Comparison of Practical Feedback Algorithms for Multiuser MIMO

Maryam Modir Shanechi
Dept. EECS, MIT
Cambridge, MA
Email: shanechi@mit.edu

Ron Porat
NextWave Wireless
San Diego, CA
Email: rporat@nextwave.com

Uri Erez
Tel Aviv University
Ramat Aviv, Israel
Email: uri@eng.tau.ac.il

***Abstract*— We consider the problem of channel state information (CSI) transmission on a fast feedback link for a multiuser MIMO system.We examine the relative merits of channel feedback schemes based on Shannon's source-channel separation theorem and non-separation based schemes and show that the latter are preferable in this application. For the non-separation based schemes, we first consider a simple analog transmission and then develop a hybrid digital-analog transmission scheme which quantizes the CSI using a few bits and sends these bits and also the quantization error using analog transmission. We show that the hybrid scheme achieves a higher throughput compared to both analog and digital transmissions and has a much lower computational complexity compared to a digital scheme.**

I. Introduction

In closed loop multiple-input multiple-output (MIMO) cellular systems, the base station must obtain knowledge of the downlink channel state information (CSI). For single user MIMO applications, this information is used to direct the transmitted power towards the mobile station and provides an SNR gain which results in significant throughput gains. For multiuser (MU) MIMO applications on the other hand, accurate CSI at the transmitter (CSIT) is much more crucial since having approximate CSIT results in residual interference, incurs a significant loss in throughput, and does not achieve the full multiplexing gain [1]. Obtaining accurate CSIT at the base station is a difficult problem especially in frequency-division duplex (FDD) systems, where channel reciprocity cannot be used and the CSI must be fed back to the base station on a fast feedback link. Hence, designing good channel feedback coding algorithms are of significant interest in these systems.

The channel state information, e.g., the channel matrix, is an analog source in nature. The goal of the feedback algorithm is to encode this information at the mobile station to enable its reconstruction with minimum distortion at the base station. Thus, the feedback coding problem is a joint source-channel coding problem. In addition, the requirement of low delay dictates the use of short block lengths.

There are two main approaches one could take in designing the CSI feedback link. One that has traditionally received most attention is based on Shannon's source-channel separation theorem that proves the asymptotic optimality of separate source and channel coding under certain conditions. In these schemes, which we refer to as "digital schemes", the source (CSI) is first quantized and then the quantization bits are coded using a channel code to recover the quantized source with low error probability at the receiver. The main drawback of a digital scheme however, is the threshold effect which means that the system achieves the desired performance only at the designed SNR. At lower SNR, system performance is severely sacrificed and at higher SNR, it does not improve. Hence this approach is suboptimal in a scenario where the exact SNR is known only to lie in some range. Another shortcoming of this approach even for a single (and known) SNR is that long source and channel codes are needed to achieve the optimal performance which is not possible over a fast feedback link.

The other approach one could take to solve this problem is a joint source-channel approach that does not follow the separation principle. The simplest example of such a scheme is analog transmission that allows for a graceful degradation of performance at low SNR and does not saturate at high SNR. However, while it is optimal (when the criterion is that of mean-squared error (MSE)) for transmitting a Gaussian source over a Gaussian channel of equal bandwidth, it is suboptimal when the bandwidths are different. Nonetheless, more advanced joint source-channel coding schemes have been devised that make better use of the available bandwidth than purely analog transmission and at the same time are more robust than separate source and channel coding (over a channel with unknown SNR) and offer better performance for a given delay [2] [3] [4].

Here, we examine the relative merits of separation based and non-separation based approaches in the context of MIMO channel feedback, and observe that the latter are preferable in practical MIMO systems. We show that these approaches work well in MU-MIMO systems without exact knowledge of the SNR, which is needed in a digital approach as shown in [5]. They also incur a much smaller computational complexity as compared to a digital scheme. In particular, we develop a hybrid digital-analog source-channel code for the feedback link and show that it performs close to the perfect CSIT case.

