Name ________________________________ Econ 186 Fall 2014

Final

You must show all of your work to get full credit. There are a total of 110 points.

1) Consider the utility function $U(M,C) = \overline{U}$ which gives the different combinations of cookies and milk that provide a level of utility equal to the constant $\overline{U}$.

a) Find $\frac{dM}{dC}$ (5 points).

b) Now, let U have a particular functional form $U(M,C) = \frac{1}{4}ln(M) + \frac{3}{4}ln(C)$. Find $\frac{dM}{dC}$ and evaluate the derivative at the point $M = 8, C = 1$ (5 points).

2)

a) Find the inverse of the matrix (6 points)

$$P = \begin{bmatrix} 6 & 1 \\ 1 & -1 \end{bmatrix}$$

b) Show that $P^{-1}AP$ is a diagonal matrix where A is another matrix (6 points)

$$A = \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}$$

3) Consider the function $U = 5x^2 + 3xy + 2y^2$

a) Compute the total differential of the function dU (3 points).

b) Compute the second-order total differential d^2U (3 points).

c) Find the discriminant (determinant with second order partial derivatives of U) (3 points).

d) Is d^2U positive or negative definite? Why? (3 points).

4) Find the third order Taylor polynomial of $f(x) = (x+1)^{\frac{1}{2}}$ around $x = 0$ (8 points).

5) Consider the following probability density function of the random variable X:

$$f(x) = \begin{cases} \frac{1}{\sqrt{x}} & 0 \leq x \leq \frac{1}{4} \\ 0 & otherwise \end{cases}$$

a) Show that the total probability is equal to 1 (the axiom of probability holds for this pdf) (3 points).

b) Find $E(X)$ (3 points).

c) Find $Var(X)$ (3 points).

d) Now consider the probability density function of the random variable Y

$$g(y) = \begin{cases} e^{3y} & y < 0 \\ 0 & otherwise \end{cases}$$

Find the mean and variance of Y using the moment generating function (3 points).

e) Find $Var\left(3X + 4Y + 1\right)$. Assume X and Y are correlated, and that $Cov(X, Y) = 2$ (3 points).

6) An individual gains utility by consuming two goods, x_1 and x_2. Their utility function is Cobb-Douglas:

$$u = x_1^{0.4} x_2^{0.5}$$

x_1 costs \$3 and x_2 costs \$4 per unit. The individual has a total of \$108 to spend and since they only get utility from these two goods, they spend all of their money on buying x_1 and x_2.

a) If the individual seeks to maximize their utility, write the Lagrangian function for this constrained optimization problem (3 points).

b) Find the first-order conditions and solve for the optimal values of x_1 and x_2 $(x_1^* \; and \; x_2^*)$, respectively (8 points).

c) Show that x_1^* and x_2^* are the values that maximize utility using the second-order sufficient conditions (4 points).

7) Solve the following differential equations (solve for the explicit solution $y(x)$).

a) $xy' - 2y = x^2$ (8 points).

b) $y' = \frac{3x^2+4x-4}{2y-4}$ $\quad y(1) = 3$ (9 points).
(Hint: To obtain the explicit solution, treat the $x's$ as constants and use the quadratic formula.)

8) Suppose you are a researcher trying to determine the effect of quantitative easing on international capital flows. Specifically, you are trying to determine the correlation between a change in the interest rate and capital flows back into the United States. You run an ordinary least squares regression on a cross-section of 31 countries and find a coefficient of $\widehat{B} = 2.3$ with a sample variance of 25. You want to test whether this effect is statistically different than 0 in either direction.

a) Set up the null and alternative hypotheses (3 points).

b) Calculate the t-statistic and perform a t-test at the 5% significance level (3 points).

c) Approximate the p-value using the t-table (3 points).

d) Interpret the p-value (3 points).

e) Construct a 99% confidence interval. Can you reject the null hypothesis at the 99% confidence level? (3 points)

9) Suppose a firm has a Cobb-Douglas production function $Q = Q(K, L) = K^{\alpha} L^{\beta}$.

a) Assume $\alpha = 1$ and $\beta = \frac{1}{2}$. What is the degree of homogeneity of Q? What returns to scale is the firm getting? (2 points)

b) Assume $\alpha = \frac{1}{4}$ and $\beta = \frac{1}{4}$. What is the degree of homogeneity of Q? What returns to scale is the firm getting? (2 points)

c) Now suppose that the firm's production also depends on time so that their production function is $Q = Q(K, L, t) = K^{\alpha} L^{\beta} + t$. Assume $\alpha = \frac{1}{4}$ and $\beta = \frac{1}{3}$. What is the degree of homogeneity of Q? (2 points)