated by moving window Fourier analysis of the recorded data, and the open circles show the estimates by the zero-crossings method also using the recorded data. The solid line and solid points show the corresponding quantities, but for the simulated response using the model in Fig. 2.14. The vertical arrows show the times where the gaps in the soil springs were closed to simulate increase in the system frequency observed in the center parts of Figs 4.6 through 4.18.

Figure 4.19 summaries the time dependent changes of \( f_p \). Fig. 4.20 shows the trends of \( \Delta f_{\text{max}} \) versus \( |v_{\text{roof}} - v_{\text{base}}|_{\text{max}} \) and \( v_{G,\text{max}} \), and Figures 4.21a and b show the amplitude dependent changes of the rocking angle \( \theta \) versus \( f_p \). Further discussion of these results can be found in Trifuanc et al. [2001b,c].

4.2 Energy of the Soil-Structure System Response

The bottom parts of Figs 4.6 through 4.18 show the time evolution of different energies in the computed model response. Top solid line represents the sum of the different partitions of the energy resulting in the “total” system energy, \( E_I \). The dotted line represents \( a_0 \int_0^t v^2 d\tau \), with \( a_0 \) determined by least squares fit of the “total” energy in terms of \( \int_0^t v^2(\tau)d\tau \).

Figure 4.22 shows the trend of computed \( E_I \) (“total” energy) versus \( a_0 \int_0^t v^2 d\tau \) for the EW (solid circles) and NS (solid triangles) responses for all twelve earthquakes. It is seen that \( a_0 \) is between 0.5x10^4 kg/s and 3x10^4 kg/s.

For vertically incident plane shear waves, and neglecting the waves scattered from the foundation, the coefficient \( a_0 \) should be equal to \( \rho A \beta \). Assuming that \( \rho \approx 1.2 \times 10^3 \text{ kg/m}^3 \), \( \beta \approx 300 \text{ m/s} \) and \( A = 873 \text{ m}^2 \), we compute \( \rho A \beta = 3.0 \times 10^8 \text{ kg/s} \). It is necessary to divide \( \rho A \beta \) by \( 10^4 \) to convert velocities squared from m/s into cm/s. It is seen that for \( a'_0 = \rho A \beta / 10^4 = 3 \times 10^4 \text{ kg/s} \), \( a'_0 \log_{10} \int_0^t v^2 d\tau \) is an upper bound for all points shown in
\[
\overline{a_0} = 4777.50 \\
\overline{a_1} = 4534.66 , \overline{b_1} = 141.50
\]

Fig. 4.22 Total computed response energy \( E_I \) (kN m) versus input wave “energy” 
\[
en = \int_0^t v^2(\tau)d\tau \quad \text{(cm}^2/\text{s)}
\]

for twelve earthquakes.
Fig. 4.23 Phase velocities of the first five Love-wave modes for a profile typical of San Fernando Valley.
Fig. 4.24 Shear $V_S$ and compression $V_P$ wave velocities for a typical profile in San Fernando Valley.
Table 4.3 Percentages of the energy components of the predicted response at the end of shaking, for VN7SH.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Code</th>
<th>N-S Comp. E I (kN·m)</th>
<th>N-S Comp. E I* (kN·m)</th>
<th>N-S Comp. E I/Ei (%)</th>
<th>N-S Comp. E i/Ibldg (%)</th>
<th>E-W Comp. E I (kN·m)</th>
<th>E-W Comp. E I* (kN·m)</th>
<th>E-W Comp. E I/Ei (%)</th>
<th>E-W Comp. E i/Ibldg (%)</th>
<th>(E K+E P) max/E i (%)</th>
<th>E I/PK (kN·m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Fernando</td>
<td>1</td>
<td>3340</td>
<td>2319</td>
<td>14.44</td>
<td>0.04</td>
<td>1491</td>
<td>860.0</td>
<td>7.82</td>
<td>0.00</td>
<td>71.69</td>
<td>20.58</td>
</tr>
<tr>
<td>Whittier Narrows</td>
<td>2</td>
<td>120.1</td>
<td>68.06</td>
<td>25.23</td>
<td>0.11</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Whittier 12th Aft.</td>
<td>3</td>
<td>15.09</td>
<td>12.69</td>
<td>20.64</td>
<td>1.03</td>
<td>8.84</td>
<td>6.98</td>
<td>19.35</td>
<td>0.41</td>
<td>63.85</td>
<td>16.64</td>
</tr>
<tr>
<td>Pasadena</td>
<td>4</td>
<td>3.26</td>
<td>1.06</td>
<td>34.95</td>
<td>7.04</td>
<td>3.25</td>
<td>3.58</td>
<td>26.95</td>
<td>1.25</td>
<td>49.75</td>
<td>21.96</td>
</tr>
<tr>
<td>Malibu</td>
<td>5</td>
<td>3.96</td>
<td>3.02</td>
<td>65.20</td>
<td>14.00</td>
<td>0.57</td>
<td>3.27</td>
<td>30.70</td>
<td>3.55</td>
<td>49.71</td>
<td>16.08</td>
</tr>
<tr>
<td>Montebello</td>
<td>6</td>
<td>0.27</td>
<td>0.07</td>
<td>29.64</td>
<td>0.33</td>
<td>0.10</td>
<td>0.10</td>
<td>78.36</td>
<td>1.38</td>
<td>10.11</td>
<td>7.01</td>
</tr>
<tr>
<td>Sierra Madre</td>
<td>7</td>
<td>61.68</td>
<td>46.97</td>
<td>20.64</td>
<td>1.03</td>
<td>42.10</td>
<td>28.89</td>
<td>22.14</td>
<td>0.00</td>
<td>62.49</td>
<td>12.67</td>
</tr>
<tr>
<td>Landers</td>
<td>8</td>
<td>766.0</td>
<td>621.5</td>
<td>10.62</td>
<td>0.01</td>
<td>377.0</td>
<td>75.12</td>
<td>13.67</td>
<td>0.10</td>
<td>56.39</td>
<td>29.83</td>
</tr>
<tr>
<td>Big Bear</td>
<td>9</td>
<td>49.58</td>
<td>32.60</td>
<td>10.11</td>
<td>0.05</td>
<td>70.65</td>
<td>45.93</td>
<td>10.60</td>
<td>0.49</td>
<td>59.50</td>
<td>28.88</td>
</tr>
<tr>
<td>Northridge</td>
<td>10</td>
<td>3531</td>
<td>2926</td>
<td>5.11</td>
<td>0.00</td>
<td>4850</td>
<td>3396</td>
<td>5.50</td>
<td>0.01</td>
<td>61.93</td>
<td>32.65</td>
</tr>
<tr>
<td>Northridge Mar. Aft.</td>
<td>11A</td>
<td>18.09</td>
<td>39.04</td>
<td>9.18</td>
<td>0.02</td>
<td>18.09</td>
<td>39.04</td>
<td>9.18</td>
<td>0.02</td>
<td>18.09</td>
<td>39.04</td>
</tr>
<tr>
<td>Northridge Mar. Aft.</td>
<td>11B</td>
<td>5.21</td>
<td>3.04</td>
<td>31.88</td>
<td>0.46</td>
<td>6.66</td>
<td>8.71</td>
<td>24.56</td>
<td>0.01</td>
<td>40.20</td>
<td>35.22</td>
</tr>
<tr>
<td>Northridge Dec. Aft.</td>
<td>12</td>
<td>6.18</td>
<td>3.07</td>
<td>17.70</td>
<td>0.10</td>
<td>6.18</td>
<td>3.07</td>
<td>17.70</td>
<td>0.10</td>
<td>6.18</td>
<td>3.07</td>
</tr>
</tbody>
</table>

*: record is not available.