II. Channel Model and Transmission Scheme

We consider a MU-MIMO system with K users each with N_r antennas and a base station (transmitter) with N_t antennas. The downlink channel model of interest takes the form

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x} + \mathbf{w}_k,\ k = 1, \cdots, K \tag{1}$$

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where channel input $\mathbf{x}$ and channel output at user k, $\mathbf{y}_k$, are N_t- and N_r-dimensional vectors, respectively, and where the associated noise vectors $\mathbf{w}_k$ are $\mathcal{CN}(0, N_0\mathbf{I})$ and independent. Here, $\mathbf{H}_k$ is the channel matrix from the base station to the kth user whose elements we assume are $\mathcal{CN}(0, 1)$ and independent, i.e., Rayleigh faded. The channel input is constrained to an average power P, i.e., $E[|\mathbf{x}|^2] = P$ and hence for downlink, $\mathrm{SNR} = P/N_0$. We assume that users have perfect CSI at their receivers (CSIR). In this work we set the number of users equal to the number of transmit antennas which is the regime of interest for most next generation wireless systems [5].

For concreteness, we study the performance of the feedback schemes for a specific (zero-forcing based) transmission approach (similar to the one in [6]), but none of the schemes is specifically tailored to this approach. Our transmission scheme is based on sending a single data stream to each user using a simple regularized zero-forcing (ZF) beamforming that performs better than a ZF beamforming [7]. Our scheme is hence similar to that in [6]. Assuming K active users in the system and denoting the strongest right singular vector of the kth user by $\mathbf{v}^{(k)}$, the regularized ZF beamformer for user k, $\mathbf{f}_k$, is given by the normalized kth column of the matrix

$$\Omega = \Gamma^\dagger \left(\Gamma\Gamma^\dagger + \frac{K}{\mathrm{SNR}} I \right)^{-1} \tag{2}$$

where $\Gamma = [\mathbf{v}^{(1)}, \cdots, \mathbf{v}^{(k)}]^\dagger$. Hence, $\mathbf{f}_k = \boldsymbol{\omega}_k / |\boldsymbol{\omega}_k|$, where $\boldsymbol{\omega}_k$ is the kth column of Ω. ZF beamforming, even though suboptimal, offers a good balance between performance and complexity, and is reasonably close to capacity (achieved by dirty paper coding) when combined with user selection [8].

In the case of channel feedback, Γ in (2) is replaced by the estimated matrix from this feedback, $\hat{\Gamma} = [\hat{\mathbf{v}}^{(1)}, \cdots, \hat{\mathbf{v}}^{(k)}]^\dagger$. We assume equal power allocation is used at the base station for all active users and hence each user only feeds back its strongest right eigenvector.

At the receiver, each user exploits a minimum mean-square error (MMSE) receiver to estimate its data as was done in [6], which further mitigates the residual interference from the other users. To concentrate on the effect of feedback schemes, we assume that the receivers have perfect CSIR of the interference terms which could be obtained using a dedicated pilot training period in the downlink. At the kth user, the MMSE receiver is thus given by

$$\mathbf{g}_k = \left(I + \sum_{i \neq k} \frac{P}{KN_0} \mathbf{H}_k \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}_k^\dagger \right)^{-1} \mathbf{H}_k \mathbf{f}_k$$

which results in the kth user SINR of

$$\mathrm{SINR}_k = \frac{P}{KN_0} \mathbf{f}_k^\dagger \mathbf{H}_k^\dagger \left(I + \sum_{i \neq k} \frac{P}{KN_0} \mathbf{H}_k \mathbf{f}_i \mathbf{f}_i^\dagger \mathbf{H}_k^\dagger \right)^{-1} \mathbf{H}_k \mathbf{f}_k$$

Finally, the ergodic system throughput is given by $R = E_\mathbf{H}[\sum_{k=1}^{K} \log_2(1 + \mathrm{SINR}_k)]$.

We implement the transmission scheme both with and without user selection where the user selection is assumed to be done with perfect knowledge (genie-aided) of the downlink channels and an exhaustive search. Thus, it gives an upper bound on the gain of user selection algorithms.

