Scheduling and Resource Allocation in 802.11ax

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***Abstract*—802.11ax introduces OFDMA to WiFi. It thus enables multiplexing users/user groups in the frequency domain. WiFi networks usually operate in a multipath environment which generates a frequency selective channel. Hence, the capacity of a user/user group changes over different subcarriers. A good scheduling and resource allocation scheme can maximize the sum rate by allocating users and user groups on subcarriers based on their CSI and other system considerations.**

In this paper we investigate how to optimally assign users and user groups to subcarriers with the goal of maximizing the user sum rate in the context of 802.11ax. We introduce a novel divide and conquer based algorithm which we prove to be optimal under the assumption that a user can be assigned to more than one resource unit (RU) which consists of one ore more subcarriers. This serves as a tight upper bound on the actual problem where users/user groups can be assigned to a single RU only per the 802.11ax standard. We then introduce two practical algorithms for the actual problem, a greedy one and a recursive one which jointly splits the bandwidth into RUs and schedules users on them. Extensive simulations comparing the performance of the aforementioned algorithms establish that our practical schemes achieve very good performance in all studied scenarios.

I. Introduction

Multi-user (MU) transmission in WiFi network is an important feature which can improve the system throughput greatly. It was first introduced in 802.11ac standard. The MU transmission in 802.11ac relies on downlink MU-MIMO, which makes use of spatial diversity to cancel the interference between users. In one MU-MIMO transmission, the frame takes the whole bandwidth.

802.11ax introduces orthogonal frequency-division multiple access (OFDMA) in WiFi network for the first time [1]–[4]. It does not improve peak data rate but allows efficient transmissions of small frames to a group of users simultaneously. In the OFDMA transmission of 802.11ax, the whole bandwidth is divided into multiple subsets of subcarriers, each subset called a resource unit (RU). Each RU is assigned with a user or a user group which is typically referred to as user scheduling [1]. 802.11ax supports three types of MU transmissions: MU-MIMO, OFDMA and Joint MU-MIMO and OFDMA [4], the latter two being the ones we study in this paper as they involve OFDMA.

WiFi networks usually work in a multipath environment where the wireless channel of the whole bandwidth can be modeled as a frequency selective channel in both downlink (DL) and uplink (UL). The channel capacity of each user or user group changes over subcarriers, especially when MU-MIMO is used. A good user schedule can assign RUs to different users or user groups based on their channel state information (CSI) such that the sum rate is maximized [3]. To efficiently utilize the available bandwidth, one needs to optimally solve this scheduling and resource allocation (SRA) problem while taking into consideration the capabilities and limitations of real world wireless access points (APs) and systems.

In this paper, we formulate the SRA problem in the context of 802.11ax. To be able to solve the problem analytically, we relax the original problem by allowing users and user groups to be assigned to multiple RUs. (802.11ax allows users to be assigned to a single RU only, unlike long-term evolution (LTE) which may allocate multiple resource blocks (RB) to a user.) We then introduce a divide and conquer algorithm which optimally solves this relaxed version of the original problem. We also introduce a practical greedy algorithm with fast execution time and a practical recursive scheme which jointly splits the bandwidth into RUs and schedules users on them in a near-optimal fashion. Last, we conduct extensive simulations and compare the performance of our algorithms against the optimal in the original constrained setting where a users may be scheduled to a single RU only. (The optimal in this case is computed by exhaustive search.) The simulations results show that our practical greedy and recursive schemes perform very well in a variety of realistic setups with the later being consistently very close to the optimal.

The outline of the rest of the paper is as follows. Section II briefly discusses prior work and Section III motivates our work. Then, Section IV sets up the system model and formulates the optimization problem. In Section V, several algorithms of the relaxed and original problem are discussed. The performance of the algorithms is compared in Section VI, where it is shown that our recursive algorithm can efficiently solve the scheduling and resource allocation problem at hand. Last, Section VII concludes the paper.

II. Related Work

The management and allocation of resources is always a critical issue in wireless networks, since resources such as spectrum and transmit power are limited. Motivated by this, there is a large body of work on managing such resources and improving the overall performance of the system, see, for example, [5]–[17] and references therein.

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Specifically, [5] optimizes the scheduling duration for OFDMA-based 802.11ax WLANs and [6] offers a summary of resource allocation and scheduling algorithms in connection with the quality of service (QoS) at the MAC layer.

A large part of this prior work, [7]–[14], shows that effective SRA can improve the throughput of OFDMA systems. For example, [7] proposed suboptimal algorithms for the SRA problem in a multiuser MIMO-OFDMA system which maximizes the system capacity, [8] generalized the framework of SRA to support various scheduling rules and objectives, [11] optimized the SRA of LTE systems under per-user QoS constraints and then proposed an energy efficient algorithm which could achieve near optimal performance, [10] investigated both SU-MIMO and MU-MIMO scheduling problems for the downlink in LTE-A networks, and, [13], [14] formulated the SRA problem of LTE uplink as a binary-integer optimization problem and proposed some efficient suboptimal algorithms whose performance was close to the optimal solution.

In prior work dealing with the general SRA problem, see, for example, [7]–[11], the whole bandwidth is split into multiple subchannels of equal size, the SRA on each subchannel is independent (except from the coupling from the power constraint) and a single user may be assigned to multiple subchannels. In 802.11ax, however, a user can only be assigned with a single RU and the size of an RU is flexible. What is more, the SRA problem for LTE uplink, see, for example, [13], [14], can assign a user with multiple subchannels if these subchannels are adjacent to each other and the locations of the subchannels are flexible, while the locations of RUs in 802.11ax are restricted to some specific locations. Therefore, existing SRA algorithms cannot be applied to 802.11ax networks directly.