$E_i^*$: total energy at the instant when $(E_K+E_P)$ is maximum.
Fig. 4.25 Percentages of the various response energies versus peak ground velocity, $v_{G,\text{max}}$. 
Fig. 4.26 Percentages of the various response energies versus peak “rocking” velocity, $\dot{\theta}_{\text{max}}$. 
For increasing amplitudes of shaking, the effective $\rho A \beta$ reduces to $\approx 0.5 \times 10^4$ kg/s. As already noted in Section 3.4, one reason why the effective factor $\rho A \beta$ is reduced is because the strong ground motion does not consist of plane vertically arriving S waves. Previous studies of phase delays between recording channels 1 and 13 (see Fig. 4.4 bottom) have shown that the apparent velocities of strong motion propagating along the longitudinal (EW) axis of this building are consistent with high frequency surface phase velocities in the soil and in the sediments beneath this building [Todorovska et al., 2001a, b].

Figure 4.23 illustrates Love wave phase velocities at this site, for the velocity structure shown in Fig. 4.24. In Table 4.3, we summarize the distribution of $E_I$ among $E_{\text{bldg}}^D$, $E_{\text{bldg}}^S$, $E_{\text{soil}}^D$ and $E_{\text{soil}}^{S+Y}$ at the end of the excitation, and $(E_K + E_P)_{\text{max}}$ for all twelve earthquakes. Figures 4.25 and 4.26 show the trend of these energies versus $v_{G,\text{max}}$ at ground level and versus $\dot{\theta}_{\text{max}} = \left| v_{\text{roof}} - v_{\text{base}} \right|_{\text{max}} / H$. 
5. POWER AND ELEMENTARY ASPECTS OF TRANSIENT DESIGN

As already noted in the introduction, much of the earthquake resistant design is based on the linear concepts of relative response spectrum, and mode superposition. However, the modal approach has a low-pass filtering effect on the end result (the computed peak relative displacement at each floor) because the higher modes are usually neglected. Therefore, the modal approach is not appropriate to represent the early transient response, particularly for excitation by high frequency pulses, with duration shorter than the travel time for an incident wave to reach the top of the building ($t < H/\beta$; $H$ and $\beta$ are the height and vertical shear wave velocity of the building). As the modes of vibration are standing waves and result from constructive interference of the incoming wave and the wave reflected from the top of the building, the building starts vibrating in the first mode only after time $t = 2H/\beta$ has elapsed. Although, in principle, the representation of the response as a linear combination of the modal responses is mathematically complete, short “impulsive” representation would require considering many modes (infinitely many for a continuous model), which is impractical. The wave propagation methods are more natural for representation of the early transient response, and therefore should be explored further and used to solve the problems where the modal approach is limited.

Wave propagation models of buildings have been used previously, but are only recently beginning to be verified against actual observations [Todorovska et al., 2001a,b]. Continuous, 2-D wave propagation models (homogeneous, horizontally layered and vertically layered shear plates) were employed to study the effects of traveling waves on the response of long buildings [Todorovska et al., 1988; Todorovska and Trifunac, 1989; 1990a,b; Todorovska and Lee, 1989], and discrete-time 1-D wave propagation models were proposed to study the seismic response of tall buildings [Safak, 1998].

In the following we use the elementary principles of wave propagation through a homogeneous shear beam model to derive approximate relationships between the power of an incident pulse of strong ground motion (with peak velocity $v_{G,max}$) and the building response, $v_b$.

5.1 Velocity Pulses

Strong ground motion can be viewed as resulting from a sequence of pulses emitted from failing asperities on the fault surface [Trifunac, 1972a,b; 1998]. Through multiple arrivals with different source to station paths and scattering, the strong motion observed at a site assumes the appearance of irregular oscillations in time, but may preserve one or several larger and long velocity “pulses”. These pulses are “spread out” in time due to multiple arrival paths and dispersion, but do appear systematically in adjacent stations, at
epicentral distances approaching 100 km [Todorovska and Trifunac 1997a,b; Trifunac et al., 1998].

As a first approximation, let us consider the foundation motion to be a velocity pulse with amplitude $v_b$ and duration $t_0$ (Fig. 5.1). For small $t_0$, this pulse approximates a delta function, and can be used as a building block to represent more general velocity pulses in input motion (Fig. 5.2). Figures 5.3 and 5.4 illustrate large velocity pulses in the near-field recorded during the 1971 San Fernando (Fig. 5.3) and 1994 Northridge (Fig. 5.4) earthquakes in California.

For an elastic building on rigid soil (i.e. no soil structure interaction), a velocity pulse with amplitude $v_b = v_{G,\text{max}}$, will create a wave propagating up the building with velocity $c$ (see Fig. 5.5). For times shorter than $H/c$ and for elastic strain $\gamma$ ($\gamma = \partial u/\partial x = v_b/c$), i.e. displacement $u(x,t)$ smaller than the elastic limit $u_y$ (Fig. 5.6), the wave propagating up into the building will be defined by a straight line

$$u(x,t) = \begin{cases} v_{b,t} - \frac{v_b}{c} x, & \text{for } 0 \leq x \leq c t \\ 0, & \text{for } x > c t \end{cases} \quad (5.1)$$

During reflection from the stress-free top of the building model, the incident wave from below and the reflected wave from above will interfere leading to double amplitude at the roof. The propagation of the energy of the pulse will continue downward as a linear wave as long as the incident wave amplitude is smaller than $u_y/2$.

Figure 5.7 illustrates the peak drift amplitudes ($v_b/c$) in a shear beam model of a building, assuming $c=100$ m/s short transient pulses, and linear response. For all twelve earthquakes recorded in the VN7SH building (Table 4.1), the maximum drift of the base of structure is plotted versus $v_b$ (solid points). For the Landers, San Fernando and Northridge earthquakes, the maximum drift, at the roof, is also shown ($2v_b/c$). It is seen that the maximum drift at base occurs during the Northridge earthquake and is approximately 0.5%, while at roof it is equal to about 1%.

For a building supported by flexible soil, the soil-structure interaction will lead to horizontal and rocking deformations of the soil, and in general this will reduce the amplitude $v_{b}$ of the strong motion pulse entering the structure. Partitioning of the incident wave energy into horizontal and rocking motions of the building-foundation-soil system and scattering of the incident wave from the foundation will thus reduce the energy available to cause relative deformation of the structure. Detailed temporal
Fig. 5.1 A velocity pulse with amplitude $v_b$ and duration $t_0$.

Fig. 5.2 A general velocity pulse $v(t)$. 

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Fig. 5.3 S16°E component of strong motion acceleration, velocity and displacement, recorded at Pacoima dam; during the San Fernando, California, earthquake of 1971 [Trifunac and Hudson, 1971].
Fig. 5.4 S48°W component of strong motion acceleration, velocity and displacement, recorded at Rinaldi Receiving Station, during the Northridge, California, earthquake of 1994 [Trifunac et al., 1998].
Fig. 5.5 A wave caused by sudden movement at the base of the shear building, for constant velocity pulse with amplitude $v_b$, for time $t < t_0$ (=pulse duration).

