We design a coordination mechanism for truck drivers that uses pricing schemes to alleviate traffic congestion in a general transportation network. We consider the user heterogeneity in Value-Of-Time (VOT) by adopting a multi-class model with stochastic Origin-Destination (OD) demands for the truck drivers. A basic characteristic of the mechanism is that the coordinator asks the truck drivers to declare their desired OD pair, as well as their individual VOT from a set of $N$ available options, and guarantees that the resulting pricing scheme is Pareto-improving, i.e. every truck driver will be better-off compared to the User Equilibrium (UE) and that every truck driver will have an incentive to truthfully declare his/her VOT, while leading to a revenue-neutral (budget balanced) on average mechanism. We show that the Optimum Pricing Scheme (OPS) can be calculated by solving a nonconvex optimization problem. To achieve computational efficiency, we additionally propose an Approximately Optimum Pricing Scheme (AOPS) and we prove that it satisfies the aforementioned characteristics. Both pricing schemes are compared to the Congestion Pricing with Uniform Revenue Refunding (CPURR) scheme through extensive simulation experiments. Initially, we experimentally show for the single OD pair with two routes network, CPURR does not provide a significantly better solution compared to the UE in terms of expected total monetary cost whenever the OD demand is stochastic. For the same network, we also show that the difference in the expected total monetary cost of truck drivers between the OPS and the CPURR solutions becomes higher as the difference between the distinct classes of VOT becomes larger. Finally, the simulation results using the Sioux Falls network demonstrate that both OPS and AOPS consistently outperform CPURR both in expected total travel time and in expected total monetary cost while concurrently approaching the System Optimum (SO) solution.

Key words: Road Pricing; Traffic Equilibrium; Congestion Pricing; Freight Routing; Mechanism Design; User Heterogeneity

* Declarations of interest: none
1. Introduction.

Measuring the contribution to the United States (U.S.) economy as the share of all expenditures in transportation-related final goods and services, the transportation sector contributed $1,489.7 billion to U.S. Gross Domestic Product (GDP) (Bureau of Transportation Statistics 2018) while in the European Union (EU) it accounts for almost 5% of the GDP (European Commision 2019). In EU, road transport has the largest share of EU freight transport accounting for 76.7% of the total inland freight transport (Eurostat 2019). Alan Hooper (2018) found that the trucking industry experienced nearly 1.2 billion hours of delay on the National Highway System (NHS) of the U.S. as a result of traffic congestion making the operational costs incurred by the trucking industry due to traffic congestion to be $74.5 billion. These statistics demonstrate that an optimized routing system is essential and could significantly contribute to the global economy.

Drivers usually make their routing decisions using GPS routing apps in an effort to minimize their individual travel time (Wardrop 1952). This phenomenon is known as User Equilibrium (UE) or the first Wardrop Principle. However, it is known that UE deviates from an optimized road usage (Beckmann et al. 1956, Pigou 1920) and it is a sub-optimal behavior compared to the socially optimum policy that could be achieved through a centrally coordinated system (Youn et al. 2008). Recent studies (Monnot et al. 2017, Zhang et al. 2016a) estimated the Price Of Anarchy (POA) (Koutsoupias and Papadimitriou 1999), i.e. the inefficiency between a selfish routing strategy and a system optimum policy in realistic transportation networks using real traffic data, demonstrating the necessity for its reduction. Based on the idea of Connected Automated Vehicles (CAVs) (Rios-Torres and Malikopoulos 2016, Zhang et al. 2016b), Zhang et al. (2016a) proposed to reduce the POA by recommending to all drivers socially optimum routes. However, such a strategy would raise several fairness and equity issues since in a System Optimum (SO) solution, some drivers may benefit while some others may be harmed compared to the UE.

One of the most common techniques addressing the problem of the inefficiency between the UE and the SO solutions is congestion pricing (Beckmann et al. 1956, Pigou 1920, Vickrey 1969) where each driver is assigned a fee corresponding to the additional cost his/her presence causes to the network. Several other works have studied congestion pricing under user heterogeneity in VOT, e.g. (Yang and Huang 2004, Yang and Zhang 2002), the problem of management of the revenue collected from the application of congestion pricing (Guo and Yang 2010, Small 1992) and the impact of congestion pricing schemes on emissions of freight transport (Chen et al. 2018). Recently, there is also a growing research interest for studies related to pricing schemes in the presence of autonomous vehicles (Lazar et al. 2019, Mehr and Horowitz 2019, Simoni et al. 2019, Tscharaktschiew and Evangelinos 2019).
Another well studied strategy addressing the problem of the inefficiency between an equilibrium flow pattern and the SO is the application of a Tradable Credit Scheme (TCS) among the drivers of the network (Yang and Wang 2011). In this case, a central coordinator initially distributes a certain number of credits to all eligible drivers and free credit trading is allowed among travelers. Wang et al. (2012) and Zhu et al. (2014) studied the application of TCS under user heterogeneity in VOT. Recently, Xiao et al. (2019) studied a Cyclic Tradable Credit Scheme (CTCS), where the credits never expire but circulate within the system, and derived a sufficient condition for the existence of a Pareto-improving CTCS in a general network.