We compare the different feedback coding algorithms in the above multiuser setup and in terms of the overall system throughput. Each user transmits its N_t dimensional CSI over $\beta_{fb} N_t$ uses of the uplink channel, where β_{fb} is the bandwidth expansion factor. In this work, we consider both an AWGN and a more realistic slow faded uplink channel generated according to the ITU Pedestrian B channel in [9]. We assume an uplink SNR of SNR_{UL} with an SNR offset between downlink and uplink of $\Delta\mathrm{SNR}$ dB$=$ SNR dB$-\mathrm{SNR}_{UL}$ dB.

III. Separation Based Approaches

Separation based or digital approaches for channel feedback have received the greatest traction so far in the literature [5] [10] [8]. It was shown in [8] that using a digital approach and any fixed number of feedback bits per user and without user selection, the overall system throughput is bounded which results in substantial throughput loss. Fig. 3(a) shows the perfect CSIT and the 4 bit digital ZF throughputs for a 1×4 channel with an AWGN uplink and no selection (details in Section V). We have also shown upper-bounds on the performance of 8 and 16 bits digital approaches that were obtained by introducing noise on $\mathbf{v}^{(k)}$s according to the quantization error distribution derived in [8] and assuming error-free transmission of the quantization bits (hence loose at low SNR). It can easily be seen that system throughput saturates with increasing SNR. This saturation is the result of the system becoming interference limited [5].

It was shown in [5] that to achieve the full multiplexing gain in a digital approach, number of feedback bits per user should be increased at least with a factor of $(N_t - 1)\log_2(\mathrm{SNR})$. Indeed, for 1 antenna users, using a random vector quantizer and uncoded quadrature amplitude modulation (QAM) achieves a performance close to perfect CSIT if and only if the number of feedback bits is increased this way. [5]

It is important to note however, that in terms of practical implementation, this scheme has the following difficulties making it of limited use in wireless systems. First, the quantization codebook has to be decided based on SNR. In many applications this SNR is known only to lie in some range and the mobile station does not have its accurate estimate. Since the performance of the system strongly depends on the correct choice of the number of feedback bits, this incurs a significant loss in practical systems and leads to a conservative design.

Moreover, the number of bits required for this digital scheme is very large for medium and high SNR. For example, for a 4 antenna base station and 1 antenna users at an SNR of 20dB, at least 20 quantization bits are needed per user [5]. This requires the design of multiple (for different SNRs) good quantization codebooks of very large cardinality which is a difficult problem. Having a large codebook also incurs a large computational complexity on the users for codeword selection and thus codebooks with large cardinality would need to be highly structured. These limitations of separation-based approaches in the context of MIMO channel feedback,

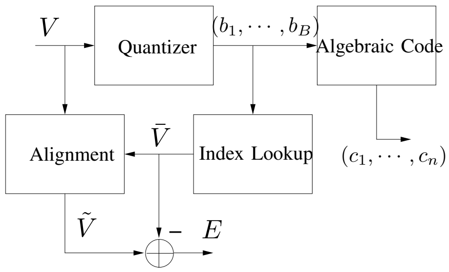


Fig. 1. Hybrid Feedback Algorithm.

motivate the study and design of feedback coding algorithms which do not adhere to the separation principle.

IV. Non-Separation Based Approaches

Here we look at two non-separation based schemes for channel feedback. We first look at an analog transmission scheme and then develop a special hybrid digital-analog feedback algorithm motivated by the source-channel codes in [2]. We will show that both schemes are preferable to a digital scheme in a practical setting and moreover the hybrid algorithm has an advantage over a pure analog approach and could be used to close the gap further with respect to the perfect CSIT case.

A. Analog Feedback Algorithm

In analog transmission, the elements of the CSI are sent over the channel using simple unquantized QAM. For the bandwidth expansion case, i.e., $\beta_{fb} > 1$, the same analog signal is repeated over the channel. At the receiver an MMSE estimator obtains the best estimate of the CSI symbols.

B. Hybrid Feedback Algorithm

Here, we develop the hybrid feedback algorithm. This algorithm is based on first quantizing the CSI using a few bits, sending those using a short (e.g., algebraic) code and then sending the quantization error using analog transmission. Here we explain the algorithm based on feeding back the right singular vectors of each user channel, $V_k = [\mathbf{v}_1^{(k)} \cdots \mathbf{v}_r^{(k)}]$, where r is the transmission rank. Note that this algorithm is applicable to any other channel feedback quantity conditioned on the existence of good vector quantizers for it. The hybrid scheme is shown in Fig. 1 and proceeds as follows:

(1) Decide on the transmission rank, r_{opt}, based on the criterion of interest, e.g., based on the MU transmission algorithm (rank 1 in our MU transmission scheme).