Fig. 5.6 Bilinear force-deformation representation of a shear-beam building model experiencing nonlinear response.
Fig. 5.7 Drift amplitudes in a shear-beam building model, with linear shear wave velocity $c = 100 \text{ m/s}$. 

$$\text{DRIFT} = \frac{v_b}{c}$$

$c = 100 \text{ m/s}$
Fig. 5.8a  Ratio of peak foundation velocity, $v_{f,\text{max}}$, at point $O$ (see Fig. 2.14) and “incident” strong motion velocity $v_{G,\text{max}}$ versus $v_{G,\text{max}}$ for strong motion recorded in Hollywood Storage Building (see Table 3.1).

Fig. 5.8b  Same as Fig. 5.8a except that the ratio $v_{f,\text{max}}/v_{G,\text{max}}$ is now plotted versus $\dot{\theta}_{\text{max}}$, an approximation of the building rocking velocity.
Fig. 5.9a Ratio of peak foundation velocity, $v_{f,max}$, at point $O$ (see Fig. 2.14) and “incident” strong motion velocity $v_{G,max}$, versus $v_{G,max}$ for strong motion recorded in VN7SH (see Table 4.1).

Fig. 5.9b Same as Fig. 5.9a except that the ratio $v_{f,max}/v_{G,max}$ is now plotted versus $\dot{\theta}_{max}$, an approximation of the building rocking velocity.
Fig. 5.10a Top: Ratio of \( \theta_{\text{rel}}^{\text{nls}} \) to \( \theta_{\text{rel}}^{\text{fix}} \) versus \( v_{G,\text{max}} \), where \( \theta_{\text{rel}}^{\text{rel}} \) is the relative response of the model shown in Fig. 2.14 and \( \theta_{\text{rel}}^{\text{fix}} \) is the same but in absence of soil-structure interaction. Bottom: Ratio \( (E_K + E_P)_{\text{max}} \big/ E_p^{\text{m}} \) versus \( v_{G,\text{max}} \), for strong motion data recorded in Hollywood Storage Building and summarized in Table 3.1. The open circles and triangles are for computed response, during events for which the roof record is not available.
Fig. 5.10b  Same as Fig. 5.10a, but with $\dot{\theta}_{\text{max}}$ in place of $v_{G,\text{max}}$. 
Fig. 5.11a Same as Fig. 5.10a, but with for strong motion data recorded in VN7SH (see Table 4.1).
Fig. 5.11b  Same as Fig. 5.10b, but with for strong motion data recorded in VN7SH (see Table 4.1).
analysis of this reduction is beyond the scope of this work. Here we only illustrate this indirectly and approximately.

The presence of the foundation within the soil creates an impedance jump for incident wave motion, and this causes scattering of the incident waves [Trifunac 1972c; Lee et al., 1982; Moslem and Trifunac, 1987; Todorovska and Trifunac 1990a,b; 1991, 1992, 1993]. For the simple model employed in this work and shown in Fig. 2.14, this scattering is not present because the strong motion excitation is not described as incident wave motion, but as horizontal and vertical translation of the “box” containing the foundation model. Consequently, the dissipation of the incident strong motion energy in this work is modeled only via the assumed radiation damping and by the hysteretic work of soil springs. This leads to negligible to minor reduction of the translational components of the incident peak ground velocity $v_{G,\text{max}}$, viewed through computed velocities $v_{f,\text{max}}$ of the foundation at point $O$ (see Fig. 2.14) representing the base of superstructure. This is illustrated in Figs 5.8 and 5.9, versus $v_{G,\text{max}}$ and $\dot{\theta}_{\text{max}}$ representing peak free-field velocity and an approximation of the peak rocking velocity of the relative response.

Figures 5.10 and 5.11 show $\theta_{\text{rel}}^{\text{rel}}/\theta_{\text{fix}}^{\text{rel}}$ and $(E_K + E_P)_{\text{max}} / E_I^*$ respectively for the Hollywood Storage Building excited by nine earthquakes (Table 3.1), and for the VN7SH building excited by twelve earthquakes (Table 4.1). The amplitudes of $(E_K + E_P)_{\text{max}} / E_I^*$ are plotted versus the peak “ground velocity”, $v_{G,\text{max}}$ and $\dot{\theta}_{\text{max}}$. It is seen that $(E_K + E_P)_{\text{max}}$ is in the range from 0.1 to 0.9 $E_I^*$. This implies that the amplitudes, $v_b$, of velocity “pulses” entering the structure and causing the relative response have been reduced, in this example by factors ranging from 1.1 to 3.5 relative to the “incident” pulses $v_{G,\text{max}}$. The top parts of Figs 5.10 and 5.11 show the reduction of the relative peak response $\theta_{\text{rel}}^{\text{rel}}$ affected by soil-structure interaction, soil damping, and the energy absorbed by hysteretic response of soil, $\theta_{\text{rel}}^{\text{rel}}$, relative to $\theta_{\text{fix}}^{\text{rel}}$ computed for the fixed-base model (without soil-structure interaction), that is for the case of “rigid” soil.
5.2 Power Demand and Absorption Capacity of the Structure

In the following, we assume that the velocity pulse entering the structure and deforming it (as shown in Fig. 5.5) has amplitude $v_b$. When the soil-structure interaction can be neglected, $v_b = v_{G,max}$, and when it redistributes the incident wave energy, $v_b = P v_{G,max}$ where $P \equiv 0.3–0.95$ for the examples considered here. Assuming that the soil foundation system has equivalent density $\rho_e$ and shear wave velocity $\beta_e$, the energy carried by the incident waves, per unit time and across unit area normal to the direction of propagation, is $\rho_e \beta_e v_b^2$.

With reference to Fig. 5.6, the hysteretic work per one complete cycle of nonlinear relative response of the structure is

$$W = 4 F_0 (u_u - u_y)$$

(5.2)

Since

$$F_0 = F_y - k_1 u_y = (1 - \alpha) k_0 u_y$$

(5.3)

defining

$$k_0 u_y \equiv F_y = m a_y$$

(5.4)

where $a_y$ is the static acceleration which produces deflection $u_y$, $k_1 = \alpha k_0$, and using the standard definition of ductility

$$\mu = u_u / u_y$$

(5.5)

one can write

$$W = 4 (1 - \alpha) m_b a_y (\mu - 1) / u_y$$

(5.6)

We approximate the equivalent stiffness of the nonlinear system by the secant modulus (see Fig. 5.6)

$$k_e = k_0 \left[ \frac{1 + \alpha (\mu - 1)}{\mu} \right]$$

(5.7)

This gives the approximate period of nonlinear oscillator

$$T_e = T_n \xi$$

(5.8)
where

$$\xi = \left[ \frac{\mu}{1 + \alpha(\mu - 1)} \right]^{1/2} \quad (5.9)$$

The dependence of $\xi$ on $\mu$ and $\alpha$ is illustrated in Fig. 5.12.