In this paper, we are addressing the problem of the inefficiency of an equilibrium flow pattern by studying pricing schemes under a centrally coordinated routing system that can alleviate traffic congestion and drive the network as close as possible to a SO solution. Even though the resulting pricing schemes can be applied to any class of vehicles, we specifically study their application on trucks which consist an ideal candidate subclass of vehicles for coordinated routing (Kordonis et al. 2019). To this end, we consider a non-atomic game theoretic model whose users are the truck drivers and their demand is assumed to be stochastic (Kordonis et al. 2019). In the case where the planning horizon is split into discrete non-overlapping time intervals and the drivers choose both their OD pair as well as their desired departure time interval, Papadopoulos et al. (2019a,b) derived sufficient conditions for the existence of revenue-neutral (budget balanced) and Pareto-improving pricing schemes that can additionally provide individual incentives to the drivers to truthfully declare their desired departure time. In this work, we take into account the user heterogeneity in the VOT. For the fixed demand case, Guo and Yang (2010) derived sufficient conditions for the existence of Pareto-improving and revenue-neutral pricing schemes. However, since they could not find a way to identify the VOT of each user, they proposed class-anonymous pricing schemes based on the idea of Congestion Pricing with Uniform Revenue Refunding (CPURR). In contrast with the work of Guo and Yang (2010), we design a coordination mechanism for the truck drivers where the central coordinator asks the users to declare their desired OD pair and additionally pick their VOT from a set of $N$ available options. Under this structure, we prove the existence of Pareto-improving and revenue-neutral pricing schemes that can additionally provide incentives to the drivers to truthfully declare their VOT. This additional information enables us to design personalized (class-specific) pricing schemes. More specifically, we propose an Optimum Pricing Scheme (OPS) that can be calculated by solving a nonconvex optimization problem. To reduce the computational time needed to calculate OPS, we propose a second pricing scheme called Approximately Optimum Pricing Scheme (AOPS) and we prove that it satisfies the desired properties. The simulation experiments demonstrate that both OPS and AOPS provide a much lower expected total travel time and
expected total monetary cost to the users compared to the CPURR scheme, while concurrently approaching the SO solution.

The rest of the paper is organized as follows. In Section 2, we present the model used and we formulate the User Equilibrium (UE) and the System Optimum (SO) problems. In Section 3, we present the Optimum Pricing Scheme (OPS) and the Approximately Optimum Pricing Scheme (AOPS) and we additionally formulate the CPURR scheme in the form of an optimization problem with complementarity constraints. In Section 4, the simulation results of our approach are provided while in Section 5, we present the Conclusion of this work.

2. Problem Formulation.

We consider a model with a continuum of users where the Origin-Destination (OD) demand of the truck drivers is assumed to be stochastic. More specifically, we assume that truck drivers know the number of passenger vehicles at each road segment of the transportation network. This assumption is not restrictive since passenger traffic has a repetitive behavior during the same day and time of the week in the absence of unexpected incidents (Papadopoulos et al. 2019a). Additionally, they know the probability distribution of the OD demand for the rest of the truck drivers but not the exact realization of the demand. This is a symmetric information model since each truck driver has the same amount of information. A similar model was also used in Kordonis et al. (2019). In this paper, we extend this model by considering user heterogeneity in VOT of the truck drivers.

Let $G = (V, L)$ denote a transportation network where $V$ is the set of nodes and $L$ is the set of links in the network. Let $C_{lT}(X_{lp}, X_{lT}(\alpha))$ be a known nonlinear function representing the travel time of a truck driver traversing road segment $l$ when there exist $X_{lp}$ passenger vehicles and $X_{lT}(\alpha)$ trucks on it where $\alpha$ is a set of variables defined as follows:

$$\alpha = \{\alpha_{jw,r} : w = 1, \ldots, N, j = 1, \ldots, v, r \in R_j\}$$

where $j$ is the index corresponding to a specific Origin-Destination (OD) pair, $w$ is the index corresponding to a class of users with VOT $s_w$, $r \in R_j$ expresses a specific route among the set of available routes $R_j$ connecting OD pair $j$, $N$ is the number of distinct classes of users and $v$ is the number of OD pairs in the network. Therefore, $\alpha_{jw,r}$ expresses the proportion of truck drivers belonging to class $w$ with a desired OD pair $j$ who choose route $r$ for their trip. Additionally, let $d^j_w$ be random variables denoting the number of truck drivers belonging to the class $w$ with desired OD pair $j$. Then, the number of trucks traversing the road segment $l$ can be defined as:

$$X_{lT}(\alpha) = \sum_{j=1}^{v} \sum_{w=1}^{N} \sum_{r \in R_j, d \in r} d^j_w \alpha_{jw,r}$$

The above formulation will be used in subsequent sections to study the User Equilibrium (UE) and System Optimum (SO) flow patterns as well as the existence of Pareto-improving pricing schemes.
2.1. User Equilibrium (UE).
In the absence of pricing schemes, the drivers are trying to minimize their own individual travel
time, e.g. through the usage of GPS routing apps. This behavior drives the network in a state
called User Equilibrium (Wardrop 1952) where no driver has an incentive to unilaterally change
his/her routing decision since he/she is not going to benefit from such a change.

In a transportation network with heterogeneous users, the equilibrium conditions can be either
calculated in time units or in cost units (Guo and Yang 2010). Since the equilibrium conditions
expressed in cost units can be obtained by multiplying each class’ travel time by its corresponding
VOT, we formulate the UE problem in time units without any loss of generality. Therefore, let
$F_{j,r}^w(\alpha)$ be the expected travel time of a truck driver with VOT belonging to the class $w$, travelling
in OD pair $j$ and following route $r$. Then, $F_{j,r}^w(\alpha)$ is given by:
\[
F_{j,r}^w(\alpha) = E \left[ \sum_{l \in r} C_l(\alpha) \right] \tag{3}
\]
where $X(\alpha)$ is given by (2). Note that in a UE solution, it holds that $F_{j,r}^w(\alpha) = F_{j,r'}^w(\alpha), \forall w \neq w'$, i.e.
the equilibrium travel time is identical for all user classes between the same OD pair. Additionally,
in an equilibrium condition, it holds that:
\[
F_{j,r}^w(\alpha) \leq F_{j,r'}^w(\alpha), \forall r' \neq r \tag{4}
\]
where $r, r' \in R_j$. Inequality (4) states that in an equilibrium condition, drivers are choosing the
route $r$ that minimizes their individual expected travel time.

