

# Operational Information Content Sum Capacity: Formulation and Examples

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**Abstract**—This paper considers Quality-of-Information (QoI) aware rate allocation policies for multiple access channels. QoI is a recently introduced composite metric which is impacted by a number of attributes including accuracy and timeliness of delivery of information communicated from the source(s) to the destination(s), and as such differs from traditional quality-of-service metrics considered to date. The focus of this work is defining the Operational Information Content Sum Capacity (OICC-S) of a network, achieved by the set of QoI-vectors supported which maximize sum utility of the system. This utility is defined as a function of the QoI attributes provided by the source input, as well as the channel induced attributes that impact the QoI delivered to the destination(s). Optimum rate allocation to maximize the output sum utility and achieve OICC-S of the network for various settings is provided, and demonstrated to differ from the solution that provides maximum throughput.

**Keywords:** Quality of Information, Rate Allocation, Network Utility Maximization, Scheduling

## I. INTRODUCTION

Traditional approaches based on Quality of Service (QoS) perform network operations that are agnostic to the application or content of data. This may not lead to best design strategies for tactical networks, where the main goal is sound decision making. To this end, a new paradigm which emphasizes the quality of information by viewing the network as an information source, and developing methods to satisfy quality requirements at the end user is necessary. To characterize information quality, there is growing interest in moving from traditional QoS metrics as throughput, packet delivery ratio, fairness, and delay towards new notions of quality associated with information.

The notion of *Quality-of-Information* (QoI) [1] [2] has been introduced to formally describe this new class of attributes, including provenance [2], accuracy and precision [1] [2], reliability [1], corroboration [2] [3], age/freshness and timeliness [1] [2]. These attributes specify how and by whom information was gathered, under what conditions, how and by whom it was processed. Given the recent interest in defining QoI metrics, it is a natural direction to explore the impact of this new paradigm on fundamental networking operations. Efforts to date in this direction are specific to event detection, see for example [4]. Detection-oriented assessment of QoI has also lead to its joint consideration [1] [4] [5] with data

fusion. The question of how to make optimal control decisions that optimize performance with respect to these new metrics has been addressed in [6], where scheduling mechanisms are proposed for different mission arrival scenarios on a single link, trading off gain in accuracy with reduction in freshness.

We consider the following scenario. A tactical network is sent missions sequentially from an end user and other users with sensing capabilities respond to the mission. We are interested in the set of QoI-vector pairs a network can support, and identifying which of these QoI-vector pairs are most useful in terms of decision making through a utility function. We denote the maximum sum utility achieved by these QoI-vector pairs supported by the network as the *Operational Information Content Sum Capacity (OICC-S)* of the network. Proposed recently, the notion of *Operational Information Content Capacity (OICC)* is an indicator of the decision making capability that the collection of sources and links, i.e., the network can provide [2]. This is for instance unlike the Network Utility Maximization (NUM) framework where the utility is a function of the flow rates [7]. Although QoI by itself is associated with information generated by a single source, OICC-S captures the interaction of multiple sources or flows and the physical layer they share. We leverage the rather large body of work on capacity of wireless networks in order to determine OICC-S. More specifically, we address the problem of sum utility maximization via optimal rate allocation for cases where some of the QoI attributes are given.

Among the attributes which can effect QoI and OICC-S, we particularly focus on the effects of *accuracy* and *timeliness*. Accuracy, generally describing the specificity relative to need of the operation, is an indicator of the initial information content and the success of generating information at the sources. On the other hand, timeliness, which measures the availability of information relative to the time it is needed, is related with success of network delivery. These two attributes possess a trade-off such that improving accuracy degrades timeliness for a given network. In Section IV, we develop a model for QoI utility that depends on these two metrics.

In this paper, we consider the multiple access channel (MAC), with the objective of maximizing the sum utility of the system, i.e., achieving the OICC-S. The main issues we address are obtaining the optimum balance between accuracy

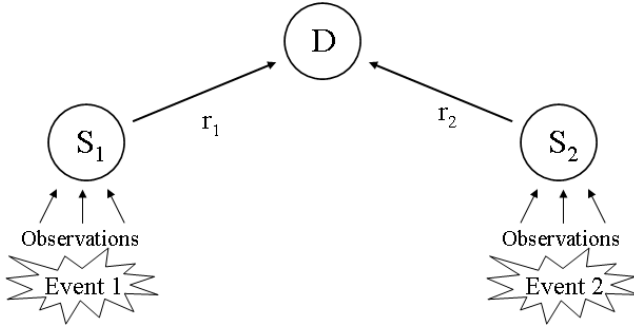


Figure 1. Two-user MAC channel for QoI-based network.

and timeliness for the given network, by selecting the *rate point* to allocate. It is well known that max weight scheduling [8] maximizes throughput for this model by operating at one of two corner points for the MAC capacity region [9]. In contrast, here, we demonstrate that arbitrary points on the dominant face of the rate region can be optimal rate points to attain OICC-S.