The maximum power the oscillator can absorb during one cycle of response is then

$$\frac{W}{T_e} = 4(1-\alpha)(\mu-1) m_b a \ y / T_n \xi \quad (5.10)$$

and since for the first mode of vibration $T_n = 4H/b$$

$$\frac{W}{T_e} = 4(1-\alpha)(\mu-1) \frac{m_b a \ y / b \beta}{4H \xi} \quad (5.11)$$

To avoid damage, we may require

$$A_b \beta_e \xi \leq (1-\alpha)(\mu-1) \frac{m_b a \ y / b \beta}{4H} \quad (5.12)$$

With $m_b = \rho b H A_b$, eqn (5.12) gives

$$v_b \leq (1-\alpha)(\mu-1) \frac{\rho b \beta_e a \ y}{\rho_e \beta_e \xi} \quad (5.13)$$

As an illustration, let $u_y = \psi H$, where $\psi$ is the drift angle and $H$ the height of the single story structure. Let $\psi = 0.05$ and $H = 3.5$ m. This gives $u_y = 0.05 \times 3.5 = 0.175$ m. Also, let $a_y = 0.25 \times 9.81$ m/s$^2 = 2.45$ m/s$^2$, $\mu = 2$, $\alpha = 0.05$, $\rho_b / \rho_e \sim 0.1$, $\beta_b / \beta_e \sim 0.3$ and $\xi \sim 1$. Then $v_b \leq (0.95)(1)(0.1)(0.3)(0.175)2.45 = 122$ cm$^2$/s$^2$ or

$$v_b \leq 11 \text{ cm/s} \quad (5.14)$$

It is interesting to note that for various transient excitations, including earthquakes, serious damage of buildings begins to occur for peak ground velocities exceeding 10-20 cm/s (e.g. see Trifunac and Todorovski [1997a]).

Equation (5.13) can be viewed only as a form of a dimensional analysis of the problem, since $v_b$ will oscillate in time, and since we did not solve the response problem explicitly. Furthermore it is not probable that the incident motion will be so regular to allow
\[ \xi = \left[ \frac{\mu}{1 + \alpha(\mu - 1)} \right]^{1/2} \]

Fig. 5.12 Plot of \( \xi \) versus ductility \( \mu \).

\[ \chi = \frac{\frac{1}{2} + (\mu - 1) + \frac{1}{2} \alpha(\mu - 1)^2}{(1 - \alpha)(\mu - 1)} \]

Fig. 5.13 Plot of \( \chi \) versus ductility \( \mu \).

\[ \frac{\chi}{\xi} \]

Fig. 5.14 Plot of \( \chi/\xi \) versus ductility \( \mu \).
monotonic completion of the complete hysteretic cycle. Instead, it is more likely that the pulse $v_B$ will be one-directional, with low frequency content, and of considerable duration causing monotonic increase of the relative displacement $u$. Therefore it is also of interest to examine the relationship of the input power demand relative to the capacity of the structure to absorb this power along the path $OYU$ (as shown in Fig. 5.6). The work accompanying nonlinear response in going from $O$ to $U$ is

$$W_x = \frac{1}{2} k_y u^2 + \left( u_a - u_y \right) k_y u_y + \frac{1}{2} k_y \left( u_a - u_y \right)^2$$

(5.15)

or

$$W_x = k_y \left[ \frac{1}{2} + \left( \mu - 1 \right) + \frac{1}{2} \alpha (\mu - 1)^2 \right] u_y^2$$

$$= \left[ \frac{1}{2} + \left( \mu - 1 \right) + \frac{1}{2} \alpha (\mu - 1)^2 \right] m_y a_y u_y$$

(5.16)

The time required to reach $U$ starting at $O$ is approximately $T_e/4$, and this gives the associated power absorbing capacity of the structure

$$4W_x / T_x \xi = \left[ \frac{1}{2} + (\mu - 1) + \frac{1}{2} \alpha (\mu - 1)^2 \right] m_y a_y u_y 4\beta_b / 4H \xi$$

(5.17)

Again, recalling that $m_b = A_b H \rho_b$, this requires

$$v_b^2 < \left[ \frac{1}{2} + (\mu - 1) + \frac{1}{2} \alpha (\mu - 1)^2 \right] \rho \beta_b / \rho \beta_e \xi$$

(5.18)

Comparison with eqn (5.13) shows that

$$\chi = \frac{1}{2} + (\mu - 1) + \frac{1}{2} \alpha (\mu - 1)^2$$

$$\left( 1 - \alpha \right) (\mu - 1) > 1$$

(5.19)

for $\mu \geq 1$ and $\alpha \geq 0$ (see Fig. 5.13).

In Fig. 5.14 we show $\chi / \xi$ versus $\mu$ and for $0 < \alpha < 0.2$. It is seen that $\chi / \xi$ monotonically decreases for increasing $\mu$. For $\mu < 4$, it is greater than 0.5 and for $\alpha \geq 0.2$ it is greater than 1.0. Thus combining eqns (5.13) and (5.18) and simplifying, we arrive at a remarkably simple criterion for design
where $C \sim 1$ for $\alpha \geq 0.2$. It is seen that as long as $a_y u_y$ satisfies eqn (5.20), the capacity of the structure to absorb the incident power will be adequate and no major damage is to be expected. For large velocity pulses, the damage will occur progressively and will continue to increase with each new additional pulse not satisfying eqn (5.20). With progressing damage, the system parameters will also change ($a_y, u_y, \beta_b$) and eqn. (5.20) will continue to apply, subject to appropriate modifications.

5.3 Application to the Case Study - Van Nuys Hotel (VN7SH)

Figure 2.15 shows $\log_{10} (a_y u_y / v_b^2)$ plotted versus $v_{G,\text{max}}$ (assuming that $v_b \approx v_{G,\text{max}}$) for the EW response of VN7SH for the 12 events in Table 4.1. Assuming $a_y = 0.13g$, $u_y = 15.3$ cm (see Fig. 5.17, $C \sim 1$, $\alpha \sim 0.2$, $\rho_e / \rho_b \sim 10$ and $\beta_e / \beta_b \sim 3$) would result in the condition $a_y u_y / v_b^2 > 30$ for no damage to occur. The observations in VN7SH suggest $a_y u_y / v_b^2 \geq 5$ as a criterion for no damage to occur. This is equivalent to the requirement that $v_b \leq 20$ cm/s, and shows that the design value of $u_y a_y$ for VN7SH should have been at least 5 to 10 times larger to avoid damage during the Northridge earthquake of 1994.

Figures 5.16a and b show the velocities recorded in the basement of VN7SH during the 12 earthquakes listed in Table 4.1. It is seen that during the 1971 San Fernando earthquake, the EW ground velocity exceeded 20 cm/s during very short time intervals and only slightly. During the 1994 Northridge earthquake, the EW peak ground velocities were larger, and two large peaks, at 3.45 s and 8.45 s, had peak amplitudes approaching 50 cm/s.

The above values of $a_y = 0.13g$ and $u_y = 15.3$ cm have been estimated based on nonlinear static push-over analysis of the EW response of VN7SH by Islam [1996] and Li and Jirsa [1998]. Their results are summarized in Fig. 5.17, showing the base shear coefficient, $V/W$, where $V$ is the computed base shear, and $W$ is total weight of the building ($W \sim 10^4$ kips), plotted versus roof displacement for triangular and uniform load distribution patterns. Also shown in this figure are the “maximum roof displacement” determined by Islam [1996] and by Li and Jirsa [1998] computed from the recorded data, and the computed UBC-94 base shear $V = 0.154 W$.