The SRA of 802.11ax also requires good user grouping algorithms which select some users to form an MU-MIMO user group, since 802.11ax supports MU-MIMO on RUs which are larger than 106 subcarriers. User grouping for MU-MIMO has been well studied for 802.11ac networks [18]–[21]. These algorithms select the best user group based on the CSI over all subcarriers and allocates the whole bandwidth to this group. These user grouping algorithms can optimize the user group under the assumption that one transmission takes the whole bandwidth. In 802.11ax, the sum rate can be further improved by allocating different user groups to different RUs.

Thus, despite the large body of work on the general SRA problem as well as on specific instantiations of the problem, e.g. for LTE networks, there is no existing solution that is applicable to the specific characteristics of the 802.11ax standard.

III. Motivation

A. The Varying Nature of the Channel and its Capacity

In a wireless network, wireless signals from APs propagate to the users through different paths and thus incur different path losses. Generally speaking, users with small path loss have higher received power and their data rate is higher. However, wireless networks usually operate in a multipath environment where the coherent bandwidth is small such that the wireless channel for the whole bandwidth is considered as a frequency selective channel. The CSI of a user changes over subcarriers and the capacity on each subcarrier, decided by CSI, also changes [3]. The channel capacity on a subcarrier is not only decided by the path loss, but also influenced by the multipath effect. If the variations over subcarriers are high enough, a user with larger path loss may have higher channel capacity than others even if their path loss is smaller. Figure 1 shows the channel capacity of two MU-MIMO groups in a wireless network where an AP and users are equipped with 4 and 1 antennas respectively, under a typical indoors wireless channel. The path loss of users in user group 1 is smaller and the average channel capacity of the two user groups is 9.2 bps/Hz and 6.8 bps/Hz respectively. However, the channel capacity of user group 2 is higher than user group 1 on some subcarriers due to variations of CSI. In summary, the channel capacity is a function of both large scale fading and small scale fading, and it changes over users and subcarriers.

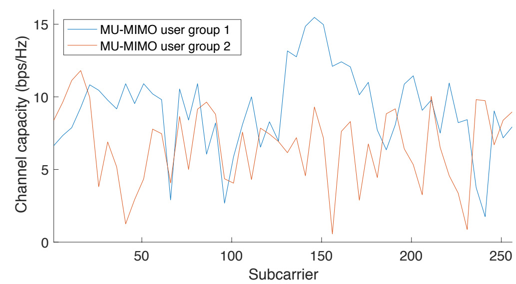


Fig. 1: The channel capacity of a frequency selective channel

B. Importance of Scheduling and Resource Allocation in 802.11ax

OFDMA is a multiple access technique based on orthogonal frequency-division multiplexing (OFDM). Like OFDM, OFDMA divides the whole bandwidth into subcarriers. The subcarrier spacing is small enough such that each subcarrier can be seen as a narrowband subchannel, even if the whole bandwidth is a frequency selective channel. The difference is that OFDM allocates all the subcarriers to a single user or user group, while in OFDMA, a user or user group is only assigned with a subset of subcarriers and their data are carried on these subcarriers only. In this way, one frame can multiplex multiple users/user groups simultaneously.

802.11ax introduced OFDMA to alleviate the intensive contentions in dense scenarios and improve the efficiency of resources. The variation of channel capacity over users and subcarriers makes it important to select a good user schedule. To see this, consider a wireless network in an indoors environment where one AP and 30 users are equipped with 4 and 1 antennas respectively and zero-forcing beamforming (ZFBF) is used for SU-MIMO and MU-MIMO. We compare the performance of random and optimal user schedules where for each subcarrier, the optimal SRA selects the best user/user group, while the random SRA selects a user/user group randomly. The simulation results, see Figure 2, show that if the users are scheduled randomly without optimization, the sum rate of the system is significantly reduced, especially for joint OFDMA and MU-MIMO transmissions. This motivates us to find a good SRA algorithm for 802.11ax OFDMA transmissions.

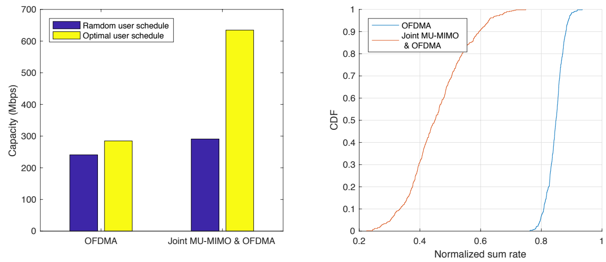


Fig. 2: Sum rate distribution and average value

IV. System Model

A. 802.11ax Primer

802.11ax supports the following bands: 20MHz, 40MHz, 80MHz, 80+80MHz (combines two 80 MHz channels) and 160MHz (a single 160 MHz channel) [1], [22]. In an OFDMA transmission, the spectrum band is divided into multiple RUs [1], [2]. In the time domain, an RU spans the entire data portion of a high efficiency (HE) PLCP protocol data unit (PPDU). In the frequency domain, it consists of a subset of contiguous subcarriers except the RUs which "straddle DC" (where some nulls are placed in the middle of the band). The size of an RU in frequency domain can be 26, 52, 106, 242, 484 or 996 subcarriers. The RUs in an HE MU PPDU using OFDMA transmission can only be any of these sizes. The locations of RUs in an HE PPDU are fixed. Each RU of size larger than 26 can be further divided into 2 smaller RUs. For example, the locations of the RUs in a 40MHz HE PPDU in frequency domain are shown in Figure 3. The whole bandwidth can be used as a single 484-tone RU, or it can be divided into two 242-tone RUs, each of which may be further divided into smaller RUs until 26-tone RUs are reached. Once the RUs have been generated, the AP allocates one RU to each user/user group for transmission. If the bandwidth is split into RUs and each of them is allocated to an individual user, then the transmission is referred to as pure OFDMA one; if an RU is equal or larger than 106 subcarriers it can also be used for MU-MIMO, and then the transmission is referred to as a joint MU-MIMO and OFDMA one.