The organization of the paper is as follows. In Sections II, we present the basic model and QoI definitions. Next, in Section III we formally define the OICC-S. We provide the formulation and give example rate allocation and QoI attribute optimization problems to achieve the OICC-S for different settings in Section IV. We provide numerical results in Section V, and conclude the paper in Section VI.

## II. SYSTEM MODEL

For clarity of exposition, we shall concentrate on two transmitter MAC (Fig. 1). The results can be readily generalized to more than two users. This constitutes a basic and inspiring model for OICC-S characterization, which involve multiuser issues as proper rate allocation between users and QoI-vectors accordingly.

We consider a scenario where missions are issued from an end user in a tactical network. Missions arrive with a random interarrival time greater than  $T_{min}$ . We assume that at most one mission is processed by the network at any time. Information sources  $S_1$  and  $S_2$  respond to the mission and focus on independent events and possibly possess or generate different types of information related with the mission. This can correspond to separate phenomena related with the environment.

We characterize the overall importance of the information to the mission as the QoI of the piece of information. We note that QoI does *not* say anything about information content: for example, it can have a high-quality image of a blank wall, or a high-quality audio clip of silence. We define two types of QoI:

- *delivered-QoI* is the QoI associated with a piece of information generated and delivered by the network.
- *desired-QoI* is the QoI requested of the network.

Both types of QoI can be represented by a *QoI-vector*, which is a vector of attribute-value pairs: for example, [ $type = image, timeliness = 10s, accuracy = 800 \times 600, FOV = 100 \text{ mm per meter} \dots$ ], where FOV is the *field of view* is the

is the (angular or linear or areal) extent of the observable world that is seen at any given moment. Here, the linear FOV is given with specified in a ratio of lengths. The first term of accuracy attribute specifies the resolution [2]. A subtle distinction between the two is that a *desired-QoI-vector* may consist of a vector of logical expressions, e.g.: [ $type = image, timeliness \leq 10s, accuracy \leq 1024 \times 768, FOV = 100 \text{ mm per meter}, \dots$ ].

Some attributes of a QoI-vector may be upper bounded due to source capabilities as processing and reception quality. A *QoI-Flow* refers to the transfer of (one piece of) information from a specific source to a specific destination. As a result of network delivery, a delay will be introduced until the information is utilized at the destination. We are interested in the effect of delay due to network delivery on the *timeliness* of the information at the destination. The effects of delivery, more specifically the delays introduced can be characterized by the amount of bits corresponding to the information and the rates of transmitting from sources to the destination.

Sources perform rate allocation and the information contents are delivered to the destination. Once the decision of transmission is made by the sources, the information available is fed into the wireless channel to the destination with a certain rate. The *boundary* of the achievable rate region defines a set of rate pairs  $(r_1, r_2)$ , which can be assigned to QoI-flows  $f_1$  and  $f_2$  respectively, such that any increase to  $r_1$  or  $r_2$  will result in instability. For our two-user model, transmission rates can be upper bounded by the capacity region of a Gaussian multiple access channel given by [10]:

$$r_i \leq W \log_2 \left( 1 + \frac{h_i P}{N_0 W} \right) = c_i, \quad i = 1, 2 \quad (1)$$

$$r_1 + r_2 \leq W \log_2 \left( 1 + \frac{(h_1 + h_2) P}{N_0 W} \right) = c_s, \quad (2)$$

where  $r_i$  is the rate from  $S_i$  to destination,  $\sqrt{h_i}$  denotes the channel gain from  $S_i$  to the destination node,  $P$  is the power constraint for all nodes, the  $\frac{N_0}{2}$  is the noise spectral density and  $2W$  is the two-sided bandwidth. We assume that channel gains are static throughout a specific mission. We also assume that the time scales of interest due to timeliness requirements are large enough, along with a large operational bandwidth, allowing usage of possibly multiple codewords with sufficiently large block lengths to approach the bounds in (1)-(2) during delivery of information from the sources. Essentially, we the available rate options are within a convex pentagonal region (Fig. 2), where two of the corner points correspond to different decoding orders at the destination. The significance of this rate region is that source rates are coupled via the third common constraint in (2). We emphasize that (1)-(2) constitute upper bounds for any practical protocol, as well as transmission schemes with any physical layer coding and modulation scheme.

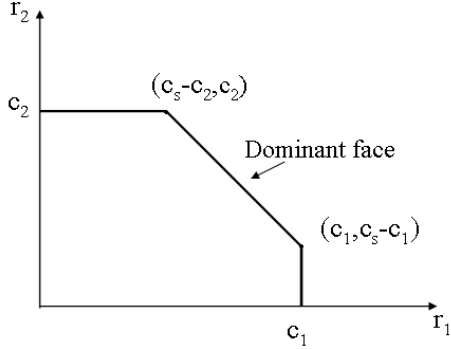


Figure 2. Capacity region for Two-user MAC channel.