For fixed-base EW response, the two independent estimates of $F_y$ are $F_y = 1300$ kips (5780 kN) [Li and Jirsa, 1998] and $F_y = 1140$ kips (5070 kN) [Islam, 1996]. Assuming $u_y = 15.3$ cm, these two estimates imply $F_y u_y = 775$ to 884 kNm. During 1/4 cycle of the
Fig. 5.15  Plot of $\log_{10} \left( \frac{a_y u_y}{v_b^2} \right)$ versus $v_b$, assuming $a_y = 0.13g$, $u_y = 15.3$ cm (see Fig. 5.17) and $v_b \sim v_{G,\text{max}}$, for the twelve events in Table 4.1.
Fig. 5.16a  EW component of corrected velocity recorded at at ground level of VN7SH during eleven earthquakes (see Table 4.1). During the Whittier-Narrows, 1987 earthquake, the EW transducer at the ground floor did not record.
Fig. 5.16b  Same as Fig. 5.16a but for the NS component of motion.
Fig. 5.17 Base shear (V) coefficient normalized by 1/3 of the total building weight, W, versus EW roof displacement of VN7SH (after Islam [1996] and Li and Jirsa [1998]).

Fig. 5.18 The shaded areas represent: (left-top) maximum potential energy associated with linear response; (left-bottom) hysteretic energy associated with monotonic nonlinear response; and (right) hysteretic energy associated with oscillatory (periodic, one cycle) excitation.
Fig. 5.19a  Comparison of the EW response of VN7SH in the presence and absence of soil-structure interaction. Top: relative velocities of the building response. Center: energies of the relative response. Bottom: power of the relative response.
Fig. 5.19b  Enlarged window in Fig. 5.19a.
response, assuming linear deformation in the building would result in maximum accumulated potential energy equal to 387 to 442 kNm (see Fig. 5.18, left-top). For $T_\mathrm{n} \sim 0.8$ s, we can estimate the largest power of the EW component of the incident waves which the VN7SH building can take without damage to be 1932 to 2208 kNm/s.

The time dependent evolution of the energy dissipated by nonlinear building response will depend on the history of the excitation, but several characteristic milestone values can nevertheless be estimated a priori. This is illustrated in Fig. 5.18. The shaded area in the top-left of this figure illustrates the largest potential energy in the oscillator, which is still responding in the linear range of response ($u < u_y$) when $\mu = 1$ and when $F = F_y$. For VN7SH, using static push over analyses of Islam [1996] and Li and Jirsa [1998], for EW response, as indicators of the possible range of $F_y$, we obtain the estimates of 5070 kN and 5783 kN respectively. A larger and longer lasting incident velocity pulse might force the equivalent oscillator to deform monotonically to say $u = 2 u_y$ ($\mu = 2$), during $T_e/4$. This case is illustrated in the left-bottom part of Fig. 5.18. Assuming that $\alpha = 0.2$ implies the work dissipated by the hysteresis in going from $O$ to $Y$ to $U$ to be 1240 to 1414 kNm (see eqn (4.16)), and the associated power $4W \to /T_\xi$ to be in the range from 4816 to 5492 kNm/s. The right part of Fig. 5.18 shows the closed hysteretic loop, starting at $OYU$ and returning to $Y$ after one complete cycle lasting $T_\xi$ s. The work dissipated by such a loop, assuming $F_y$ as above, ($\mu = 2$ and $\alpha = 0.2$) is 2480 to 2829 kNm. The corresponding maximum power this oscillator can dissipate along this path is then 2407 to 2746 kNm/s. These estimates of the building capacity to absorb energy and power are shown by the gray bands in Fig. 5.19a.

Figure 5.19a (top) shows the relative response of the model in Fig. 2.14 with parameters chosen to represent VN7SH. In this figure we compare the relative response assuming fixed-base (“no SSI”), and flexible base (“with SSI”) using the model in Fig. 2.14. In the center of Fig. 5.19 we show the sum of all energies in the relative building response (kinetic, potential and hysteretic, when the building model is linear and nonlinear), for the fixed base model (“no SSI”) and in the presence of soil-structure interaction (“with SSI”). It is seen that for the “no SSI” case, a large ground motion pulse starting at about 3.4 s (see Fig. 5.16a) would have resulted in energy jump of about 520 kNm, during about 0.22 s, resulting in input power approaching 3000 kNm/s (see bottom of Fig. 5.19). This pulse would have deformed the building beyond its linear response range, between 3.5 and 4 s into the earthquake (see also Islam, 1996). In the presence of soil structure interaction (see model in Fig. 2.14), the amplitude of the incident wave is reduced, and the building continues to respond in essentially a linear manner until 8.4 s into the earthquake. At about 8.9 s, the SSI model experiences sudden jump in the energy of the relative response (e.g. at 8.9 and 9.7 s) during short “stiff” episodes of response, for example
Fig. 5.20 Same as Fig. 5.19a but for the EW response of VN7SH building during the San Fernando earthquake of 1971.
Fig. 5.21  Comparison of the relative response energies in the EW response of VN7SH building to eleven earthquakes (see table 4.1).
Fig. 5.22 Same as Fig. 5.21 but for the NS response.
Fig. 5.23 Comparison of the total (end) relative response energies in the VN7SH building for the response computed with and without soil-structure interaction, plotted versus the peak velocity of the ground floor response $v_{G,max}$. 
during closure of the gaps between the foundation and the nonlinear springs representing soil. Nevertheless, the benefits of not ignoring SSI are apparent from Fig. 5.19a (center), which shows that the response energy in the building in this case is reduced by a factor of about 3 due to SSI. The challenge for future research is to quantify such reductions and to show how those can be estimated for use in the design process.

Figure 5.20 shows the EW response of VN7SH during the 1971 San Fernando earthquake. It is seen that both the energy in the building and the power of the relative EW response were smaller than ~ 300 kNm and ~ 800 kNm/s respectively, implying that the building responded essentially in a linear manner. The building experienced only minor damage during the San Fernando earthquake [Blume et al., 1973], which may have been caused by its response to NS excitation. Analysis of the NS response of VN7SH is beyond the scope of this paper, and will be presented in our future studies.

Figures 5.21 and 5.22 compare the EW and NS energies of the relative building response during 12 (11) earthquakes listed in table 4.1. The two gray zones (387-442 kNm and 1204-1373 kNm) described in the above discussion of Fig. 5.19a are also shown in Fig. 5.21. A complete set of figures showing further details of this response are presented in Appendix C.

Figure 5.23 summarizes the total (end) relative response energies in the VN7SH building in the presence and absence of SSI, versus $v_{G,max}$ and during the 12 earthquakes listed in Table 4.1.
6. SUMMARY AND CONCLUSIONS

In this report, we described the earthquake energy flow, from the earthquake source to a structure, with the objective to define a new methodology for design of structures for transient excitation. Starting with the earthquake source and ending with the power of the waves propagating through a structure, we attempted to identify the principal stages of seismic energy flow and of the accompanying energy dissipation mechanisms. For the wave energy to become a viable design tool, it is necessary to understand and to quantify empirically all stages of this flow, and to show that it provides better, or at least equivalent, results as the currently accepted design methods. One of the principal weaknesses of the classical Biot’s response spectrum method has been its dependence on the peak response amplitude alone, without explicit consideration of the duration of strong shaking and of the rate of arrival of the incident strong motion energy.