B. Physical Layer Modeling

1) The channel model: Consider an 802.11ax BSS where the AP and users are equipped with N_T and N_R [1] ($N_T > N_R$) antennas respectively. The downlink CSI to all users is transmitted to the AP through channel sounding (CSIT). Then the AP applies ZFBF for SU-MIMO or MU-MIMO. The users

[1]It is easy to extend our work to the scenario where users have different number of antennas.

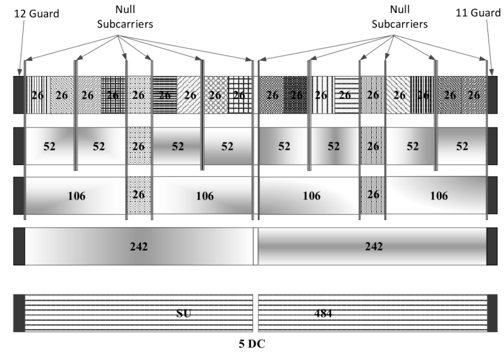


Fig. 3: RU locations in a 40MHz HE PPDU [1]

are indexed by the set $U = \{1, 2, ..., N\}$. In a downlink transmission, the AP decides to transmit to a set of users $U_s \subset U$ on subcarrier s. The received signal at user k on subcarrier s is

$$y_{k,s} = h_{k,s} w_{k,s} \sqrt{P_{k,s}} x_{k,s} + \sum_{\substack{j \in U_s \\ j \neq k}} h_{k,s} w_{j,s} \sqrt{P_{j,s}} x_{j,s} + z_{k,s}, \quad (1)$$

where $x_{k,s}$, $h_{k,s}$, $w_{k,s}$, $P_{k,s}$ and $z_{k,s}$ are the data symbol, channel response, beamforming weight vector, transmit power and additive white Gaussian noise (AWGN) for user k at subcarrier s, where the noise power is normalized such that $z_{k,s} \sim \mathcal{CN}(0, \boldsymbol{I})$. The transmit power $P_s = \sum_{k \in U_s} \gamma_{k,s}^{-1} P_{k,s}$ is constant over all subcarriers, where

$$\gamma_{k,s} = \frac{1}{\|w_{k,s}\|^2}. \quad (2)$$

The beamforming matrix $W_s = [w_{k,s}, k \in U_s]$, consisting of all beamforming weight vectors $w_{k,s}$, is the pseudo-inverse of $H_s = [h_{k,s}^T, k \in U_s]^T$, that is

$$W_s = H_s^H (H_s H_s^H)^{-1}. \quad (3)$$

Last, the sum rate on subcarrier s equals

$$R_{ZFBF}(s) = \sum_{k \in U_s} \log_2(1 + P_{k,s}), \quad (4)$$

where $P_{k,s}$ can be optimized by waterfilling [18], or set by equal power allocation for simplicity [23]. (Recall that the noise power is normalized to one.)

2) User grouping of MU-MIMO: If the AP transmits in joint MU-MIMO and OFDMA mode, the scheduling and resource allocation requires to perform user grouping. The global optimal solution to user grouping in MU-MIMO is hard to obtain and previous studies such as [18]–[21] have proposed many efficient algorithms to provide a suboptimal solution whose performance is very close to the global optimal one. It is beyond the scope of this paper to work on user grouping. Instead, we use prior work to select user groups.

3) Abstraction of RUs: As mentioned in Section IV-A, each RU larger than 26 subcarriers can be split into two smaller RUs. As shown in Figure 3, the whole bandwidth can be split at most $L-1$ times, where L is the number of levels. L is related to the whole bandwidth as larger bandwidth can be split more times. 802.11ax supports 20MHz, 40MHz, 80MHz and 160MHz, and L varies from 4 to 7 respectively.[2]

Each RU is denoted by $RU(l,i)$, where l is the number of splits from the original RU to the current one and i is the index of an RU at its level. Note that $RU(0,0)$ refers to the RU occupying the whole bandwidth. The whole bandwidth can be split into 2^l RUs of equal size at level l ($l \in \{0,1,...,L-1\}$), labeled as $0,1,...,2^l-1$. Each RU $RU(l,i)$ with $l < L-1$ can be split into two RUs $RU(l+1,2i)$ and $RU(l+1,2i+1)$. Using this notation, Figure 4 shows an example where we label the RUs of a 20MHz HE PPDU.

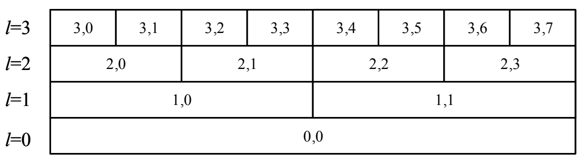


Fig. 4: RUs of a 20MHz HE PPDU

4) Scheduling and resource allocation: In one OFDMA transmission, the whole bandwidth is divided into a combination of RUs from different levels. Let $p = \{p_j ...\}$ be a valid partition of the whole bandwidth where $p_j = RU(l_j, i_j)$ is the j^{th} RU in p and let $\mathcal{P}$ be the universal set of all partitions. For example, one possible partition is shown in Figure 5. It includes RUs from level 1 to 3, specifically, $p_0 = RU(2,0)$, $p_1 = RU(3,2)$, $p_2 = RU(3,3)$, and $p_3 = RU(1,1)$.