### III. OPERATIONAL INFORMATION CONTENT SUM CAPACITY (OICC-S)

In this definition of OICC-S, there are three main steps, illustrated with 2-flow cases. The descriptions will generalize to  $n$  flows or  $m$ -element QoI-vectors.

We assume the existence of two functions:

- one which maps each possible *QoI-vector* to a rate in bits/sec. For example, [*type = image, timeliness* > 10s, *accuracy* > 1024 × 768, *FOV* = 100mm per meter, ...] may require at least 200 Kbps. We call this function the *QoI-rate function*.
- one which maps each possible *QoI-vector* to a scalar quantity that we will call *utility*. For example, [*type = image, timeliness* = 10s, *accuracy* = 1024 × 768, *FOV* = 100 mm per meter, ...] may have a utility of 10 (on some application-specific scale), but [*type = image, timeliness* = 5s, *accuracy* = 1024 × 768, *FOV* = 100 mm per meter, ...] may have a utility of 20. We call this the *QoI-utility function*.

We further specify these two functions in subsequent sections.

Then the OICC-S of a given network can be derived as follows (for two QoI-flows  $f_1$  and  $f_2$ ):

For each pair  $(r_1, r_2)$  on the boundary of the achievable rate region

- let  $(\mathbf{q}_{f_1}^1, \mathbf{q}_{f_1}^2, \dots)$  (respectively for flow  $f_2$ ) be the set of all QoI-vectors whose rates (obtained from the QoI-rate function) are less than or equal to  $r_1$  (respectively  $r_2$ ).
- let  $\mathbf{q}_{f_1}^j$  (respectively  $\mathbf{q}_{f_2}^k$ ) be the QoI-vector whose utility is the highest in the set  $(\mathbf{q}_{f_1}^1, \mathbf{q}_{f_1}^2, \dots)$  (respectively for  $f_2$ ).

Then, the set of all pairs  $(\mathbf{q}_{f_1}^j, \mathbf{q}_{f_2}^k)$  are candidate QoI-vector pairs for the rate pair  $(r_1, r_2)$  to attain the OICC-S of the network. Note that this set of all pairs  $(\mathbf{q}_{f_1}^j, \mathbf{q}_{f_2}^k)$  can also be equivalently identified as the set of QoI-vectors whose sum utility is the highest among any of the feasible  $(\mathbf{q}_{f_1}, \mathbf{q}_{f_2})$  pairs defined above.

Ultimately, the OICC-S of the network is defined as the *maximum of the sum utilities attained among all rate allocation options*.

### IV. OICC-S FORMULATION

We next provide an OICC-S formulation for a given network. We first note that the definition of OICC-S entitles

the QoI-utility and QoI-rate functions. While more specific relations could be specified for these two functions for applications as face recognition, speech recognition, in this work we provide a general formulation that can be applied to various applications. Next, we propose a candidate utility function which reflects the trade-off between accuracy and timeliness. More specifically, consider the following utility function of the form for QoI-vector  $\mathbf{q}$ :

$$u(\mathbf{q}) = ag(t_d), \quad (3)$$

where  $t_d$  is the timeliness, i.e. delivery time of  $\mathbf{q}$ , and  $a$  is a scalar capturing the overall instantaneous *accuracy metric* of the resolution and FOV of the information specified by  $\mathbf{q}$ .  $l(a)$  is a function corresponding the amount of bits required to represent information of accuracy metric  $a$ . While the exact relationship depends on the type of the information under consideration, a natural assumption is that  $l(a)$  is a non-decreasing function of  $a$  for a specific type of information. Notice that  $t_d \geq \frac{l(a)}{r}$ , where  $r$  is the rate. This equivalently relates to the QoI-rate function through  $r \geq \frac{l(a)}{t_d}$  as a rate requirement to support a QoI-vector  $\mathbf{q}$  with the given attributes.  $g(t_d)$  is a degradation function reflecting the reduction in utility due to latency. We can also express (3) in terms of  $a$  and  $r$  as follows:

$$u_r(a, r) = ag\left(\frac{l(a)}{r}\right). \quad (4)$$

A function to reflect the traditional notion of timeliness could have the form that the output utility is preserved when delivered within the timeliness requirements, and reduces after some critical deadline [11]. Note that this differs from strict delay constraints which would reduce utility to zero. Piecewise linear functions can be defined for that goal. However, we rather focus on smooth functions which are twice differentiable and concave within the domain of interest in order to pursue more systematic solution methods. As a utility function approximating the desired property, let us consider:

$$g(t_d) = k(\gamma, D)(1 - e^{\gamma(t_d - D)}), \quad (5)$$

for  $t_d \leq D$ . Example utility degradation curves depicting the effect of timeliness for some different parameters are illustrated in Fig. 3. Note that the general behavior of the utility function is that it initially stays relatively unchanged for low delivery time and decays to zero as the delivery time approaches  $D$ .  $D \leq T_{min}$  can be thought as a maximum tolerable delay deadline in which the information is regarded useless afterwards, and the exact behavior of the utility curve can be adjusted by varying  $\gamma$ .  $k(\gamma, D) = \frac{1}{1 - e^{-\gamma D}}$  is a normalization parameter.