It has been shown by Kordonis et al. (2019) that there are possibly many non-equivalent UE
solutions. In this work, we calculate a specific equilibrium solution by solving an optimization
problem with complementarity constraints (Facchinei and Pang 2007) which is a nonconvex optimi-
zation problem. Before formulating the problem, let us first define the expected total travel time
of the truck drivers in the network as:
\[
E[T_{tr}(\alpha)] = E \left[ \sum_{l \in r} X_{IT}(\alpha)C_l \right] \tag{5}
\]
where $X(\alpha)$ is given by (2). Under the assumption that the demand of the truck drivers follows
a probability distribution with finite support, we can define the expected total monetary cost of
the truck drivers in the network as:
\[
E[T_{tr}^{mon}(\alpha)] = \sum_c \sum_{j=1}^N \sum_{w=1}^W \sum_{r \in R_j} p_c d_{j,w}^{c} \alpha_{j,w,r}^{UE} s_{w} J_{c,j,r}^{UE} \tag{6}
\]
where $c$ and $p_c$ correspond to a specific realization of the demand $d_{j,w}^{c}$ and its associated probability,
respectively, $\alpha_{j,w,r}^{UE}$ is the proportion of truck drivers belonging to class $w$ with a desired OD pair $j$
who follow route \( r \) at the UE, \( s_w \) is the VOT of the class \( w \) and \( J^{UE}_{c,j,r} \) is the travel time of a truck driver with an OD pair \( j \) who follows route \( r \) during the demand realization \( c \) at the UE. Note that at the UE, drivers make their own independent routing decisions. Therefore, in our formulation, given the assumption that truck drivers only know the probability distribution of the demand for the rest of the truck drivers and not the exact realization of it, their routing decisions \( \alpha^{UE}_{j,w,r} \) do not depend on the exact demand realization \( c \). Given the aforementioned definitions, we can formulate the optimization problem through which we can calculate a UE solution as follows:

\[
\begin{align*}
\text{minimize} \quad & \lambda E[T_{tr}(\alpha)] + (1 - \lambda) E[T_{\text{mon}}^{tr}(\alpha)] \\
\text{subject to} \quad & 0 \leq \alpha^j_{w,r} \perp F^w_{j,r}(\alpha) - \zeta^j_w \geq 0, \quad \forall j, w, r \\
& \sum_{r \in R_j} \alpha^j_{w,r} = 1, \quad \forall j, w
\end{align*}
\]  

(7)

where \( \zeta^j_w \) is a set of free variables that are used in order to solve the equilibrium optimization problem (7) and \( F^w_{j,r}(\alpha) \) is given by (3). Additionally, the notation \( \perp \) means that either \( \alpha^j_{w,r} = 0 \) or \( F^w_{j,r}(\alpha) - \zeta^j_w = 0 \) and finally, \( \lambda \) is a weighting factor such that \( \lambda \in [0, 1] \). Therefore, in the equilibrium optimization problem (7), among the possibly nonequivalent UE solutions, we are looking for the one that minimizes a weighted combination of the expected total travel time and the expected total monetary cost of the truck drivers. Viewing the expected total travel time of the truck drivers as a uniformly weighted expected total cost and given the fact that \( E[T_{\text{mon}}^{tr}(\alpha)] \) is equal to the expected total travel time of the truck drivers weighted by the corresponding VOT of each class \( w \), the overall objective of (7) can be expressed in cost units. The reasoning behind choosing the UE solution which minimizes the objective function of (7) is the following. First, as also mentioned in Guo and Yang (2010), \( E[T_{tr}(\alpha)] \) has long been accepted as a standard index of system performance in a transportation context while \( E[T_{\text{mon}}^{tr}(\alpha)] \) is a more appropriate system measure from an economic viewpoint. Second, we use the solution of (7) as a benchmark for designing Pareto-improving pricing schemes. Note that in order to create a Pareto-improving pricing scheme, i.e. a pricing scheme that can make everyone better-off compared to the UE, we first need to guarantee that the expected total travel time and the expected total monetary cost of the truck drivers using the proposed pricing scheme are lower than their best possible corresponding values at the UE.

Recently, to study how close a real traffic scenario is to a UE, for the static traffic assignment problem (Patriksson 2015), Cabannes et al. (2019) defined the average marginal regret as the expected time-saving drivers have in the network if they change their path to an optimal one. They proved that as the number of routing apps used is increased, the observed traffic assignment converges to a UE. The simulation results using real data for the whole Los Angeles network showed that the minimum travel time of a driver can be achieved whenever the ratio of GPS routing app
users reaches 100% where the network converges to the UE. Therefore, ensuring that the designed pricing schemes can make every driver better-off compared to the best possible travel time he/she could have at the UE, we also make sure that the reduction in his/her travel time will be even bigger compared to the real traffic conditions, even in the realistic scenario where the ratio of drivers using routing apps is less than 100%.

2.2. System Optimum (SO).

In a System Optimum (SO) solution, drivers are making routing decisions in a manner that contributes to the minimization of a socially optimum cost compared to the UE where they are trying to minimize their own individual travel time. Letting $E[T_p(\alpha)]$ denote the expected total travel time of the passenger vehicles in the network, we can define the expected total travel time of the network as:

$$E[T_s(\alpha)] = E[T_p(\alpha)] + E[T_{tr}(\alpha)]$$

(8)

Note that in a SO solution, the routing decisions of the truck drivers depends on the exact realization of the OD demands and therefore, we can define the expected total monetary cost of the truck drivers as:

$$E[T_{mon}^{tr}(\alpha)] = \sum_{c} \sum_{j=1}^{N} \sum_{w=1}^{w} \sum_{r \in R_j} p_c d_{j,w}^c \alpha_{w,r} c_{j,r}^c s_{w,j} J_{c,j,r}$$