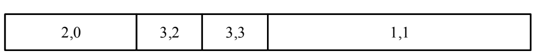


Fig. 5: A valid partition of the bandwidth

Having obtained a valid partition of the bandwidth, we need to allocate users to RUs. Say $g = \{(p_j, u_j)\}$ is a valid user schedule where $p_j = RU(l_j, i_j)$ is the jth RU in a valid partition of the whole bandwidth and u_j is the user set allocated to p_j. For example, one valid user schedule with the partition in Figure 5 is shown in Table I.

j	p_j	u_j
0	(2, 0)	{3}
1	(3, 2)	{16}
2	(3, 3)	{5}
3	(1, 1)	{1, 7, 10, 15}

TABLE I: A valid user schedule

[2]Even though $L \in \{4,5,6,7\}$ in 802.11ax, we may set L to other positive integers in the analysis afterwards, but the conclusions are true for all positive integers, including $L \in \{4,5,6,7\}$.

The ZFBF capacity on an $RU(l,i)$ can be computed by summing the achieved rates at each subcarrier which is part of this RU, that is,

$$R_{ZFBF}(RU(l,i)) = \sum_{s \in RU(l,i)} R_{ZFBF}(s). \tag{5}$$

Then, the ZFBF capacity of g is

$$\begin{aligned} R_{ZFBF}(g) &= \sum_j R_{ZFBF}(p_j) \\ &= \sum_j \sum_{s \in p_j} \sum_{k \in U_s} \log_2(1 + P_{k,s}). \end{aligned} \tag{6}$$

C. The Optimization Problem

The SRA problem consists of two tasks: (i) split the bandwidth into one or multiple RUs and (ii) allocate the RUs to users (SU-MIMO) or user groups (MU-MIMO). The constraints are:

1) A user or user group can only be assigned with no more than one RU.
2) MU-MIMO transmission only applies to RUs larger than 106 subcarriers, in other words, $l \leq L-3$.
3) The number of users allocated on $RU(l,i)$ is between 1 and $M(l)$.

$M(l)$ is the maximum number of users allowed on $RU(l,i)$ and it is a function of l. If the AP transmits in joint MU-MIMO and OFDMA mode and $l \leq L-3$, $RU(l,i)$ can be used for MU-MIMO and $M(l) = \lfloor N_T / N_R \rfloor$, otherwise $RU(l,i)$ is used for SU-MIMO and $M(l) = 1$. With all the above constrains, the SRA of 802.11ax can be formed as an optimization problem

$$\begin{aligned} & \max_{g \in \mathcal{G}} R_{ZFBF}(g) \\ st \quad & 0 \leq \sum_j c_{j,k} \leq 1 \\ & 1 \leq \sum_k c_{j,k} \leq M(l_j) \\ & c_{j,k} \in \{0,1\}, \end{aligned} \tag{7}$$

where $c_{j,k}$ indicates if user k is allocated on the jth RU, and $\mathcal{G}$ is the set of all valid user schedules.

V. Scheduling algorithms

A. The Relaxed Scheduling and Resource Allocation Problem

Constraint 1) makes the SRA problem complicated since it requires taking into consideration all users scheduled on each RU. Now consider the problem which relaxes Constraint 1), i.e., a user or user group can be assigned with multiple RUs and the first constraint in Problem (7) is gone. Now, the SRA on different RUs can be solved independently.

Consider the relaxed SRA problem on the subcarriers of $RU(l,i)$ in Figure 6. We can use the subcarriers as a single RU $RU(l,i)$ and allocate some users to $RU(l,i)$, otherwise, we need to split the subcarriers into multiple RUs at higher levels and allocate users to each of them. Note that no matter how we split the subcarriers of $RU(l,i)$, we need to first split

Parameters	Description
N_T	Number of antennas at the AP
N_R	Number of antennas at the user
k, s, j	User, subcarrier and RU indexes
U	The user set
N	The number of users
U_s	The user set allocated on subcarrier s
$y_{k,s}$, $x_{k,s}$	The received and transmitted signals of user k on subcarrier s
$z_{k,s}$	The noise signals of user k on subcarrier s
$h_{k,s}$, $w_{k,s}$	The channel response and beamforming weight vector of user k on subcarrier s
H_s, W_s	The channel response and beamforming weight matrix for all users on subcarrier s
$P_{k,s}$	The transmit power allocated to user k on subcarrier s
P_s	The total transmit power allocated to subcarrier s
$R_{ZFBF}(\cdot)$	The ZFBF capacity function
$RU(l,i)$	The ith RU at level l
L	The max number of levels
p_j	The jth RU in partition p, equivalent to $RU(l_j, i_j)$
$c_{j,k}$	A binary number indicating if user k is allocated on p_j
$M(l)$	The max number of users allowed on an RU at level l
g	A valid user schedule and resource allocation
$\mathcal{P}$	The set of all valid partitions
$\mathcal{G}$	The set of all valid user schedules

TABLE II: Notation glossary

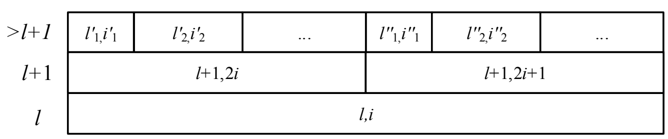


Fig. 6: The relaxed SRA problem on $RU(l,i)$

$RU(l,i)$ into two RUs, $RU(l+1,2i)$ and $RU(l+1,2i+1)$. Each of them can be further split into more RUs. After we split the subcarriers of $RU(l,i)$ into one or multiple RUs, we can select optimal user groups for each RU independently, since the relaxed SRA problem does not require Constraint 1). Note that we need to make sure the subcarriers are split optimally and the user grouping on each RU is also optimal. As we already mentioned, we use prior work such as [18]–[21] to obtain a good user group. Our contribution is on how to split the subcarriers into RUs and how to allocate users/user groups to those RUs optimally. We do this by a novel algorithm we refer to as the *divide and conquer* algorithm.