#### A. General Problem Statement

Given a network, it is essential to optimally allocate its resources in order to achieve the OICC-S. To that end, in this subsection we first express the general formulation leading to the OICC-S of a network. First, the OICC-S of a network is the maximum sum utility attained over all rate allocation

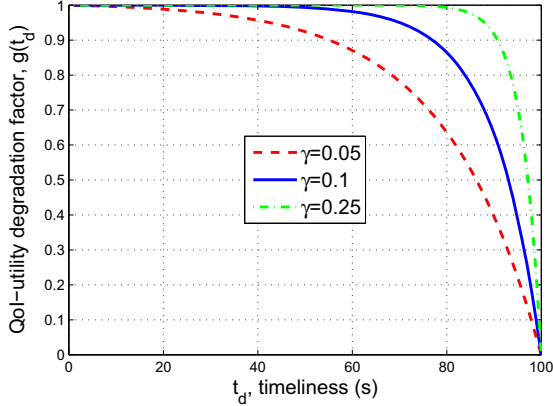


Figure 3. Utility degradation as a function of delivery time,  $D = 100s$ .

options and QoI-vector attributes:

$$\max_{r_1, r_2, a_1, a_2} a_1 k_1 (1 - e^{\gamma_1 (\frac{l_1(a_1)}{r_1} - D_1)}) + a_2 k_2 (1 - e^{\gamma_2 (\frac{l_2(a_2)}{r_2} - D_2)}) \quad (6)$$

$$\text{s.t.} \quad r_i \leq c_i, i = 1, 2 \quad (7)$$

$$r_1 + r_2 \leq c_s, \quad (8)$$

where  $r_i$  are the rates allocated to source  $i$ ,  $a_i$  are the accuracy metrics related with QoI-vector  $\mathbf{q}_{f_i}$ , for  $i = 1, 2$ . Timeliness parameters  $D_i$ ,  $\gamma_i$ , and constants  $k_i$   $i = 1, 2$  are specific to the application. We first note that the objective function is not jointly concave in all decision variables ( $r_1, r_2, a_1, a_2$ ), so standard Karush-Kuhn-Tucker(KKT)-based optimization [12] is not readily applicable. Accordingly, we rely on iterative optimization methods based on alternating maximization [13] discussed in Section IV-D. Before presenting the general solution, we next consider a rate allocation problem which attains the OICC-S for special restrictions on the QoI-vector pairs. This problem is followed by the alternative problem of maximizing output sum utility by QoI attribute adaptation for a given rate pair in Section IV-C. The solutions of these two problems will also constitute building blocks for the solution to achieve the OICC-S of a network for the generalized case.

### B. OICC-S Based Rate Allocation

Consider the special case the following optimization problem defined where accuracy metrics  $a_i$ ,  $l_i(a_i)$ ,  $i = 1, 2$ , of the information from each source are given:

$$\max_{r_1, r_2} a_1 k_1 (1 - e^{\gamma_1 (\frac{l_1(a_1)}{r_1} - D_1)}) + a_2 k_2 (1 - e^{\gamma_2 (\frac{l_2(a_2)}{r_2} - D_2)}) \quad (9)$$

$$\text{s.t.} \quad r_i \leq c_i, i = 1, 2 \quad (10)$$

$$r_1 + r_2 \leq c_s, \quad (11)$$

where timeliness parameters  $D_i$ ,  $\gamma_i$ , and constants  $k_i$   $i = 1, 2$  are specific to the application. Hence, we are interested in the optimal rate allocation to maximize sum utility, which will in-turn define the timeliness attributes of  $\mathbf{q}_{f_1}$  and  $\mathbf{q}_{f_2}$ . We first note the separability of the sum utility  $r_1$  and  $r_2$ . In order to assess the applicability of standard optimization methods, we

check for concavity:

$$\frac{\partial u_r(a, r)}{\partial r} = ka \frac{\gamma l(a)}{r^2} e^{\gamma (\frac{l(a)}{r} - D)}, \quad (12)$$

Next,

$$\frac{\partial^2 u_r(a, r)}{\partial r^2} = kal(a)\gamma \left[ \frac{-2}{r^3} + \frac{-\gamma l(a)}{r^4} \right] e^{\gamma (\frac{l(a)}{r} - D)} \quad (13)$$

$$< 0. \quad (14)$$

Hence the utility function is concave in rate  $r$ . We also note that the feasible region for  $r$  (MAC rate region) is a convex set.