(9)

where the main difference between (6) and (9) is the fact that in (9), the routing decisions of the truck drivers $\alpha_{w,r} c_{j,r}^c$ depend on the demand realization $c$. Using the aforementioned definitions, we can calculate the SO solution of the network by solving the following optimization problem:

$$\min_{\alpha(\cdot)} \quad \lambda E[T_s(\alpha)] + (1 - \lambda) E[T_{mon}^{tr}(\alpha)]$$

subject to

$$\sum_{r \in R_j} \alpha_{w,r} c_{j,r}^c = 1, \forall c, j, w$$

$$\alpha_{w,r} c_{j,r}^c \geq 0, \forall c, j, w, r$$

(10)

In (10), we minimize a weighted combination of the expected total travel time of the network (passenger vehicles + truck drivers) and the expected total monetary cost of the truck drivers. The reasoning behind the selection of this objective function is that even though we are providing routing suggestions only to the truck drivers, simultaneously, we want to additionally improve the overall traffic congestion of the network. Furthermore, using (8), the first term of the objective function of (10) can be written as $\lambda E[T_s(\alpha)] = \lambda (E[T_p(\alpha)] + E[T_{tr}(\alpha)])$. i.e. we equally weight the travel time of the passenger vehicles and the truck drivers. By introducing another weighting factor, i.e. by converting the objective function of (10) into $\lambda_1 E[T_p(\alpha)] + \lambda_2 E[T_{tr}(\alpha)] + \lambda_3 E[T_{mon}^{tr}(\alpha)]$ where $\lambda_1, \lambda_2, \lambda_3 \geq 0$ and $\lambda_1 + \lambda_2 + \lambda_3 = 1$, one could also adjust the weight put at each category of vehicles.
In a UE solution, every driver makes his/her own individual routing decisions which leads to an inefficient road usage. On the other hand, in a SO solution, some drivers may benefit while some others may be harmed compared to the UE solution making the SO hard to be applied in practice. In this section we study pricing schemes that are Pareto-improving, i.e. they can make every user better-off compared to the UE while at the same time, they can drive the network as close as possible to the SO solution.

We design a coordination mechanism that can be applied to truck drivers taking into account the user heterogeneity in their Value Of Time (VOT). More specifically, the coordinator asks the truck drivers to declare their desired OD pair and additionally choose their VOT from a set of $N$ available options. After collecting this information, the coordinator provides routing suggestions and additionally designs pricing schemes that are Pareto-improving and that can guarantee that every driver will have an incentive to truthfully declare his/her VOT while concurrently leading to a revenue-neutral (budget balanced) on average mechanism. This is in contrast with the previous literature studying pricing schemes, e.g. (Liu and Nie 2017, Tian et al. 2013), that makes assumptions about the distribution that the user heterogeneity might follow. However, note that it is important to guarantee that a user will truthfully declare his/her VOT in order to avoid the exploitability of the designed mechanism. This is mainly because many truck drivers would be willing to declare a high VOT in order to be assigned to the fastest possible route. In the next two subsections, we design pricing schemes that mathematically satisfy this property.

3.1. Optimum Pricing Scheme (OPS).
Let $\pi^{c,j}_{w,r}$ be the payment (made or received) of a truck driver belonging to the class $w$ with an OD pair $j$ who follows route $r$ during demand realization $c$. Then, we can calculate the optimum way to route the truck drivers $\alpha^*$ as well as the the optimum pricing scheme $\pi^*$ by solving the following nonconvex optimization problem:

$$\begin{align*}
\text{minimize} & \quad \lambda E[T_s(\alpha)] + (1 - \lambda)E[T_{tr}^{mon}(\alpha)] \\
\text{subject to} & \quad \sum_c \sum_{r \in R_j} p_c \alpha^{c,j}_{w,r} (J^{M,c,j}_{w,r} + \frac{1}{s_w} \pi^{c,j}_{w,r}) \leq \sum_c p_c A^{UE}_{c,j}, \forall j, w \\
& \quad \sum_c \sum_{r \in R_j} p_c \alpha^{c,j}_{i,r} (J^{M,c,j}_{i,r} + \frac{1}{s_i} \pi^{c,j}_{i,r}) \leq \sum_c \sum_{r \in R_j} p_c \alpha^{c,j}_{k,r} (J^{M,c,j}_{k,r} + \frac{1}{s_k} \pi^{c,j}_{k,r}), \forall j, i, k \\
& \quad \sum_{c=1}^{N} \sum_{j=1}^{W} \sum_{r \in R_j} p_c d^{w}_{c,j} \alpha^{c,j}_{w,r} \pi^{c,j}_{w,r} = 0 \\
& \quad \sum_{r \in R_j} \alpha^{c,j}_{w,r} = 1, \forall c, j, w \\
& \quad \alpha^{c,j}_{w,r} \geq 0, \forall c, j, w, r
\end{align*}$$

(11)
where $J_{w,r}^{M,c,j}$ is the travel time of a truck driver belonging to class $w$ with a desired OD pair $j$ who follows route $r$ during the demand realization $c$ under the mechanism routing suggestions $M$ and $A_{c,j}^{UE}$ is the average travel time of a truck driver with OD pair $j$ during demand realization $c$ at the UE. Therefore, the first constraint of (11) guarantees that every truck driver will be better-off compared to the UE in time units. The second constraint of (11) guarantees that a truck driver which belongs to class $i$ and truthfully declares class $i$ to the coordinator will be better-off on average compared to a truck driver who originally belongs to class $i$ but declares class $k$ to the coordinator. Therefore, the second constraint of (11) guarantees that every user will have an incentive to truthfully declare his/her VOT. Last, the third constraint of (11) guarantees that the expected total payments made and received by the coordinator are equal to zero and therefore, the resulting mechanism satisfies the budget balanced on average property. Note that the UE solution where no pricing scheme is applied to the users satisfies the constraints of (11) and therefore, a solution to (11) always exists.

In order to reduce the dimensionality of (11), in the next subsection, we present an Approximately Optimum Pricing Scheme (AOPS) and we show that we can assign routes to the drivers so that the proposed pricing scheme meets the desired goals.