(2) For each user, quantize $V_k = [\mathbf{v}_1^{(k)} \cdots \mathbf{v}_{r_{\text{opt}}}^{(k)}]$ using a B bit codebook, $\mathcal{Q} = \{Q_1, \cdots, Q_{2^B}\}$, with the mapping criterion of interest such as maximum correlation criterion [10], i.e., $\bar{V}_k = \arg\max_i ||Q_i^\dagger V_k||_F$. Denote the resulting quantization bits by $(b_1^{(k)}, \cdots, b_B^{(k)})$.

(3) Align V_k to $\bar{V}_k$ by performing a unitary transformation from right on V_k. Since the transmission scheme (and capacity of the channel) is invariant to a unitary transformation from right on V_k for any user (for example for the one dimensional rank 1 transmission this is just an equal phase rotation on every element of $\mathbf{v}^{(k)}$ which does not affect its direction), the goal is to find the unitary transformation matrix $\Phi_{\text{opt}}^{(k)}$ that minimizes the variance of the error term, i.e., solving for each user

$$\Phi_{\text{opt}}^{(k)} = \arg\min_{\Phi \in \mathcal{U}} ||\bar{V}_k - V_k \Phi||_F^2 \tag{3}$$

where $\mathcal{U}$ is the set of all unitary matrices. We can prove that this transformation is given by $\Phi_{\text{opt}}^{(k)} = V_{\text{corr}} U_{\text{corr}}^\dagger$ where V_{corr} and U_{corr} are the right and left singular vectors of the correlation matrix $\bar{V}_k^\dagger V_k$, respectively, i.e., $\bar{V}_k^\dagger V_k = U_{\text{corr}} \Sigma V_{\text{corr}}^\dagger$. We denote the aligned V_k by $\tilde{V}_k = V_k \Phi_{\text{opt}}^{(k)}$. For example for our rank 1 transmission, $\Phi_{\text{opt}}^{(k)}$ is just a phase rotation on every element by $-\angle(\bar{\mathbf{v}}^{(k)\dagger}\mathbf{v}^{(k)})$.

(4) Construct the correction or error signal, $E_k = \tilde{V}_k - \bar{V}_k$.

(5) Possibly use a short (n, B, d) algebraic code such as a Reed Muller code to encode the B quantization bits for each user into a codeword $(c_1, \cdots, c_n)$.

(6) Send E_k and $(c_1, \cdots, c_n)$ for each user over the uplink channel. This should be done by splitting the resources in the channel, i.e., power and bandwidth, between the analog error signal and the digital code depending on the application. We will show an example of such splitting and the corresponding receiver, using the uplink control channel in the IEEE802.16e standard [11].

At the base station, digital data is decoded using a maximum likelihood (ML) decoder. This will not incur much complexity since the number of quantization bits is small. The analog error signal is reconstructed using an MMSE estimator. Denoting the estimated E_k by $\hat{E}_k$ and assuming correct decoding of the digital quantized data, $\hat{V}_k$ for user k is reconstructed as $\hat{V}_k = \bar{V}_k + \hat{E}_k$.

V. Simulations

Here we implement the digital, analog, and hybrid feedback algorithms in two scenarios: First, in a simple case where the uplink channel is AWGN and the users each have 1 antenna. Then in the realistic case where the uplink channel is faded and modeled using the ITU Pedestrian B channel in [9] and each user has 2 receive antennas and transmits with 1 antenna in the uplink (as is the case in most wireless devices).

A. Simulation Setup

We use the uplink control channel in the IEEE 802.16 standard [11] for our implementation. This channel consists of basic units, called tiles, that each consists of 12 uses of the uplink channel, 4 for pilots and 8 for data as shown in Fig. 2.

