Name _____ Econ 186 Early Fall 2015

Final

You must show all of your work to get full credit. There are a total of 90 points.

1) Find the values of x_1 and x_2 that maximize the function $C = -x_1^2 - x_1x_2 - x_2^2$ subject to $x_1 - 2x_2 \leq -1$ and $2x_1 + x_2 \leq 2$ (15 points).

2) Consider the matrix

$$A = \begin{bmatrix} 1 & 5 & 2 \\ 1 & 1 & 7 \\ 0 & -3 & 4 \end{bmatrix}$$

a) Find A^{-1}, the inverse of A. (8 points)

b) Confirm that the inverse you found in part a) is actually the inverse by showing that $A^{-1}A = I$ (4 points)

3) Consider the following probability density function of the random variable X:

$$f(x) = \begin{cases} \frac{1}{\sqrt{x}} & 0 \leq x \leq \frac{1}{4} \\ 0 & otherwise \end{cases}$$

a) Show that the total probability is equal to 1 (the axiom of probability holds for this pdf) (2 points).

b) Find $E(X)$ (2 points).

c) Find $Var(X)$ (2 points).

d) Now consider the probability density function of the random variable Y

$$g(y) = \begin{cases} 3e^{3y} & y < 0 \\ 0 & otherwise \end{cases}$$

Find the mean and variance of Y using the moment generating function (4 points).

e) Find $Var\,(3X + 4Y + 1)$. Assume X and Y are not independent, and that $Cov(X, Y) = 2$ (2 points).

4) Consider the function $U = 5x^2 + 3xy + 2y^2$

a) Compute the total differential of the function dU (3 points).

b) Compute the second-order total differential d^2U (3 points).

c) Find the discriminant (determinant with second order partial derivatives of U) (3 points).

d) Is d^2U positive or negative definite? Why? (3 points).

5) An individual gains utility by consuming two goods, x_1 and x_2. Their utility function is Cobb-Douglas:

$$u = x_1^{0.4} x_2^{0.5}$$

x_1 costs \$3 and x_2 costs \$4 per unit. The individual has a total of \$108 to spend and since they only get utility from these two goods, they spend all of their money on buying x_1 and x_2.

a) If the individual seeks to maximize their utility, write the Lagrangian function for this constrained optimization problem (2 points).

b) Find the first-order conditions and solve for the optimal values of x_1 and x_2 (x_1^* *and* x_2^*), respectively (4 points).

c) Show that x_1^* and x_2^* are the values that maximize utility using the second-order sufficient conditions (4 points).

6) Solve the following differential equations (solve for the explicit solution $y(x)$).

a) $xy' - 2y = x^2$ (6 points).

b) $y' = \frac{3x^2+4x-4}{2y-4}$ $\qquad y(1) = 3$ $\qquad$ (6 points).
(Hint: To obtain the explicit solution, treat the $x's$ as constants and use the quadratic formula $y = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$)

7) Find the third order Taylor polynomial of $f(x) = (x+1)^{\frac{1}{2}}$ around $x = 0$ (7 points).

8) Suppose you are a researcher trying to determine the interest rate that banks in the United States are paying on their deposits. To do this, you find out the interest rate that 31 banks are paying on deposits and calculate the mean interest rate as 2.3 (call it $\overline{X}$), with a variance of 25. Due to quantitative easing, you suspect that the true population mean interest rate is 0 (call it μ). So, you want to test whether your sample mean is different from 0 in either direction.

a) Set up the null and alternative hypotheses (2 points).

b) Calculate the t-statistic and perform a t-test at the 5% significance level (2 points).

c) Approximate the p-value using the t-table (2 points).

d) Interpret the p-value (2 points).

e) Construct a 99% confidence interval. Can you reject the null hypothesis at the 99% confidence level? (2 points)

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
	Confidence Level										