### 3.2. Approximately Optimum Pricing Scheme (AOPS)

For a given routing decision $\alpha$, let us define the following pricing scheme:

$$
\pi_{c,j,w,r}^{AOPS} = s_w (A_{c,j}^{UE} - J_{w,r}^{M,c,j}) + \frac{s_w}{\sum_{l=1}^{N} s_l} \sum_{j=1}^{N} d_{w,c,j} \left( E\left[ T_{mon,M}^{tr}\right] - E\left[ T_{mon,UE}^{tr}\right] \right)
$$

The pricing scheme given by (12) initially makes each driver to pay (or receive a payment) such that his/her travel time under the mechanism routing suggestions $J_{w,r}^{M,c,j}$ becomes equal to his/her average travel time at the UE $A_{c,j}^{UE}$. Then, after calculating the expected total monetary benefits of the truck drivers $E\left[ T_{mon,M}^{tr}\right] - E\left[ T_{mon,UE}^{tr}\right]$ obtained from the application of the mechanism, it distributes those benefits to the different classes proportionally to the VOT that each class has. Finally, each class benefits are uniformly shared among the truck drivers of the class.

Let us now formulate the following optimization problem:

$$
\begin{align*}
\text{minimize} & \quad \lambda E[T_s(\alpha)] + (1 - \lambda) E[T_{tr}^{mon}(\alpha)] \\
\text{subject to} & \quad E[T_{tr}^{mon}(\alpha)] \leq E[T_{tr}^{mon,UE}] \\
& \quad H_{i,k}^{j}(\alpha) \leq N_{i,k}^{j}(\alpha), \forall j, i, k \\
& \quad \sum_{r \in R_j} \alpha_{w,r}^{c,j} = 1, \forall c, j, w \\
& \quad \alpha_{w,r}^{c,j} \geq 0, \forall c, j, w, r
\end{align*}
$$

(13)
where \( E[T_{tr}^{mon,UE}] \) is the expected total monetary cost of the truck drivers at the UE and \( H_{i,k}^j(\alpha) \) and \( N_{i,k}^j(\alpha) \) are given by the following equations:

\[
H_{i,k}^j(\alpha) = \left(1 - \frac{8k}{s_i}\right) \sum_c p_c A_{c,j}^{UE} + \frac{1}{\sum_{w=1}^N s_w} \sum_c p_c \frac{E[T_{tr}^{mon,M}] - E[T_{tr}^{mon,UE}]}{\sum_{j=1}^{d_{c,j}} d_{c,j}} (14)
\]

\[
N_{i,k}^j(\alpha) = \left(1 - \frac{8k}{s_i}\right) \sum_c \sum_{r \in R_j} p_c \alpha_{c,j}^{i,r} J_{M,c,j}^{i,r} + \frac{8k}{s_i} \sum_{w=1}^N s_w \sum_c p_c \frac{E[T_{tr}^{mon,M}] - E[T_{tr}^{mon,UE}]}{\sum_{j=1}^{d_{c,j}} d_{c,j}} (15)
\]

Note that a solution to the optimization problem (13) always exists since the UE satisfies all of its constraints. Let us call the optimum solution of the optimization problem described by (13)-(15) as \( \alpha^{AOPS}_* \). Now, we are ready to state the following theorem.

**Theorem 1.** The pair \((\alpha^{AOPS}_*, \pi^{AOPS}_{c,j,w,r})\) makes everyone better-off compared to the UE, guarantees that every user will have an incentive to truthfully declare his/her VOT and leads to a budget balanced on average mechanism.

**Proof.** To prove the statement of the theorem, we equivalently prove that \( \pi^{AOPS}_{c,j,w,r} \) is Pareto-improving, guarantees that every user will have an incentive to truthfully declare his/her VOT and creates a budget balanced on average mechanism if and only if the first and the second constraint of (13) together with (14)-(15) hold. Note that a user will be better-off compared to the UE if the first constraint of (11) holds. Therefore, substituting (12) into the first constraint of (11), we get:

\[
\sum_c \sum_{r \in R_j} p_c \alpha_{c,j}^{i,r} \left( J_{M,c,j}^{i,r} + A_{c,j}^{UE} - J_{M,c,j}^{i,r} \right) + \frac{1}{\sum_{l=1}^N s_l} \frac{E[T_{tr}^{mon,M}] - E[T_{tr}^{mon,UE}]}{\sum_{j=1}^{d_{c,j}} d_{c,j}} \leq \sum_c p_c A_{c,j}^{UE} \iff \\
\iff \frac{1}{\sum_{l=1}^N s_l} \left( E[T_{tr}^{mon,M}] - E[T_{tr}^{mon,UE}] \right) \sum_c p_c \sum_{j=1}^{d_{c,j}} d_{c,j} \leq 0
\]

which holds true if and only if \( E[T_{tr}^{mon}(\alpha)] \leq E[T_{tr}^{mon,UE}] \) which is equivalent to the first constraint of (13). Additionally, a user will have an incentive to truthfully declare his/her VOT if the second constraint of (11) holds. Therefore, substituting (12) into the second constraint of (11), we get:

\[
\sum_c \sum_{r \in R_j} p_c \alpha_{c,j}^{i,r} \left( J_{M,c,j}^{i,r} + A_{c,j}^{UE} - J_{M,c,j}^{i,r} \right) + \frac{1}{\sum_{l=1}^N s_l} \frac{E[T_{tr}^{mon,M}] - E[T_{tr}^{mon,UE}]}{\sum_{j=1}^{d_{c,j}} d_{c,j}} \leq \\
\leq \sum_c \sum_{r \in R_j} p_c \alpha_{c,j}^{i,r} \left( J_{M,c,j}^{i,r} + A_{c,j}^{UE} - J_{M,c,j}^{i,r} \right) + \frac{1}{\sum_{l=1}^N s_l} \frac{E[T_{tr}^{mon,M}] - E[T_{tr}^{mon,UE}]}{\sum_{j=1}^{d_{c,j}} d_{c,j}} \iff \\
\iff \left(1 - \frac{8k}{s_i}\right) \sum_c p_c A_{c,j}^{UE} + \frac{1}{\sum_{l=1}^N s_l} \sum_c p_c \frac{E[T_{tr}^{mon,M}] - E[T_{tr}^{mon,UE}]}{\sum_{j=1}^{d_{c,j}} d_{c,j}} \leq \\
\leq \left(1 - \frac{8k}{s_i}\right) \sum_c \sum_{r \in R_j} p_c \alpha_{c,j}^{i,r} J_{M,c,j}^{i,r} + \frac{8k}{s_i} \sum_{l=1}^N s_l \sum_c p_c \frac{E[T_{tr}^{mon,M}] - E[T_{tr}^{mon,UE}]}{\sum_{j=1}^{d_{c,j}} d_{c,j}}
where the last inequality is equivalent to the second constraint of (13). Last, a mechanism is budget balanced on average if the third constraint of (11) holds. Substituting (12) into the third constraint of (11), we get:

\[
\sum_{c} \sum_{j=1}^{N} \sum_{r \in R_j} p_c d_{c,j}^{w} \alpha_{w,r}^{c,j} \left( s_w (A_{c,j}^{UE} - J_{w,r}^{M,c,j}) + \frac{s_w}{\sum_{l=1}^{N} s_l} \sum_{j=1}^{N} d_{c,j}^{w} \alpha_{w,r}^{c,j} \pi_{r} \right) = \\
= E[T_{tr}^{mon,UE}] - E[T_{tr}^{mon,M}] + E[T_{tr}^{mon,M}] - E[T_{tr}^{mon,UE}] = 0
\]

Since the UE satisfies the constraints of (13), a solution to (13) always exists. Therefore, we have proved that by solving the optimization problem described by (13)-(15), we can calculate \( \alpha_{AOPS}^* \) such that the pricing scheme \( \pi_{c,j,w,r}^{AOPS} \) given by (12) satisfies all the statements of Theorem 1 and this concludes the proof. Q.E.D.

Theorem 1 states that one can get a sub-optimal solution to the original optimization problem (11) by solving the optimization problem described by (13)-(15) in order to assign routes to the truck drivers and by subsequently applying the pricing scheme given by (12). We prefer to call this method as Approximately Optimum Pricing Scheme (AOPS). The main advantage of this approach is the fact that we significantly reduce the dimensionality of the problem by calculating a pricing scheme using a simple algebraic equation. As we will later show experimentally, AOPS achieves a significant improvement compared to the UE and can additionally provide a solution close to the SO.

3.3. Congestion Pricing with Uniform Revenue Refunding (CPURR).

Under a congestion pricing scheme, each driver is assigned a fee depending on the OD pair and the route he/she follows. Guo and Yang (2010) proposed to combine Congestion Pricing with a Uniform Revenue Refunding (CPURR) scheme, i.e. the fees collected from congestion pricing are uniformly distributed among the participant drivers irrespective of the class they belong. Therefore, the whole scheme is class-anonymous. The CPURR scheme can be calculated by solving the following optimization problem with complementarity constraints:

\[
\begin{align*}
\text{minimize} & \quad \lambda E[T_s(\alpha)] + (1 - \lambda) E[T_{tr}^{mon}(\alpha)] \\
\text{subject to} & \quad 0 \leq \alpha_{w,r}^j \perp F_{w,r}^j(\alpha, \pi) - \zeta_w^j \geq 0, \forall j, w, r \\
& \quad \sum_{r \in R_j} \alpha_{w,r}^j = 1, \forall j, w \\
& \quad \sum_{c} \sum_{j=1}^{N} \sum_{r \in R_j} p_c d_{c,j}^{w} \alpha_{w,r}^{c,j} \pi_r = 0
\end{align*}
\]

where \( \zeta_w^j \) is a set of free variables that are used in order to solve the equilibrium optimization problem (16) and \( F_{w,r}^j(\alpha, \pi) \) is given by the following equation:

\[
F_{w,r}^j(\alpha, \pi) = \sum_{c} p_c \left( J_{w,r}^{M,c,j} + \frac{1}{s_w} \pi_r \right)
\]
Note that under a congestion pricing scheme, the network users still make their own individual routing decisions as in the UE but with the main difference being that now, they are concurrently taking into account the fees corresponding to each route. Since in our model the truck drivers only know the probability distribution of the demand for the rest of the truck drivers and not its exact realization, the way that the drivers choose their routes $\alpha^j_{w,r}$ does not depend on the exact realization $c$. Additionally, note that the variables $\pi^j_{w,r}$ corresponding to the pricing scheme do not depend on $w$ and $c$. The independence of $w$ can be justified by the fact that the CPURR scheme is class-anonymous. On the other hand, the coordinator of the CPURR scheme who is responsible for assigning fees to each route and then uniformly distribute the collected revenue back to the participant drivers, could design a pricing scheme that depends on the exact realization $c$ of the demand. However, even in that case, since none of the constraints of (16) depends on $c$, the optimum solution of (16) would not change.

4. Experimental Results.

The experimental results section is divided into two subsections. In the first subsection, for the single Origin-Destination (OD) with 2 routes network of Figure 1, we experimentally show that in the case where the demand is stochastic, i.e. if the truck drivers know the exact number of passenger vehicles in the network and only the probability distribution of the demands for the rest of the truck drivers, then the Congestion Pricing with Uniform Revenue Refunding (CPURR) scheme cannot decrease the expected total monetary cost of the truck drivers $E[T_{mon}^{\text{tr}}]$ compared to the UE solution. Additionally, for the same network, we experimentally show that in the case where there are only two classes with different VOT, the benefits in the expected total monetary cost of the truck drivers by applying the Optimum Pricing Scheme (OPS) rather than the CPURR scheme become greater as the difference in the VOT of the two classes becomes larger. The aforementioned results were obtained using the nonconvex optimization solver BARON (Sahinidis 2017, Tawarmalani and Sahinidis 2005). In the next subsection, we compare the User Equilibrium (UE), System Optimum (SO), Optimum Pricing Scheme (OPS), Approximately Optimum Pricing Scheme (AOPS) and the Congestion Pricing with Uniform Revenue Refunding (CPURR) scheme by applying them in the Sioux Falls network (LeBlanc et al. 1975).