*Theorem 1:* Given accuracy metrics ( $a_1, a_2$ ) of QoI-flows, the optimal rate allocation ( $r_1^*, r_2^*$ ) is given by one of:

- 1)  $(r_1, r_2) = (c_1, c_s - c_1)$
- 2)  $(r_1, r_2) = (c_s - c_2, c_2)$
- 3)  $(r_1, r_2)$  on dominant face ( $r_1 + r_2 = c_s$ ) with:

$$\frac{r_1^2}{r_2^2} = \frac{k_1 \gamma_1 a_1 l_1(a_1) e^{\gamma_1 (\frac{l_1(a_1)}{r_1} - D_1)}}{k_2 \gamma_2 a_2 l_2(a_2) e^{\gamma_2 (\frac{l_2(a_2)}{r_2} - D_2)}}, \quad (15)$$

and the exact operating point solution can be determined by evaluating the total output QoI utilities. Moreover, timeliness attributes attaining the OICC-S are given by  $t_{di}^* = \frac{l_i(a_i)}{r_i^*}$ , for  $i = 1, 2$ .

*Proof:* Let us introduce Lagrange multipliers  $\lambda_1, \lambda_2, \lambda_3$ , all greater than or equal to 0, for constraints (10)-(11). The Lagrangian can be expressed as:

$$L(r_1, r_2, \lambda_1, \lambda_2, \lambda_3) = - \sum_{i=1}^2 a_i k_i (1 - e^{\gamma_i (\frac{l_i(a_i)}{r_i} - D_i)}) + \sum_{i=1}^2 \lambda_i (r_i - c_i) + \lambda_3 (r_1 + r_2 - c_s). \quad (16)$$

KKT conditions dictate:

$$-k_i a_i \frac{\gamma_i l_i(a_i)}{r_i^2} e^{\gamma_i (\frac{l_i(a_i)}{r_i} - D_i)} + \lambda_i + \lambda_3 = 0, \quad (17)$$

$$\lambda_i (r_i - c_i) = 0, i = 1, 2 \quad (18)$$

$$\lambda_3 (r_1 + r_2 - c_s) = 0. \quad (19)$$

Hence we have:

$$k_i a_i \frac{\gamma_i l_i(a_i)}{r_i^2} e^{\gamma_i (\frac{l_i(a_i)}{r_i} - D_i)} = \lambda_i + \lambda_3, i = 1, 2. \quad (20)$$

Note that these imply that  $\lambda_1 + \lambda_3 > 0$  and  $\lambda_2 + \lambda_3 > 0$ . First, assume  $\lambda_3 = 0$ . Then,  $r_1 + r_2 < c_s$  and  $\lambda_1 > 0, \lambda_2 > 0$  should be satisfied leading to  $r_1 = c_1, r_2 = c_2$  but this combination is not feasible ( $c_s < c_1 + c_2$ ). Hence it is required that  $\lambda_3 > 0$ , and accordingly we have  $r_1 + r_2 = c_s$ .

As for  $\lambda_1$  and  $\lambda_2$ , we have the option that only one of them is positive, which would correspond to one of the corner points of the rate region. The other option is that when  $\lambda_1 = \lambda_2 = 0$ , which implies that  $r_1 < c_1$  and  $r_2 < c_2$ . Along with  $r_1 + r_2 = c_s$ , this results in an operating point on the dominant face of the rate region (which is achieved by strict time-sharing between the two corner points corresponding to

different decoding order at the receiver). From (20), with  $\lambda_1 = \lambda_2 = 0$  we have

$$k_1 \gamma_1 a_1 \frac{l_1(a_1)}{r_1^2} e^{\gamma_1(\frac{l_1(a_1)}{r_1} - D_1)} = k_2 \gamma_2 a_2 \frac{l_2(a_2)}{r_2^2} e^{\gamma_2(\frac{l_2(a_2)}{r_2} - D_2)} \quad (21)$$

leading to equation (15). In other words, the operating point is the point on the dominant face satisfying (15). The specific point will depend on multiple parameters, including accuracy attributes and timeliness parameters. ■

*Remark 1:* Throughout the analysis we made the assumption that both QoI-flows were served timely, using utilities given by (5). We point out that the solution given by Theorem 1 is sufficient to cover cases where there exists QoI-flows which cannot be served in time. In the case where the rate region cannot support either of the flows regardless of the particular rate allocation, all candidate points will result in zero utility. As for the case where only one of the flows are supported, we note that no further improvement on sum utility can be attained by considering any additional rate pairs. This is due to the fact that the corner points of MAC rate region given by Theorem 1 already provide full prioritization and maximum possible rate for the supported flow. ■

### C. OICC-S Based Attribute Optimization

Next, we focus on the following problem: Given fixed rate pair  $(r_1, r_2)$  on the MAC rate region boundary, we characterize the set of QoI-vectors  $(\mathbf{q}_{f1}, \mathbf{q}_{f2})$  that attain the OICC-S.