We started this analysis by considering the earthquake energy at the source, and showed how a simple dimensional analysis, based on plane wave representation of seismic energy flow, can be used to approximate this energy. By using integrals of the form $\int_0^t v^2(\tau) d\tau$ and the Parseval’s Theorem, we showed how empirical scaling equation for Fourier amplitude spectra could be extended to estimate the radiated seismic energy. Since attenuation of strong motion spectral amplitudes is a well-researched and documented process, we in fact also showed how the seismic wave energy can be estimated at any distance from the source. To show how the seismic wave energy attenuates in a realistic geologic environment, we presented examples of contour maps of strong motion duration and of various energy-related functionals for the Los Angeles area during the 1994 Northridge earthquake. These maps show that the attenuation of energy and of power of seismic waves are slowly varying functions of distance and of azimuth relative to the earthquake source, and that their spatial variations are affected mainly by the complex three-dimensional geology of the Los Angeles basin [Todorovska and Trifunac, 1997a,b].

To study the energy flow and dissipation through a soil-structure system, as a basic vehicle we adopted a simple model in which both the soil and structural response can be nonlinear. This model, shown in Fig. 2.14, consists of a rigid foundation supported by nonlinear soil springs, and a structure represented by a single degree-of-freedom oscillator, which can also experience nonlinear response. For case studies for this model, we used a 14-story storage building in Hollywood, and a 7-story hotel in Van Nuys, for which processed strong motion data was available. All of the recorded strong motion data was analyzed, and the simple model was used to quantify approximately the distribution of the incident wave energy. We were able to show that there is good correspondence between the estimates of the incident wave energy and the sum of all response energies in
the soil-structure system, shown in Figs 3.21 and 4.22. This result points to the need to research the transfer of the incident wave energy into soil-structure systems. These and other published results show also that a soil-structure system is capable of reflecting large fractions of the incident strong motion energy back into the soil by means of scattering (not present for the model in Fig. 2.14) and nonlinear soil response. Clearly, the nature of this powerful energy dissipation mechanism must be carefully studied to provide reliable and verifiable estimates for use in the future earthquake resistant design.

At the end, we examined some elementary aspects of transient waves propagating in a structure, and presented conceptual relationships between the amplitude of peak velocity of the wave propagating up the structure, $v_b$, and the energy and power of the response. The presented analysis is approximate and represents only a *dimensional analysis* of the problem in its most rudimentary form. Nevertheless, it shows that consideration of the energy of response is not sufficient for the purposes of earthquake resistant design. The power of the incident waves, that is the time rate of the incoming seismic wave energy, must also be considered if the damage of structures is to be controlled or eliminated.
7. REFERENCES


APPENDIX A: EVALUATION OF SOIL STIFFNESS AND DAMPING CONSTANTS

One method for estimating the deflection of a soil-foundation system involves replacing the soil by a set of independent springs and dashpots producing an equivalent reactive force to the deformation developed.

The soil dashpots represent two sources of damping: material and radiation damping. The material damping depends on the level of strain in the material. If the strains are large, the material damping can be substantial, and if they are small, it may be negligible. The radiation damping represents a purely geometric effect, which exists at small as well as large strain amplitudes. For typical foundations, the radiation damping is often larger than the material damping.

Most previous studies have used the theory of elasticity to obtain the spring constants for foundations resting on the surface of an elastic half-space. Also, there have been many studies for embedded foundations. The embedment increases the soil resistance to motion of the foundation as well as the inelasticity of the soil behavior.

According to Richart et al. [1970], for rectangular footings of dimensions $2a \times 2b$, the sliding and rocking stiffness coefficients and the radiation damping ratios are

$$k_s = 4(1 + \nu) GB_s \sqrt{ab}$$  \quad (A.1)

$$\zeta_s = \frac{0.288}{\sqrt{B_s}}$$  \quad (A.2)

$$K_r = \frac{G}{1-\nu} B_r 8ab^2$$  \quad (A.3)

and

$$\zeta_r = \frac{0.15}{(1 + B_r)\sqrt{B_r}}$$  \quad (A.4)

in which

$2a = $ width of the foundation (along the axis of rotation in the case of rocking)

$2b = $ length of the foundation (in the plane of rotation for rocking)
and $B_s$ and $B_r$ depend on the $bla$ ratio, and on the soil density and Poisson ratio, $\rho_s$ and $\nu$.

Lysmer [1965] considered a SDOF lumped parameter analog to estimate the dynamic response of a circular footing of radius $r_0$ under vertical vibrations, with spring and damping constants

$$k_z = \frac{4Gr_0}{1-\nu} \quad \text{(A.5)}$$

$$c_z = \frac{3.4}{(1-\nu)} \frac{r_0^2}{\sqrt{\rho_s G}} \quad \text{(A.6)}$$

The above expressions depend only on the area of the contact surface and on the elastic properties of the soil medium. Also, during vertical vibrations, the soil below the footing is assumed to be in a state of elastic uniform compression.

Coupled rocking and sliding vibrations of an embedded rectangular foundation produce normal stress on the soil mass adjacent to the two vertical contact faces, parallel to and on either sides of the axis of rocking. This soil mass is assumed to be in a state of compression if only the horizontal component of coupled motion is considered.

As a half of the semi-infinite soil medium is effective with respect to each vertical contact face of the foundation, it is reasonable to assume that the horizontal lateral stiffness $k_h$ and damping $c_h$ are approximately equal to the values of $k_z$ and $c_z$ given by eqns (A.5) and (A.6) respectively [Krishnaswany and Ravi, 1986].

Therefore, for an embedded rectangular foundation of base dimensions $2a \times 2b$ with embedment $D$, approximate values of $k_h$ and $\zeta_h$ can be calculated using the following expressions

$$k_h = \frac{G}{1-\nu} B_h \sqrt{2aD} \quad \text{(A.7)}$$

$$\zeta_h = \frac{0.213}{\sqrt{B_h}} \quad \text{(A.8)}$$

in which $B_h$ depends on the $D/(2a)$ ratio and on the soil properties, $\rho_s$ and $\nu$.

In this report, the initial values of the soil spring and damping constants were obtained from eqns (A.1) through (A.4), (A.7) and (A.8). Using these as initial trial values, if the predicted model response was “similar” to the recording in both the time and frequency
domains, these constants were accepted and stored. If it was not similar to the recording, we tried scaling factors to modify eqns (A.1), (A.3) and (A.7), until the model response became similar to the recorded response.
APPENDIX B: SOIL FORCE-DEFORMATION MODELS

B.1 Slip Hysteretic Force-Deformation Characteristics

The slip force-deformation characteristics of the model we employed for the analysis in this report are specified by the elastic slope, $k$, the yield level, $u_y$, and the second stiffness ratio, $\alpha$, as shown in Fig. B.1.