In the *divide* step, as shown in Figure 6, the subcarriers of $RU(l,i)$ are first split into two RUs $RU(l+1,2i)$ and $RU(l+1,2i+1)$. Then, each of these two RUs can be further split. The SRA problem on $RU(l,i)$ becomes two subproblems on $RU(l+1,2i)$ and $RU(l+1,2i+1)$ which can be solved independently. Let the optimal user schedules on $RU(l+1,2i)$ and $RU(l+1,2i+1)$ be $g_{opt}(l+1,2i)$ and $g_{opt}(l+1,2i+1)$ respectively, irrespectively of whether they are used as a single RU or split into multiple RUs.

In the *merge* step, $g_{opt}(l+1,2i)$ and $g_{opt}(l+1,2i+1)$ can be merged into a user schedule on $RU(l,i)$. Denote this user schedule as $g_m(l,i)$, where the subscript m means there are multiple RUs in this user schedule. Note that there is another optimal user schedule candidate on $RU(l,i)$, which uses all the subcarriers of $RU(l,i)$ as a single RU. Let the optimal user schedule in this case be denoted by $g_s(l,i)$, where the subscript s means there is only one RU in this user schedule. For a single RU $RU(l,i)$ all the subcarriers $s \in RU(l,i)$ are allocated to a single user or user group. Thus, this is the well known problem of user grouping over a channel, which can be solved by existing user grouping algorithms [18]–[21].

At this point, we have two user schedule candidates $g_s(l,i)$ and $g_m(l,i)$ for $RU(l,i)$. The optimal user schedule on $RU(l,i)$ is

$$g_{opt}(l,i) = \underset{g \in \{g_s(l,i), g_m(l,i)\}}{\arg\max} (R_{ZFBF}(g)). \tag{8}$$

Last, the optimal user schedule of the relaxed problem is $g_{opt} = g_{opt}(0,0)$. For more details, see the provided pseudo code.

Algorithm 1 DIVIDE-AND-CONQUER(U, l, i)

Require:
The CSI of all users;
//divide the problem into two subproblems
if $l < L-1$ **then**
  $g_{opt}(l+1,2i)$ = DIVIDE-AND-CONQUER($U, l+1, 2i$)
  $g_{opt}(l+1,2i+1)$ = DIVIDE-AND-CONQUER($U, l+1, 2i+1$)
  $g_m(l,i) =$ MERGE($g_{opt}(l+1,2i), g_{opt}(l+1,2i+1)$)
end if
//User selection on $RU(l,i)$
$g_s(l,i) =$ USER-SELECTION(U, l, i)
$G = \{g_s(l,i), g_m(l,i)\}$
//select the optimal user schedule from G
$g_{opt}(l,i) = \arg\max_{g \in G}(R_{ZFBF}(g))$
return $g_{opt}(l,i)$
function USER-SELECTION(U, l, i)
if $l \leq L-3$ **and** joint MU-MIMO and OFDMA mode **then**
  $g_s(l,i)$ = USER-GROUPING(U, l, i)
else
  Select user $u_{opt} \in U$ with max capacity on $RU(l,i)$
  $g_s(l,i) = \{(RU(l,i), u_{opt})\}$
end if
return $g_s(l,i)$
end function

Lemma 1. *The optimal user schedule of the relaxed SRA problem can be obtained by the divide and conquer algorithm.*

Proof. If $L = 1$, the lemma is obviously true. If Lemma 1 is true when the max number of levels is L, then we can also prove that it is true when the max number of levels is $L+1$. Suppose there is another user schedule g' which is different from g_{opt} obtained by the divide and conquer algorithm and $R_{ZFBF}(g') > R_{ZFBF}(g_{opt})$. If g' consists of only one RU, then its sum rate is larger than $R_{ZFBF}(g_s(0,0))$, which contradicts the fact that $g_s(0,0)$ is the optimal user group on $RU(0,0)$. If g' consists of multiple

RUs, its sum rate is $R_{ZFBF}(g'(1,0)) + R_{ZFBF}(g'(1,1))$ and $R_{ZFBF}(g'(1,0)) + R_{ZFBF}(g'(1,1)) > R_{ZFBF}(g_m(1,0)) + R_{ZFBF}(g_m(1,1))$, which contradicts the fact that $g_m(1,0)$ and $g_m(1,1)$ are optimal user schedules on $RU(1,0)$ and $RU(1,1)$ respectively. Therefore, Lemma 1 is also true for $L+1$. □

Lemma 2. *The sum rate of the optimal user schedule of the relaxed SRA problem is an upper bound for the original SRA problem.*

Proof. The optimal user schedule of the original SRA problem satisfies all the constraints of the relaxed SRA problem and thus is also a feasible solution of the relaxed SRA problem, the sum rate of the optimal user schedule of the relaxed SRA problem is no less than the sum rate of the optimal user schedule of the original SRA problem. □

Even though the optimal user schedule of the relaxed SRA problem does not satisfy all the constraints of the original problem, it can be used as an upper bound of the optimal user schedule of the original SRA problem. In the simulations section we show that this upper bound is quite tight.

B. The Original Scheduling and Resource Allocation Problem

The original SRA problem with Constraint 1) can not be solved by the divide and conquer algorithm, since the subproblems are solved independently on the user set U, thus when the user schedules of the subproblems are merged, it is possible that a user shows up on multiple RUs. The original problem can of course be solved by exhaustive search (for small scale scenarios) and by a greedy algorithm (leading to suboptimal solutions).