For the case of MU-MIMO implementation, rank 1 transmission is used and hence each user's feedback consists of 4 symbols of $\mathbf{v}^{(k)}$. Hence each tile use of the control channel corresponds to a $\beta_{fb} = 2$. In the hybrid scheme, the digital data is constructed using 4 bit codebook quantizers. To encode the 4 bits, we use the (8,4,4) extended Hamming code. These 8 coded bits are then placed on 4 (complex) channel uses within a tile and the other 4 channel uses are used for analog transmission of the error signal as shown in Fig. 2.

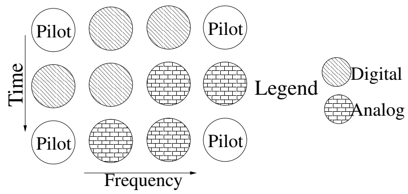


Fig. 2. Tile Assignment.

We satisfy the uplink power constraint of P_{UL} per channel use by scaling each tile with a factor α which depends on the analog error power in that tile. However since the analog error variance (average power) is much smaller than the constant digital power of 1 per symbol (QPSK constellation), we first boost the analog error signal by a *known* factor γ which we optimize over. Hence, the power equation for a tile becomes $P_{UL} = (\alpha^2/8)(P_{\text{digital}} + \gamma^2 ||E||_F^2)$, where $P_{\text{digital}} = 4$ (4 QPSK symbols) and from which we can obtain α for any tile. At the receiver we estimate α (required to retrieve the magnitude of analog error) by finding the ratio of pilot to digital average power since pilots all have power 1.

The encoding in the pure 4 bits digital scheme is the same as the digital part of the hybrid scheme discussed above. For both the pure analog and pure digital schemes, each symbol of the feedback is just repeated twice within a tile and the right power scaling is used. Since here only the direction matters, there is no need to estimate this factor at the base station.

B. Simulation Results

Fig. 3 and 4 compare the overall throughputs of the analog and hybrid schemes in the 2 simulation scenarios for $\Delta\text{SNR} = 6\text{dB}$ and 15dB, with or without user selection for $\beta_{fb} = 2$. In these figures we also show an upperbound on the performance of the system, assuming error free transmission at *capacity* on the uplink channel. To get this bound we introduced noise on the $\mathbf{v}^{(k)}$s according to the quantization error distribution derived in [8] with the number of quantization bits equal to the total uplink capacity of a tile ($\beta_{fb} = 2$). This bound shows the best one can do with *any* feedback algorithm and long block lengths. For reference, we also show the performance of a 4 bit digital scheme and upperbounds on the performance of fixed digital approaches with 8 and 16bits.

Fig. 3 compares the different algorithms for the first scenario without user selection. At $\Delta\text{SNR} = 6\text{dB}$, both analog and hybrid transmissions perform very close to perfect CSIT. Note that hybrid transmission performs better than analog and this improvement becomes more apparent at larger ΔSNR. Also the performance of the hybrid algorithm is very close to the upperbound for up to about 15dB even for $\Delta\text{SNR} = 15\text{dB}$. Fig. 4 similarly shows the throughputs for the second simulation scenario with and without user selection. We again observe that both analog and hybrid perform very close to perfect CSIT and hybrid performs better than analog as ΔSNR grows. Intuitively, the improvement of hybrid over analog repetition results from the fact that the analog error has a much lower variance (about 5dB lower) than the signal itself. Hence sending the error as opposed to repeating the signal over the same channel results in a lower MSE in signal reconstruction.

We can also see that the digital approach with any fixed number of bits and no user selection saturates in both simulation scenarios as expected [8], and with user selection, even though it does not saturate, it still has a substantial loss as compared to perfect CSIT. Note that (c.f. Section III) in order to avoid saturation and have a good performance, the number of bits should grow as $(N_t - 1)\log(\text{SNR})$. As discussed before, SNR is unknown and users may not have a good estimate of it. Also, in practice a limited number of codebooks (with different sizes) would be used and thus a single codebook would need to be used for a range of SNRs, limiting the granularity of the number of operational SNR points. Both the SNR estimation error and limited SNR granularity, will degrade the performance of such a digital approach. We conjecture that in practice, the loss from these factors will be more pronounced than that associated with even using a simple analog scheme. Moreover, by using the hybrid scheme we can further reduce the gap in performance from the perfect CSIT.