Suppose that this slip system has been loaded (the foundation pressing against the soil), and the force-deformation state has reached point $A$ in Fig. B.1. The restoring force becomes zero when the unloading reaches point $B$. At this instant, a permanent offset, $s$, is produced. During loading in the reverse direction, the $f-u$ relation takes the path $B-O$. The path between points $B$ and $O$ has no accompanying stresses. Since the soil on the side of the foundation is assumed to be non-extensible, no negative deformation develops beyond point $O$. During reloading in the positive direction from point $O$, $f = 0$ while the deformation reaches point $B$. Further positive loading follows path $B-A-C$. The deformation of point $A$ becomes the current yield level, $u_{yc}$, and the unloading occurs along the elastic path $D-E$.

Fig. B.1 Adjusted slip hysteretic force-deformation model.
Like in the case of bilinear systems, the system behaves elastically when the deflection $u$ is between $s$ and $u_{yc}$, even after having experienced a history of arbitrary inelastic strains. On the other hand, the slip system has a domain for which deformation may develop without reference to $f$, while $f = 0$. This $f$–$u$ relation applies to all springs on both sides of the foundation in Fig. 2.14.

### B.2 Bilinear Hysteretic System

The bilinear hysteretic force-deformation model used in this report is shown in Fig. B.2. Here, $u_y$ represents the initial yield level, $u_{yn}$ and $u_{yp}$ are the current yield levels (negative and positive, respectively), $s$ is the current permanent offset, $k$ is the initial elastic and unloading stiffness, and $\alpha$ is the ratio of the second stiffness (strain-hardening stiffness) to the elastic stiffness. Initially, $s = 0$, $u_{yp} = u_y$, and $u_{yn} = -u_y$. Notice that the current positive and negative yield levels are separated by a region of linear elastic deformation $2u_y$.

![Bilinear hysteretic force-deformation model](image)

Let us consider first linear elastic response following unloading. The corresponding $f$–$u$ relation is

$$f = k (u - s) \quad \text{(B.2.1)}$$

The offset $s$ in eqn (B.2.1) is computed at the instant of unloading. Following an excursion of positive yielding, the offset is given by
\[ s = (1 - \alpha)(u_{unl} - u_y) \]  
(B.2.2)

and following an excursion of negative yielding

\[ s = (1 - \alpha)(u_{unl} + u_y) \]  
(B.2.3)

In these formulae, \( u_{unl} \) is the deformation calculated at the instant of unloading. At the same time, the current yielding levels are updated. For example, following a positive yield excursion, \( u_{yp} = u_{unl} \) and \( u_{yn} = u_{unl} - 2u_y \).

Next, let us consider excursions of loading for the bilinear system beyond the current yield levels. With reference to Fig. B.2, the \( f-u \) relation for deformations greater that the current positive yield level \( u_{yp} \) is

\[ f = k(u_{yp} - s) + \alpha k(u - u_{yp}) \]  
(B.2.4)

This expression applies for \( u > u_{yp} \) until unloading is detected, when the product \( \dot{u}_i \dot{u}_{i+1} < 0 \). For an excursion of negative yielding, \( u < u_{yn} \), and eqn (B.2.4) applies with the modification

\[ f = k(u_{yn} - s) + \alpha k(u - u_{yn}) \]  
(B.2.3)
This appendix presents results of simulated EW and NS responses of VN7SH, using the model in Fig. 2.14, for the twelve earthquakes listed in Table 4.1. The solid and dashed lines correspond respectively to the cases of considering and ignoring the soil-structure interaction. The top parts show the relative velocity response, the central parts show the energies of the relative responses, and the bottom parts show the power of the relative responses.
Fig. C.1a  Comparison of EW response of VN7SH in the presence and absence of soil-structure interaction, during the 1971 San Fernando earthquake. Top: relative velocities of the building response. Center: energies of the relative response. Bottom: power of the relative response.
Fig. C.1b  Same as Fig. C.1a, but for the NS response.
Fig. C.2  Same as Fig. C.1a, but for the NS response during the 1987 Whittier-Narrows earthquake.
Fig. C.3a Same as Fig. C.1a, but for the EW response during the 4 October, 1987, Whittier-Narrows aftershock.
Fig. C.3b  Same as Fig. C.1a, but for the NS response during the 4 October, 1987, Whittier-Narrows aftershock.
Fig. C.4a  Same as Fig. C.1a, but for the EW response during the 1988 Pasadena earthquake.
Fig. C.4b  Same as Fig. C.1a, but for the NS response during the 1988 Pasadena earthquake.
Fig. C.5a Same as Fig. C.1a, but for the EW response during the 1989 Malibu earthquake.
Fig. C.5b  Same as Fig. C.1a, but for the NS response during the 1989 Malibu earthquake.
Fig. C.6a  Same as Fig. C.1a, but for the EW response during the 1989 Montebello earthquake.
Fig. C.6b Same as Fig. C.1a, but for the NS response during the 1989 Montebello earthquake.
Fig. C.7a  Same as Fig. C.1a, but for the EW response during the 1991 Sierra Madre earthquake.
Fig. C.7b  Same as Fig. C.1a, but for the NS response during the 1991 Sierra Madre earthquake.
Fig. C.8a Same as Fig. C.1a, but for the EW response during the 1992 Landers earthquake.
Fig. C.8b Same as Fig. C.1a, but for the NS response during the 1992 Landers earthquake.
Fig. C.9a Same as Fig. C.1a, but for the EW response during the 1992 Big Bear earthquake.
Fig. C.9b  Same as Fig. C.1a, but for the NS response during the 1992 Big Bear earthquake.
Fig. C.10a  Same as Fig. 5.19a.
Fig. C.10b  Same as Fig. 5.19a, but for the NS response.
Fig. C.11a  Same as Fig. C.1a, but for the EW response during the first segment of 20 March 1994 Northridge aftershock.
Fig. C.11b  Same as Fig. C.1a, but for the NS response during the first segment of 20 March 1994 Northridge aftershock.
Fig. C.11c  Same as Fig. C.1a, but for the EW response during the second segment of 20 March 1994 Northridge aftershock.
Fig. C.11d  Same as Fig. C.1a, but for the NS response during the second segment of 20 March 1994 Northridge aftershock.
Fig. C.12a  Same as Fig. C.1a, but for the EW response during the 6 December 1994 Northridge aftershock.
Fig. C.12b  Same as Fig. C.1a, but for the NS response during the 6 December 1994 Northridge aftershock.
APPENDIX D: NOTATION AND LIST OF SYMBOLS USED

Chapter 1

$M_s$ = surface wave magnitude

$M_w$ = moment magnitude

$M_0$ = seismic moment

$M_{SM}$ = strong-motion magnitude

$r$ = distance that wave travels away from its source

$E_I$ = input energy

$E_K$ = kinetic energy

$E_\zeta$ = damping energy

$E_E$ = elastic strain energy

$E_H$ = hysteretic energy

Chapter 2

$M_L$ = Richter’s local magnitude

$I$ = maximum Modified Mercelli Intensity

$h$ = hypocentral depth

$\rho$ = material density

$\alpha$ = velocity of P-waves

$\beta$ = velocity of S-waves

$v$ = particle velocity

$A$ = area that the shear wave energy is transmitted through

$T$ = duration of recorded and processed acceleration

$F(\omega)$ = Fourier amplitude spectrum of acceleration

$en$ = integral of velocity squared in the frequency band 0.01–100 Hz

$L$ = fault length

$W$ = fault width

$R$ = hypocentral distance

$\Delta$ = epicentral distance

$Q$ = quality factor

$IA2$ = integral of acceleration squared

$IV2$ = integral of velocity squared

$G$ = shear modulus of the soil medium

$\nu$ = Poisson’s ratio

$\rho_s$ = mass density of the soil medium

$\theta_{rel}^{rel}$ = relative rotational response of the equivalent SDOF oscillator