1) Exhaustive search: The exhaustive search traverses all the possible user schedules to search the optimal solution. It guarantees to find the optimal user schedule, but it is impractical in WiFi networks due to its time complexity. To get the size of the search space, consider the number of user schedules $\tau(n,l)$ which allocates *exactly* n users to the subcarriers of a RU at level l. Similar to Section V-A, there are two kinds of user schedules on a RU at level l:

Case 1: All the subcarriers make up a single RU. There is only one way to allocate the RU to the n users, if n is no larger than $M(l)$. So the number of possible user schedules in this case is

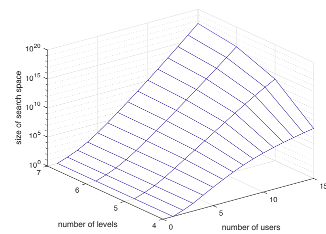
$$\tau_s(n,l) = \begin{cases} 1 & n \le M(l) \\ 0 & \text{otherwise} \end{cases}. \tag{9}$$

Case 2: The subcarriers are split into multiple RUs. Similar to Algorithm 1, the subcarriers are first split into two RUs and the number of possible user schedules in this case is

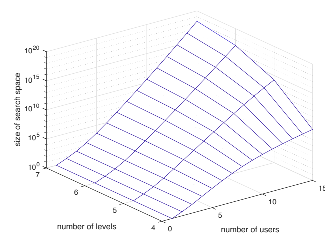
$$\tau_m(n,l) = \begin{cases} \sum_{k=1}^{n-1} \binom{n}{k} \tau(k,l+1)\tau(n-k,l+1) & l < L-1 \\ 0 & \text{otherwise} \end{cases}. \tag{10}$$

Note that $\tau(n,l) = \tau_s(n,l) + \tau_m(n,l)$ and $\tau(1,l) = 1$, and, using Equations (9) and (10), $\tau(n,l)$ can be calculated recursively.

Suppose there are N users in the network and L levels of RUs. In one transmission, the AP chooses some of the users to serve. The number of combinations of choosing n users from N users is $\binom{N}{n}$. Therefore, the number of possible user schedules is $\sum_{n=1}^{N} \binom{N}{n} \tau(n,0)$. Figure 7 shows the number of user schedules as a function of N and L. The size of the search space increases very fast as N and L increase. If a network has $N = 10$ users and works on 40MHz ($L = 5$), then the search space of OFDMA mode and joint MU-MIMO and OFDMA mode is 9.1×10^8 and 1.7×10^9 respectively. Clearly the exhaustive search is computational too expensive and thus impractical for real world 802.11ax networks.



(a) OFDMA



(b) Joint MU-MIMO and OFDMA

Fig. 7: The time complexity of exhaustive search

2) Greedy algorithm: We propose a greedy algorithm which first selects a level l to operate at and then splits the whole bandwidth such that the partition p consists of RUs of equal size. The level l is chosen such that each RU can be assigned with at least one user/user group. Given a total number of N users with say $N_R = 1$ antenna each, if every RU were to be assigned with the maximum possible number of users N_T (forming a maximum size user group), then we could assign users to N/N_T RUs. Following this rational, the algorithm chooses level $l = \min(L-1, \lfloor \log_2(N) \rfloor)$ for OFDMA mode or $l = \min(L-3, \lfloor \log_2(N \cdot N_R/N_T) \rfloor)$ for joint MU-MIMO and OFDMA mode. Looking at Figure 4, once the level l is identified, the algorithm moves from left to right within the chosen level selecting the best user or user group for each RU, and the moving on to the next RU to the right while excluding the already selected users from further consideration.

C. Recursive Scheduling

The divide and conquer algorithm violates Constraint 1) as the SRA of $RU(l+1, 2i)$ and $RU(l+1, 2i+1)$ are solved independently and some users may appear in both $g_{opt}(l+1, 2i)$ and $g_{opt}(l+1, 2i+1)$. In order to satisfy Constraint 1), we propose a new algorithm referred to as *recursive* scheduling, which excludes the users of $g_{opt}(l+1, 2i)$ from the user set when solving $g_{opt}(l+1, 2i+1)$ and vice versa. The algorithm may return a suboptimal solution since the optimal user schedule does not necessarily consist of either one of $g_{opt}(l+1, 2i)$ and $g_{opt}(l+1, 2i+1)$. Interestingly, the simulation results in Section VI show that the gap between the solution of recursive scheduling and optimal user schedule is very small. For more details on the algorithm, see the provided pseudo code.

Algorithm 2 RECURSIVE-SCHEDULING(U, l, i)

Require:
    The CSI of all users;
//divide the problem into two sub problems
if $l < L-1$ **then**
    // Solve the SRA on $RU(l+1, 2i)$ first
    $g_{opt}(l+1, 2i)$ = RECURSIVE-SCHEDULING(U, $l+1$, $2i$)
    $U_c = U - \{u | u \in g_{opt}(l+1, 2i)\}$
    $g'_{opt}(l+1, 2i+1)$ = RECURSIVE-SCHEDULING(U_c, $l+1$, $2i+1$)
    $g_m(l, i)$ =MERGE($g_{opt}(l+1, 2i), g'_{opt}(l+1, 2i+1)$)
    // Solve the SRA on $RU(l+1, 2i+1)$ first
    $g_{opt}(l+1, 2i+1)$ = RECURSIVE-SCHEDULING(U, $l+1$, $2i+1$)
    $U'_c = U - \{u | u \in g_{opt}(l+1, 2i+1)\}$
    $g'_{opt}(l+1, 2i)$ = RECURSIVE-SCHEDULING(U'_c, $l+1$, $2i$)
    $g'_m(l, i)$ =MERGE($g'_{opt}(l+1, 2i), g_{opt}(l+1, 2i+1)$)
end if
//User grouping on $RU(l, i)$
$g_s(l, i)$ = USER-SELECTION(U, l, i)
$G = \{g_m(l, i), g'_m(l, i), g_s(l, i))\}$
//Select the optimal user schedule from G
$g_{opt}(l, i) = \arg\max_{g \in G}(R_{ZFBF}(g))$
return $g_{opt}(l, i)$