The network is shown in Figure 1. For the first experiment, we assume that there are no passenger vehicles in the network and the cost of each route is described by the following nonlinear functions:

$$C_{1T} = 20 + X_{1T}^2, \quad C_{2T} = 2 + 3X_{2T}^2$$

(18)
where $C_{1T}$ and $C_{2T}$ are the costs of the first and the second route, respectively and $X_{1T}$ and $X_{2T}$ denote the number of trucks at the corresponding route. Additionally, we assume that the coordinator of the mechanism provides only two classes for the truck drivers to choose from with VOT:

$$s_1 = 100 \frac{\$}{hr}, \quad s_2 = 30 \frac{\$}{hr}$$

We also consider two different scenarios for the demand of the truck drivers. In the first scenario, we assumed that the demand of the truck drivers is deterministic and equal to $d = [2, 8]$ where the first element of the demand vector $d$ stands for the truck drivers who choose the class with VOT $s_1$ while the second element stands for the truck drivers who choose the class with VOT $s_2$. In the second scenario, we assumed that the demand of the truck drivers is stochastic and takes one of the following two equiprobable values:

$$d_1 = [2, 8], \quad d_2 = [3, 7]$$

Note that in this scenario, the truck drivers only know the probability distribution and not the exact realization of the demand vector. The value of the weighting factor was chosen to be $\lambda = 0.9$. The simulation results of this experiment are shown in Figure 2.

As can be observed in Figure 2, in the deterministic demand scenario both the OPS and the CPURR scheme can make the expected total monetary cost of the truck drivers $E[T_{\text{mon}}^{\text{tr}}]$ equal to its corresponding value in the SO solution. However, in the case where the demand vector is stochastic and the truck drivers only know the probability distribution of the demand for the rest of the truck drivers, the OPS can still achieve the same solution as the SO, while on the other hand the CPURR scheme does not provide a significantly better solution compared to the UE. Note that both UE, SO, OPS and CPURR can be calculated by solving the corresponding nonconvex optimization problems. For the values mentioned in Figure 2, UE and CPURR have been calculated with global optimality guarantees while SO and OPS are local optimum values obtained by using the optimization solver BARON (Sahinidis 2017, Tawarmalani and Sahinidis 2005).

Figure 1 Single Origin-Destination (OD) with 2 routes network.
Figure 2  The expected total monetary cost of the truck drivers $E[T_{tr}^{mon}]$ in case of a deterministic and a stochastic demand scenario for the single OD with 2 routes network of Figure 1.

For the second experiment, we assume that the cost of each route of the network shown in Figure 1 is given by the nonlinear functions described by (18). Additionally, we assume that the coordinator offers the truck drivers the possibility to choose between two classes with VOT $s_1$ and $s_2$, respectively. The demand of the truck drivers is assumed to be stochastic and takes one of the following values:

$$d(s_1) = \begin{cases} 2 & \text{w.p. 0.5} \\ 5 & \text{w.p. 0.5} \end{cases}, \quad d(s_2) = \begin{cases} 4 & \text{w.p. 0.5} \\ 8 & \text{w.p. 0.5} \end{cases}$$

where $d(s_1)$ and $d(s_2)$ are the demands for the truck drivers who choose the class with VOT $s_1$ and $s_2$, respectively. In this experiment, we measured the expected total monetary cost of the truck drivers $E[T_{tr}^{mon}]$ for the OPS and the CPURR scheme under four different scenarios. In the first scenario, we assumed that the VOT of the two classes was $(s_1, s_2) = (30 \frac{\text{hr}}{\text{hr}}, 30 \frac{\text{hr}}{\text{hr}})$, in the second scenario $(s_1, s_2) = (50 \frac{\text{hr}}{\text{hr}}, 30 \frac{\text{hr}}{\text{hr}})$, in the third scenario $(s_1, s_2) = (100 \frac{\text{hr}}{\text{hr}}, 30 \frac{\text{hr}}{\text{hr}})$ while in the fourth scenario the corresponding values were $(s_1, s_2) = (200 \frac{\text{hr}}{\text{hr}}, 30 \frac{\text{hr}}{\text{hr}})$. The value of the weighting factor was chosen to be $\lambda = 0.9$. The experimental results are shown in Figure 3.

As can be observed by the results shown in Figure 3, as the difference in the VOT between the two classes becomes larger, the benefits from applying the OPS over the CPURR scheme become greater. This result was expected since CPURR first uses the congestion pricing scheme by applying taxes/fees to the truck drivers and then uniformly distributes the total collected fees to the truck drivers. This makes CPURR a class-anonymous pricing scheme (Guo and Yang 2010). On the other hand, when a coordinator applies the Optimum Pricing Scheme (OPS), the coordinator concurrently asks the drivers to declare their VOT and by guaranteeing that every driver will have an incentive to truthfully declare his/her VOT, the coordinator can subsequently use that
Figure 3 The expected total monetary cost of the truck drivers $E[T_{tr}^{mon}]$ as the difference in the VOT between the two classes of users increases for the single OD with 2 routes network of Figure 1.

information to calculate a personalized (class-specific) pricing scheme. Therefore, as the difference in the VOT between classes becomes larger, we expect that OPS will provide a much better solution compared to the CPURR scheme in terms of the expected total monetary cost of the drivers. Note that in Figure 3, the values presented for the OPS are local optimum solutions while the values presented for the CPURR scheme are global optimum solutions obtained by using the nonconvex optimization solver BARON.