Hence, we are interested in maximizing utility by optimizing over QoI attributes. Note that the incentive of possibly preferring QoI-vectors with low accuracy is that information with high accuracy may lead to excessive delay and utility reduction due to *untimely* delivery. More specifically, we consider the following problem:

$$\max_{a_1, a_2} a_1 k_1 (1 - e^{\gamma_1(\frac{l_1(a_1)}{r_1} - D_1)}) + a_2 k_2 (1 - e^{\gamma_2(\frac{l_2(a_2)}{r_2} - D_2)}), \quad (22)$$

where rates  $r_i$ ,  $i = 1, 2$  are already given, and timeliness parameters  $D_i$ ,  $\gamma_i$ , and constants  $k_i$  for  $i = 1, 2$  all depend on the specific application. Note that by tracing over all  $r_i$ ,  $i = 1, 2$  on the MAC rate region boundary we can characterize different  $(\mathbf{q}_{f1}, \mathbf{q}_{f2})$  pairs.

While we have expressed (22) as a maximization over accuracies  $(a_1, a_2)$ , we could have equivalently expressed it as a maximization over  $(t_{d1}, t_{d2})$ . Since  $r_i$ ,  $i = 1, 2$  is fixed, the attributes can be related through  $t_d = \frac{l(a)}{r}$ .

First, we check for concavity of the utility function. Since the output utility is separable in  $a_1$  and  $a_2$ , we can focus on individual utilities for concavity.

*Proposition 1:* Let  $f'(a)$  and  $f''(a)$  denote first- and second- order derivatives of function  $f(a)$  with respect to  $a$ . The utility function is concave in  $a$  if  $l(a)$  satisfies:

$$2l'(a) + al''(a) + \frac{\gamma}{r} a(l'(a))^2 \geq 0. \quad (23)$$

Moreover, a sufficient condition for concavity in  $a$  is  $l'(a) \geq 0$  and  $l''(a) \geq 0$ .

*Proof:*

$$\frac{\partial u_r(a, r)}{\partial a} = k(1 - (1 + \frac{\gamma al'(a)}{r})e^{\gamma(\frac{l(a)}{r} - D)}), \quad (24)$$

and

$$\frac{\partial^2 u_r(a, r)}{\partial a^2} = -k[\frac{\gamma}{r}(2l'(a) + al''(a) + \frac{\gamma}{r}a(l'(a))^2)]e^{\gamma(\frac{l(a)}{r} - D)}, \quad (25)$$

which is  $< 0$  and the utility function is also concave in accuracy  $a$  if (23) is satisfied. The sufficient condition stated is readily shown to satisfy this requirement. ■

Next, we state the following theorem:

*Theorem 2:* Given operating point  $(r_1, r_2)$ , the  $a_i^*$ ,  $i = 1, 2$  for QoI-vectors on the OICC-S are given by the equation:

$$a_i = \frac{r_i(e^{-\gamma_i(\frac{l_i(a_i)}{r_i} - D_i)} - 1)}{\gamma_i l'(a_i)}. \quad (26)$$

Moreover timeliness attributes on the OICC-S are given by  $t_{di}^* = \frac{l_i(a_i^*)}{r_i}$ , for  $i = 1, 2$ .

*Proof:* The optimal point is readily obtained by equating (24) to 0. ■

### D. Joint Rate Allocation and QoI Adaptation

In Section IV-A, we noted that the objective function in (6) is not jointly concave in the rates and accuracy metrics. On the other hand, in Section IV-B, we demonstrated that the objective function is concave in the rates given fixed accuracy metrics. Conditions on concavity in the accuracy metric were also presented in Section IV-C. Motivated by the availability of the solutions of these two subproblems, we rely on iterative optimization. Specifically, we use alternating maximization [13] in order to solve (6) and achieve the OICC-S in the most general setting where QoI-vector attributes can be adapted as well in addition to rate allocation.

The method can be described as follows:

- 1) Initialize  $(r_1^0, r_2^0)$ ,  $(a_1^0, a_2^0)$ .
- 2) At step  $k$ ,  $k > 0$ :
  - Given  $(a_1^{k-1}, a_2^{k-1})$ , maximize sum the utility by optimizing over  $(r_1, r_2)$  with solution  $(r_1^*, r_2^*)$ , set  $(r_1^k, r_2^k) = (r_1^*, r_2^*)$ .
  - Given  $(r_1^k, r_2^k)$ , maximize sum the utility by optimizing over  $(a_1, a_2)$  with solution  $(a_1^*, a_2^*)$ , set  $(a_1^k, a_2^k) = (a_1^*, a_2^*)$ .
- 3) Stop iteration when convergence criteria is specified.