In terms of complexity, the large codebooks needed for a digital scheme will incur a much greater computational complexity in the system as compared to a simple analog or hybrid scheme. Note that in the hybrid scheme a fixed small (4 bits) codebook is used and the additional accuracy is obtained by sending a correction analog signal and thus the hybrid approach does not suffer from the disadvantages of a digital approach with variable codebook size.

VI. Conclusions

We examined the relative merits of the separation-based and non-separation based transmission schemes in the context of MU-MIMO channel feedback and showed that non-separation based schemes are preferable for this application. Among non-separation schemes, we developed a hybrid digital-analog feedback scheme that achieves higher throughputs compared to pure analog or digital schemes and also has a much smaller computational complexity compared to a digital approach.

References

[1] A. Lapidoth, S. Shamai(Shitz), and M. A. Wigger, "On the capacity of fading MIMO broadcast channels with imperfect transmitter side-information," *IEEE Trans. Inform. Theory*, May 2006, submitted. Available at http://arxiv.org/abs/cs.IT/0605079.

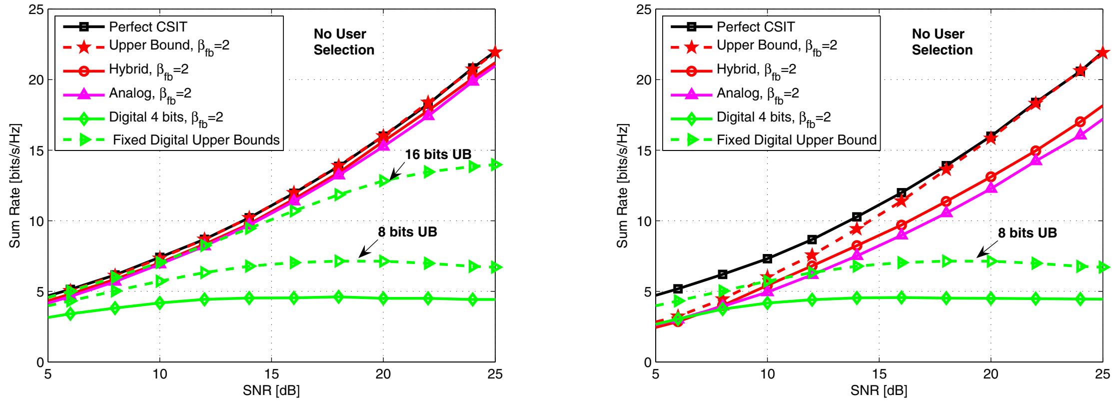
[2] U. Mittal and N. Phamdo, "Hybrid digital-analog (HDA) joint sourcechannel codes for broadcasting and robust communications," *IEEE Trans. Inform. Theory*, vol. 48, no. 5, pp. 1082–1102, May 2002.

[3] N. Phamdo, N. Farvardin, and T. Moriya, "A unified approach to tree-structured and multistage vector quantization for noisy channels," *IEEE Trans. Inform. Theory*, vol. 39, no. 2, pp. 835–850, May 1993.

[4] M. Wang and T. R. Fischer, "Trellis-coded quantization designed for noisy channels," *IEEE Trans. Inform. Theory*, vol. 40, no. 6, p. 17921802, Nov. 1994.