We now comment on the complexity of the algorithm. The basic operation of recursive scheduling is user selection. Recursive scheduling on an RU calls recursive scheduling 4 times and user selection once at level $l < L-1$, and calls user selection once at level $L-1$. The number of user selection operations, ϕ, is a function of l and equals

$$\phi(l) = \begin{cases} 4\phi(l+1)+1 & l < L-1 \\ 1 & l = L-1 \end{cases}. \tag{11}$$

Thus, the recursive scheduling has $\phi(0) = (4^L - 1)/3$ user selection operations. In OFDMA mode, the user selection selects the best user from N users with time complexity $O(N \log N)$; in joint MU-MIMO and OFDMA mode, the user selection selects the best user group from N users at level $l \leq L-3$ with time complexity is $O(N^2)$ (assuming [20] is used for user grouping). The time complexity of scheduling is $O(4^L N \log N)$ in OFDMA mode and $O(4^L N^2)$ in joint MU-MIMO and OFDMA mode. Even though the time complexity grows exponentially with L, L is no more than 7 in an 802.11ax network. As a result, the number of user selection operations is at most 5461, which makes the scheme practical.

VI. Simulations

The presented algorithms for both the relaxed SRA problem (divide and conquer based algorithm) and the original SRA problem (greedy and recursive scheduling) are evaluated in simulations. Consider downlink MU transmissions in a single 802.11ax basic service set (BSS) in a 50m×50m office area with central frequency of 5GHz, where the wireless channel can be modeled with the WINNER II model [24]–[26]. According to WINNER II, the path loss is given as

$$PL = A\log_{10}(d[m]) + B + C\log_{10}(f_c[GHz]/5.0) + X, \tag{12}$$

where d is the distance between the user and the AP, A, B, C and X are parameters related with scenarios which can be found in [24]. We choose the indoor office (A1) non-line-of-sight (NLOS) scenario for the simulations. For each simulation scenario, we keep the location of the AP fixed and create 500 different topologies by randomly distributing the users. We then report the CDF and the average value of the sum rate under both the pure OFDMA and the joint MU-MIMO and OFDMA modes. Last, in all simulations we use SIEVE [20] as the user grouping algorithm.

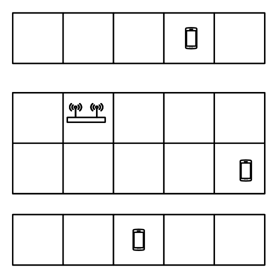


Fig. 8: The topology of the 802.11ax BSS

A. Divide and Conquer versus Exhaustive Search

The divide and conquer algorithm is compared with exhaustive search to evaluate the tightness of the upper bound. As mentioned in Section V-B1, the exhaustive search is computationally expensive, thus we consider a small scale scenario: The BSS consists of 1 AP and 7 users with $N_T = 4$ and $N_R = 1$ antennas respectively. The bandwidth is 20MHz ($L = 4$). As shown in Figure 9, the sum rate of divide and conquer is slightly higher than exhaustive search, which is expected since it allows assigning multiple RUs to a user while exhaustive search does not. Interestingly, the gap between the sum rate of exhaustive search and divide and conquer is very small. Specifically, the average sum rate is very similar (see left plot) and the CDF of the ratio of the sum rate of exhaustive search over that of divide and conquer shows that the sum rate of exhaustive search is always less than 8% off that of divide and conquer (see right plot). Thus, the sum rate achieved by the divide and conquer algorithm is a tight upper bound of the optimal user schedule and we will use it in place of exhaustive search (optimal) for the larger scale scenarios discussed below.

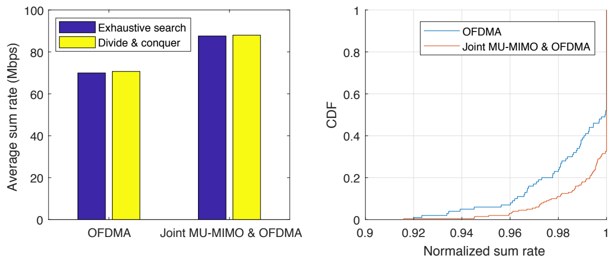
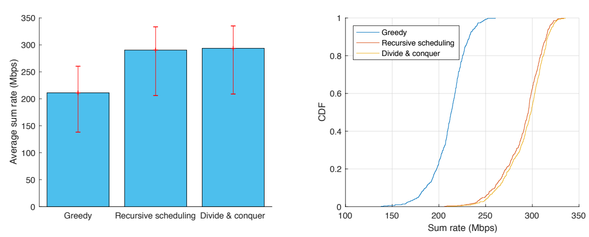


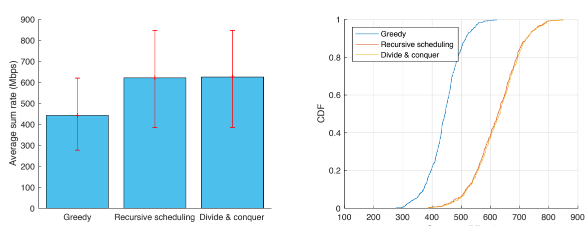
Fig. 9: The gap between exhaustive search and divide and conquer

B. Comparison between Divide and Conquer, Greedy, and Recursive Scheduling

The performance of divide and conquer, greedy, and recursive scheduling is compared in a BSS with one AP and 30 users with $N_T = 4$ and $N_R = 1$ antennas respectively. The bandwidth is 40MHz ($L = 5$). As shown in Figure 10, the sum rate of recursive scheduling is very close to divide and conquer, which means it is also close to the optimal user schedule achieved by exhaustive search. The sum rate of the greedy algorithm, however, is about 82Mbps and 183Mbps less than the optimal user schedule in OFDMA and joint MU-MIMO and OFDMA respectively.