Note that for each iteration, the rate allocation step was discussed in Section IV-B, and the QoI-vector attribute optimization was discussed in Section IV-C. Each iteration leads to an improved sum utility value, approaching to the OICC-S. The final ingredient required for convergence of these iterations is boundedness of the decision variables. Note that this is already readily imposed for the rates  $(r_1, r_2)$  by the rate region. On the other hand, it is a very natural assumption for QoI attributes as well, which can result from device capabilities as reception and processing limitations. Hence, upper bounds could be readily included as constraints in (22) without altering the convexity of the problem.

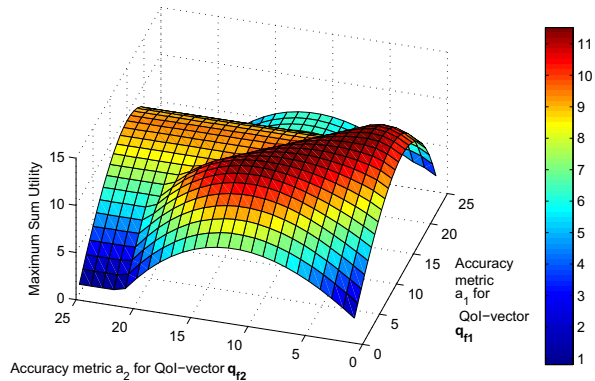


Figure 4. Maximum Sum Utilities for varying  $a_1$  and  $a_2$ .

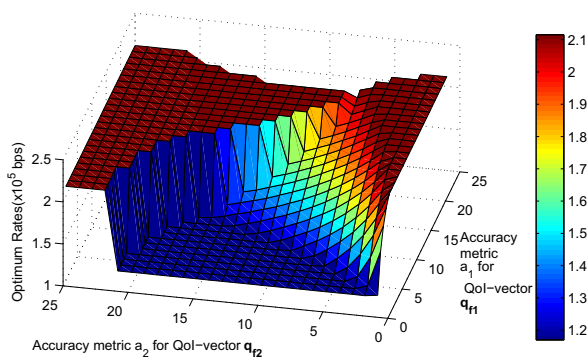


Figure 5. Optimum  $r_1$  ( $\times 10^5$ ) for varying  $a_1$  and  $a_2$ .

## V. NUMERICAL RESULTS

Next, we demonstrate that optimal rate allocation can be different from a corner point of the rate region for various scenarios.

First, consider the scenario with information types, QoI-vectors, timeliness properties, link qualities and device capabilities characterized by parameters  $\gamma_1 = 0.15$ ,  $\gamma_2 = 0.05$ ,  $D_1 = 12s$ ,  $D_2 = 15s$ ,  $c_1 = 212Kbps$ ,  $c_2 = 142Kbps$ ,  $c_s = 259Kbps$ . We assume that  $l_i(a_i) = a_i \times 10^5$ . We present the OICC-S values offered by the network as a function of the accuracy metrics in Fig. 4. Fig. 5 demonstrates the optimizing  $r_1$ , i.e. rate from source 1 to achieve the corresponding OICC-S values in Fig. 4. In essence, these two figures demonstrate that optimal rate allocation and the resulting OICC-S greatly depends on the QoI attributes, and for many cases time-sharing is the optimum rate allocation choice. For this scenario the overall OICC-S of the network is 11.518, achieved by  $(r_1, r_2) = (117.5Kbps, 141Kbps)$  and  $(a_1, a_2) = (8.4, 11.5)$ . Note that the optimizing rate point is very close to the corner point where information from source 2 is decoded later.

Next, for a more detailed explanation on factors effecting OICC-S we focus on the previous scenario with  $a_2 = 5$ . We observe the effect of varying  $a_1$  on optimal rate allocation and the sum utilities for the MAC in Fig. 6 and Fig. 7. We see that the optimal rate allocation greatly differs depending on  $a_1$ . For small  $a_1$ , strict priority is given to user 2, which

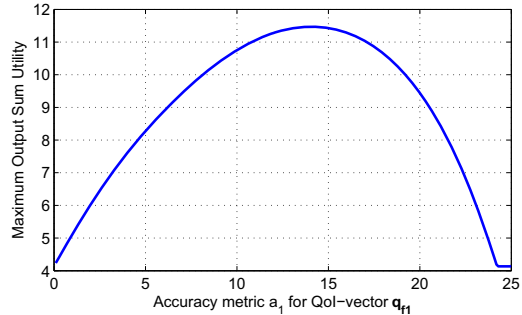


Figure 6. Maximum Sum Utilities for varying  $a_1$ .