$m_b$ = mass of the equivalent SDOF oscillator

$H$ = height of the equivalent SDOF oscillator

$r_b$ = radius of gyration of the equivalent SDOF oscillator

$K_b$ = spring stiffness of the equivalent SDOF oscillator

$C_b$ = viscous damping of the equivalent SDOF oscillator

$T_1$ = first natural period of a continuous building model

$H_{sb}$ = height of a continuous building model
\[ W_{sb} \] = width of a continuous building model
\[ D \] = depth of the foundation
\[ m_f \] = mass of the foundation
\[ I_f \] = mass moment of inertia of the foundation
\[ u \] = horizontal translation of the foundation
\[ \phi \] = rocking of the foundation
\[ k_h \] = equivalent horizontal stiffness of the soil
\[ k_s \] = equivalent shear stiffness of the soil
\[ K_r \] = equivalent rocking stiffness of the soil
\[ c_h \] = equivalent horizontal damping constant of the soil
\[ c_s \] = equivalent shear damping constant of the soil
\[ C_r \] = equivalent rocking damping constant of the soil
\[ u_g, v_g \] = horizontal and vertical ground displacements
\[ g \] = acceleration due to gravity
\[ u_{CG}, v_{CG}, \phi_{CG} \] = displacements (horizontal, vertical and rotational) of the center of mass of the foundation
\[ u_O, v_O, \phi_O \] = displacements (horizontal, vertical and rotational) of the base of the equivalent SDOF oscillator
\[ u_b, v_b, \phi_b \] = displacements (horizontal, vertical and rotational) of the center of mass of the equivalent SDOF oscillator
\[ f_{x,b}, f_{z,b}, M_{O,b} \] = interactive forces and moment between the equivalent SDOF oscillator and the foundation
\[ f_{x,f}, f_{z,f}, M_{B,b} \] = interactive forces and moment acting at the base of the foundation
\[ f_{h,i}, f_{s,j} \] = horizontal reactive forces of the soil
\[ d_i \] = distance from the \( x \)-axis to the point where \( f_{h,i} \) is acting
\[ l_j \] = distance from the \( z \)-axis to the point where \( f_{s,j} \) is acting
\[ E_K \] = kinetic energy
\[ E_p \] = potential energy of the gravity forces
\[ E_D^{\text{bldg}} \] = damping energy dissipated in the building
\[ E_S^{\text{bldg}} \] = recoverable elastic strain energy in the building
\[ E_D^{\text{soil}} \] = energy dissipated by the dashpots of the soil
\[ E_S^{\text{soil}} \] = elastic strain energy in the soil
\[ E_Y^{\text{soil}} \] = irrecoverable hysteretic energy in the soil
\[ E_S^{\text{soil}+Y} = E_S^{\text{soil}} + E_Y^{\text{soil}} \]
\[ E_I \] = total earthquake input energy

Chapter 3 and Chapter 4

\[ f_p \] = system apparent frequency
\[ a_0 \] = a constant determined by linear least squares fit \( y = a_0 x \)
\[ a_1, b_1 \] = constants determined by linear least squares fit \( y = a_1 x + b_1 \)
\[ a'_0 \] = \( a_0 / 10^4 \)
\( v \) = velocity
\( E_I \) = “total” energy of the SSI system at the end of shaking
\( E_I^* \) = “total” energy of the SSI system at the instant when \( (E_K + E_P) \) is maximum
\( \rho \) = density
\( \beta \) = shear wave velocity in the soil surrounding the foundation
\( A \) = area of the plan of the building foundation
\( v_{G,\text{max}} \) = peak ground velocity
\( (\Delta v)_{\text{max}} \) = difference between the peak velocities at the roof and at ground level of the building
\( \theta \) = rocking “angle” of the building
\( \dot{\theta}_{\text{max}} \) = peak “rocking” velocity
\( f_{\text{beg}}, f_{\text{end}} \) = apparent frequencies of response at the beginning and end of shaking
\( f_{\text{max}}, f_{\text{min}} \) = maximum and minimum apparent frequencies of response during shaking
\( (\Delta f)_{\text{max}} \) = largest difference in the apparent frequencies during shaking
\( R \) = epicentral distance

Chapter 5

\( t \) = time
\( H \) = building height,
\( \beta \) = shear-wave velocity
\( v_b \) = amplitude of the velocity pulse
\( t_0 \) = duration of the velocity pulse
\( c \) = vertical wave velocity in the building
\( \gamma \) = elastic strain
\( u(x,t) \) = displacement
\( \theta_{\text{rel}} \) = relative rotational deformation of the equivalent SDOF oscillator for a nonlinear SSI system
\( \theta_{\text{rel,fix}} \) = relative rotational deformation of the equivalent SDOF oscillator for a fixed-base system
\( u_y \) = elastic displacement limit
\( u_u \) = ultimate displacement limit
\( a_y \) = static acceleration
\( F_y \) = force at the first yielding displacement \( u_y \)
\( F_u \) = force at the ultimate displacement \( u_u \)
\( F_0 \) = force in the hysteretic system at zero displacement \( u=0 \)
\( k_0, k_1 \) = initial elastic and second stiffness
\( \alpha \) = ratio of the second stiffness to the elastic stiffness
\( \rho_e \) = equivalent density of the soil-foundation system
\( \beta_e \) = equivalent shear-wave velocity of the soil-foundation system
\( k_e \) = equivalent stiffness of a nonlinear system
\( \mu \) = ductility
\( m_b \) = building mass
$\beta_b$ = shear-wave velocity in the building
$\rho_b$ = building mass density
$\xi$ = factor describing response period prolongation in nonlinear response
$T_n$ = natural period of the building
$T_e$ = equivalent period of the nonlinear oscillator
$\psi$ = drift angle
$W_{ts}$ = maximum energy the oscillator can absorb during one cycle of hysteretic response
$W_{\rightarrow}$ = work accompanying nonlinear response in pushover analysis
$\chi$ = dimensionless ratio
$C$ = constant in eqn (5.20)
$V$ = base shear force
$W$ = total building weight

Appendix A

$k_s, \zeta_s, K_r, \zeta_r$ = sliding and rocking stiffness and the radiation damping ratios
$G$ = shear modulus of the soil
$\nu$ = Poisson’s ratio
$\rho_s$ = mass density of the soil
$2a, 2b$ = plan dimensions of the foundation
$k_c, c_c$ = vertical spring and damping constants
$r_0$ = radius of a circular footing
$k_h, c_h, \zeta_h$ = horizontal lateral stiffness and damping constants
$D$ = embedment depth of a rectangular foundation
$B_s, B_r, B_h$ = constants

Appendix B

$f$ = force
$u$ = deformation
$k$ = elastic stiffness
$u_y$ = yield level
$\alpha$ = second stiffness ratio
$s$ = permanent offset
$u_{yc}$ = current yield level
$u_{yn}, u_{yp}$ = current negative and positive yield levels
$u_{unl}$ = deformation at the instant of unloading
$\dot{u}_i, \dot{u}_{i+1}$ = velocities at instants $i$ and $i+1$