(a) OFDMA

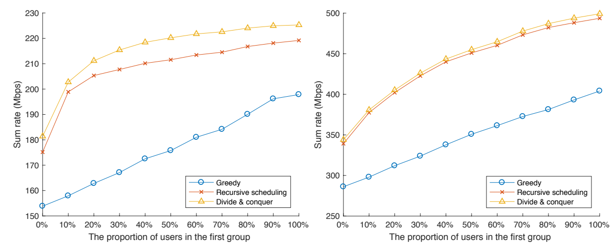


(b) Joint MU-MIMO and OFDMA

Fig. 10: Comparison of the sum rate between different algorithms

C. Impact of Different Type of Users

Consider a BSS with 1 AP and 30 users with $N_T = 4$ and $N_R = 1$ antennas respectively and a bandwidth of 40MHz ($L = 5$) like before. Suppose there are two group of users, the first group of users 10m away from the AP having a relatively small path loss, while the other group of users 20m away from the AP having a larger path loss. We gradually increase the proportion of users in the first group and compare the sum rate of the divide and conquer, greedy, and recursive algorithms. Obviously, with more users in the first group, the sum rate of the system should increase. As shown in Figure 11, the sum rate of the greedy algorithm is, as expected, lower than that of the other two algorithms. Also, the gap is larger when there are a few users with small path loss (first group). This is because the greedy algorithm allocates a smaller portion of the RUs to them in comparison to what the recursive scheduling and divide and conquer algorithm do. Also note that, as before, divide and conquer and recursive scheduling have very similar performance.

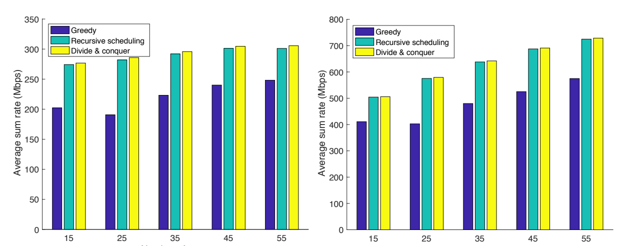


(a) OFDMA (b) Joint MU-MIMO and OFDMA

Fig. 11: Impact of different type of users

D. Impact of the Number of Users

The impact of the number of users is evaluated in a BSS which consists of 1 AP and N users equipped with $N_T = 4$ and $N_R = 1$ antennas respectively. The bandwidth is 40MHz ($L = 5$) like before. As shown in Figure 12, the sum rate increases as N increases since there are more users which are close to the AP. Similar to Section VI-C, the greedy algorithm has lower sum rate and the sum rate of the recursive scheduling is very close to the divide and conquer.



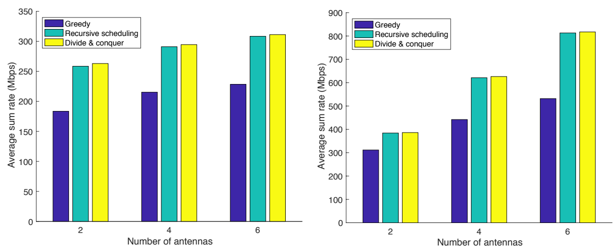
(a) OFDMA (b) Joint MU-MIMO and OFDMA

Fig. 12: Impact of number of users

E. Impact of the Number of Antennas at the AP

The impact of the number of transmit antennas is evaluated in a BSS which consists of 1 AP and 30 users equipped with

N_T and $N_R = 1$ antennas respectively and a bandwidth of 40MHz ($L = 5$) like before. As shown in Figure 13, the sum rate increases as N_T increases, since there are more spatial streams in each MU-MIMO group. The sum rate of recursive scheduling is close to that of divide and conquer as N_T increases. The sum rate increases faster in joint MU-MIMO and OFDMA mode because the channel capacity of MIMO is related to both the SNR and the rank of the channel matrix H_s: The rank of SU-MIMO in OFDMA mode is constant 1, while the rank of MU-MIMO in joint MU-MIMO and OFDMA mode changes from 2 to 6 as we vary the number of antennas from 2 to 6. Last, note that since the total transmit power P_s is constant, the sum rate does not change linearly with N_T.



(a) OFDMA (b) Joint MU-MIMO and OFDMA

Fig. 13: Impact of number of antennas

VII. CONCLUSIONS

In this paper we study how to jointly split bandwidth into RUs and schedule users/user groups within these RUs in the context of 802.11ax networks. We first investigate a relaxed version of this scheduling and resource allocation (SRA) problem which allows allocating a user to multiple RUs. It is proved that the relaxed SRA problem can be solved optimally by a divide and conquer based algorithm that we introduce and the sum rate of its optimal user schedule is a tight upper bound of the original SRA problem. However, the original SRA problem, allocating a user to at most one RU per the 802.11ax standard, can only be solved optimally by exhaustive search which is computationally expensive. Motivated by this, we propose a greedy and a recursive based algorithm which provide efficient user schedules. As shown in the simulations, the recursive scheduling algorithm yields a sum rate which is very close to the optimal.

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