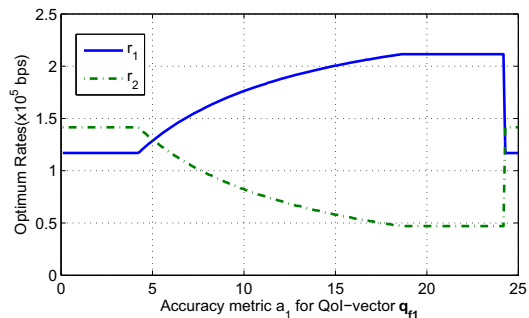


Figure 7. Optimal rates for QoI-flows for varying  $a_1$ .

has a higher accuracy. As  $a_1$  increases, timesharing between two corner points is selected. Further increase in  $a_1$  results in strict priority to user 1 due to its higher impact on utility. However, eventually increase in accuracy for source 1 results in significant degradation of utility due to untimeliness, and priority is again switched to source 2.

Hence, in many scenarios a simplified policy only focusing on corner points could not have provided the network with the maximum utility, i.e., attained OICC-S for the available information at hand. Such a policy would not provided the maximum decision making capability with the specified attributes.

Finally, we consider a case with identical parameters except  $l(a) = \alpha a^3$ , where  $\alpha = 160$ , presented in Fig. 8 and Fig. 9. This corresponds to a case where accuracy metric is a concave function of the number of bits required. The intuition is that utility gains are diminishing in return; after some level the accuracy metric and the effect to utility tends to saturate. Moreover, it satisfies the condition to preserve concavity of utility in  $a_i$  given by (23). For this scenario, the overall OICC-S of the network is 24.347 achieved by  $(r_1, r_2) = (133.1Kbps, 125.4Kbps)$  and  $(a_1, a_2) = (15.6, 15.3)$ . Note that the optimizing rate point is achieved by time-sharing. The intuition is that the increased level of nonlinearity in the objective function due to the  $l(a)$  relationship tends to cause the solution to deviate more from linear programming based solutions, i.e., corner points of the MAC rate region.

## VI. CONCLUSIONS

In this paper, we propose methods for QoI based utility evaluation in multisource networks. We characterize the max-

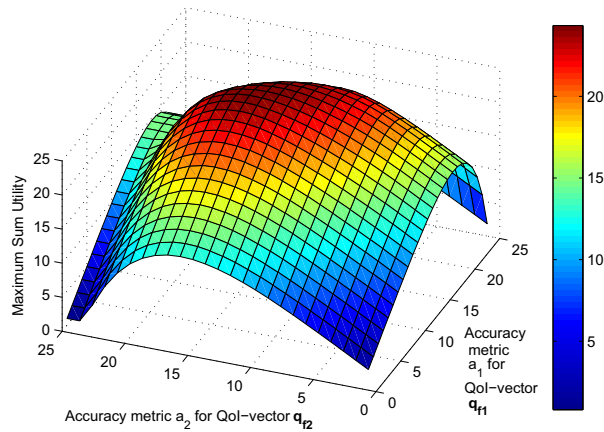


Figure 8. Maximum Sum Utilities for varying  $a_1$ ,  $l(a) = \alpha a^3$ .

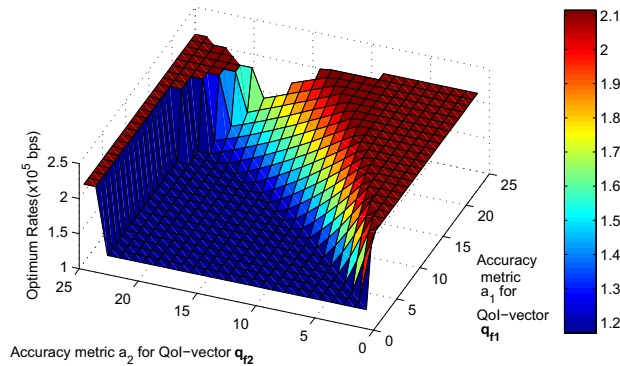


Figure 9. Optimal  $r_1 (\times 10^5)$  for varying  $a_1$  and  $a_2$ ,  $l(a) = \alpha a^3$ .

imum sum output utility provided by QoI-vectors supportable and by the network as the OICC-S. For OICC-S formulation, we focus on the effect of network delivery and timeliness on information with specific accuracy. We characterize rate allocation schemes in order to attain OICC-S for the most basic multiuser network model, specifically a two-user MAC. The formulations provided can be generalized to account for other QoI attributes which inherent similar trade-offs.

While we have focused on a two-user MAC in this work, we note that similar principles can also be used to determine rate allocation for a two-user broadcast channel. Future work includes extension of the policies for general multihop networks, and policies addressing random arrivals of information. Furthermore, generalizing the formulation to allow for correlated events and node capabilities as compression is of interest.

## VII. ACKNOWLEDGEMENT

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