4.2. Sioux Falls Network.

The Sioux Falls network (LeBlanc et al. 1975) constitutes a benchmark in the transportation research field. It consists of 24 nodes and 76 links. For the needs of our experiments, we assumed that the cost of each route corresponds to travel time and can be described by a Bureau of Public Roads (BPR) function (Sheffi 1985) of the form:

$$C_{IT}(X_{lp}, X_{IT}) = \epsilon_a + \epsilon_b \left( \frac{X_{lp} + 3X_{IT}}{\epsilon_c} \right)^4$$

where $\epsilon_a$, $\epsilon_b$ and $\epsilon_c$ are constants and their values were chosen similar to the ones adopted in Kordonis et al. (2019). Without loss of generality, we assumed that the number of passenger vehicles at each link of the Sioux Falls network was constant and equal to the values used in Kordonis et al. (2019). To retain computational tractability, we further assumed that there are 6 available OD pairs for the truck drivers, namely $(O_1, D_7), (O_1, D_{11}), (O_{10}, D_{11}), (O_{10}, D_{20}), (O_{15}, D_{5})$ and $(O_{24}, D_{10})$ and that each truck driver chooses his/her route among the 10 least congested routes corresponding to his/her desired OD pair. In this experiment, we assumed that the truck drivers choose their class
among two available options with VOT $s_1 = 200 \frac{\$/hr}{hr}$ and $s_2 = 50 \frac{\$/hr}{hr}$, respectively. Their demand was assumed to take one of the following two equiprobable values:

$$d_1 = \begin{bmatrix} 3 & 4.5 & 6 & 3 & 14 & 3.6 \\ 1 & 2.8 & 5.4 & 7 & 9 & 2 \end{bmatrix}, d_2 = \begin{bmatrix} 5 & 1.8 & 3.9 & 15 & 6.4 & 2.4 \\ 6 & 5.5 & 1.8 & 6.5 & 11 & 6 \end{bmatrix}$$

where each column of $d_1$ and $d_2$ corresponds to the demand of truck drivers for each OD pair and each row denotes a different class of users. In Figure 4, Figure 5 and Figure 6, we plot the expected total travel time of the truck drivers $E[T_{tr}]$, their expected total monetary cost $E[T_{mon}^{tr}]$ as well as the expected total travel time of the network (passenger vehicles + trucks) for different values of the weighting factor $\lambda \in [0,1]$.

![Figure 4](image)

**Figure 4**  The expected total travel time of the truck drivers $E[T_{tr}]$ for different values of the weighting factor $\lambda$ in the Sioux Falls network.

As can be observed in Figure 4, as the value of the weighting factor $\lambda$ increases, the expected travel time of the truck drivers $E[T_{tr}]$ decreases. It is worth mentioning that for all values of $\lambda$, the OPS solution closely follows the SO solution. Additionally, the AOPS solution can significantly decrease $E[T_{tr}]$ compared to the CPURR solution, especially for $\lambda > 0.25$.

In Figure 5, we observe that the expected total monetary cost of the truck drivers $E[T_{mon}^{tr}]$ increases as the value of the weighting factor $\lambda$ increases for both SO, OPS and AOPS solutions. On the other hand, we observe that for $\lambda > 0.25$, $E[T_{mon}^{tr}]$ does not significantly change for the CPURR solution. This was expected since CPURR uses a uniform refunding scheme to distribute the fees collected from congestion pricing. Note also that AOPS has a smaller increase rate compared to the SO and the OPS solutions and it provides a smaller expected total monetary cost for the truck drivers when $\lambda = 1$. This can be explained by the fact that whenever $\lambda = 1$, SO and OPS only
Figure 5  The expected total monetary cost of the truck drivers $E[T_{tr}^{mon}]$ for different values of the weighting factor $\lambda$ in the Sioux Falls network.

minimize the expected total travel time of the network while on the other hand, AOPS applies a pricing scheme during which the expected total monetary benefits $E[T_{tr}^{mon,M} - E[T_{tr}^{mon,UE}]$, are shared to the users proportionally to the VOT of the class they belong. Therefore, a truck driver with higher VOT will get reimbursed with a bigger amount of money compared to a truck driver with lower VOT in case both of the drivers are assigned to a slower route. This behavior makes AOPS better contribute to the minimization of the expected total monetary cost of the truck drivers.

Figure 6  The expected total travel time of the network (passenger vehicles + truck drivers) $E[T_s]$ for different values of the weighting factor $\lambda$ in the Sioux Falls network.
In Figure 6, it is shown that as the value of the weighting factor $\lambda$ increases, the expected total travel time of the network $E[T_s]$ decreases. Furthermore, it can be observed that OPS makes $E[T_s]$ closely follow its corresponding value at the SO solution while at the same time, both OPS and AOPS can significantly reduce the expected total travel time of the network compared to the CPURR scheme.

5. Conclusion.
In this paper, we designed pricing schemes that can be applied in a general transportation network to alleviate traffic congestion. We particularly focused on the design of a coordination system for the truck drivers in the case of stochastic OD demand considering their heterogeneity in VOT. In contrast to previous efforts that proposed class-anonymous pricing schemes for the fixed OD demand case using the idea of Congestion Pricing with Uniform Revenue Refunding (CPURR), we designed personalized (class-specific) pricing schemes. More specifically, assuming that the users are asked to declare their OD pair and additionally pick their VOT from a set of $N$ available options, we proved the existence of Pareto-improving and revenue-neutral (budget balanced) on average pricing schemes that can additionally guarantee that every driver will have an incentive to truthfully declare his/her VOT. We showed that the Optimum Pricing Scheme (OPS) can be calculated by solving a nonconvex optimization problem and we additionally proposed an Approximately Optimum Pricing Scheme (AOPS) to approximate the solution of the OPS. Finally, we experimentally showed that both OPS and AOPS can significantly reduce the expected total travel time and the expected total monetary cost of the users and concurrently approximate the SO solution.

Acknowledgments
Funding: This work has been supported by the National Science Foundation Awards CPS #1545130 and CNS-1932615.
References