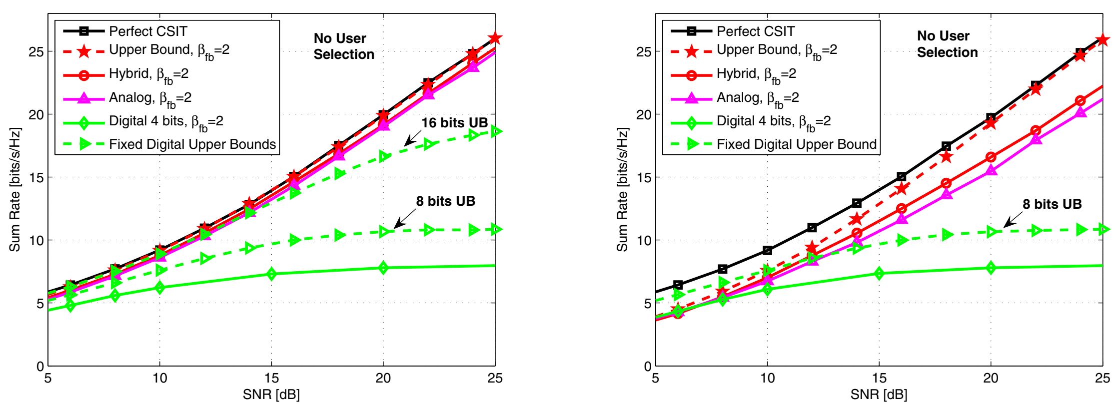
[5] G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, "Multiuser MIMO downlink made practical: Achievable rates with simple channel state estimation and feedback schemes," *IEEE Trans. Inform. Theory*, Nov. 2007, submitted for publication. See http://arxiv.org/abs/0711.2642.

[6] F. Boccardi, H. Huang, and M. Trivellato, "Multiuser eigenmode transmission for MIMO broadcast channels with limited feedback," in *SPAWC Workshop*, 2007.

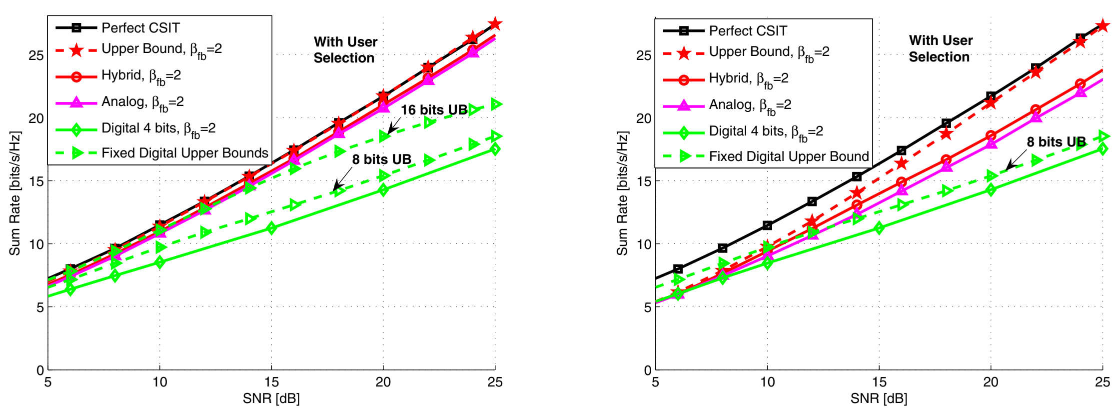


(a) (b)

Fig. 3. 1×4 channel without user selection and with (a) $\Delta\text{SNR} = 6\text{dB}$ (b) $\Delta\text{SNR} = 15\text{dB}$.



(a) (b)



(c) (d)

Fig. 4. 2×4 channel with (a) no user selection and $\Delta\text{SNR} = 6\text{dB}$ (b) no user selection and $\Delta\text{SNR} = 15\text{dB}$ (c) user selection and $\Delta\text{SNR} = 6\text{dB}$ (d) user selection and $\Delta\text{SNR} = 15\text{dB}$.

[7] C. B. Peel, B. M. Hochwald, and A. L. Swindlehurst, "A vector-perturbation technique for near-capacity multiantenna multiuser communication Part I: Channel inversion and regularization," *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 195–202, Jan. 2005.

[8] N. Jindal, "MIMO broadcast channels with finite rate feedback," *IEEE Trans. Inform. Theory*, vol. 52, no. 11, pp. 5045–5059, Nov. 2006.

[9] "Recommendation ITU-R M.1225. available at www.itu.int/itu-r/study-groups/docs."

[10] D. Love and J. Robert W. Heath, "Limited feedback unitary precoding for spatial multiplexing systems," *IEEE Trans. Inform. Theory*, vol. 51, no. 8, pp. 2967–2976, Aug. 2005.

[11] "IEEE P802.16Rev2/D6 July 2008."