Homework Day 13 Solutions - ECON 186

Problem 1.

To sketch the direction field, let's first find where the differential equation will have horizontal tangent lines, that is, the values for which the first derivative is equal to 0.

$$\frac{dv}{dt} = 9.8 - 0.196v \to 0 = 9.8 - 0.196v \to v = 50$$

Now, the graph is split into two regions, $v < 50$ and $v > 50$. So, we want to check the slope of the tangent lines, that is, the sign and magnitude of the first derivative at various values. For the lower region, let's check $v = 49$ and $v = 0$.

$$\frac{dv}{dt} = 9.8 - 0.196(49) = 0.196$$

$$\frac{dv}{dt} = 9.8 - 0.196(0) = 9.8$$

So, we know that the slope farther away from 50 is quite steep, but is nearly horizontal near $v = 50$. Furthermore, the derivative is always positive, so v is increasing as $t \to \infty$ in this region.

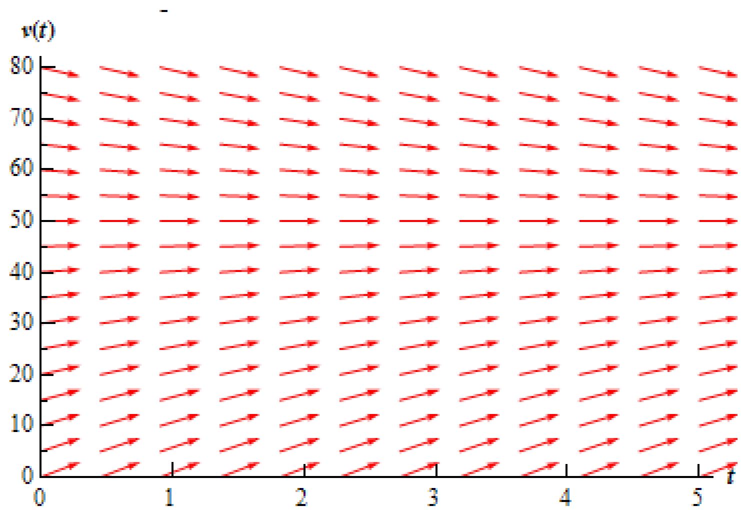
For $v > 50$, let's check $v = 51$ and $v = 80$.

$$\frac{dv}{dt} = 9.8 - 0.196(51) = -0.196$$

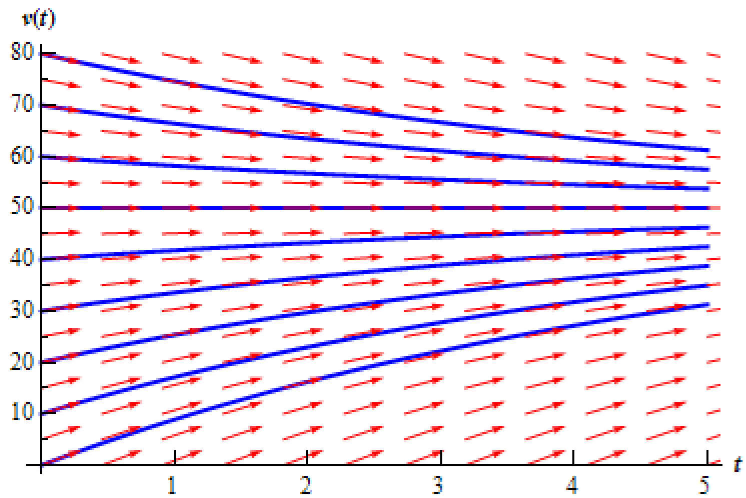
$$\frac{dv}{dt} = 9.8 - 0.196(80) = -5.88$$

The upper region is nearly exactly the same, but downward sloping. Thus, an overall description is that no matter what the initial condition of v is, v will be approaching $v = 50$ as $t \to \infty$, slowing down as v nears $v = 50$.

The direction field will look something like this



Additionally, we can also sketch some integral curves that will tell us how the solutions are moving as $t \to \infty$ given various initial conditions of v.



Problem 2.

a) First order linear nonhomogeneous

b)
First, put the differential equation in the proper form

$$\frac{dy}{dt} = 20 + 2y \to \frac{dy}{dt} - 2y = 20$$

Then apply the formulas

$$\mu(t) = e^{\int -2dt} = e^{-2t}$$

$$y(t) = \frac{\int e^{-2t}(20)dt + c}{e^{-2t}} = \frac{\frac{20}{-2}e^{-2t} + c}{e^{-2t}} = -10 + ce^{2t}$$

c)
We must find the value of c that satisfies the initial condition. To do this, simply plug in the initial condition into the general solution:

$$y(0) = -10 + ce^{2(0)} = 3 \to -10 + c = 3 \to c = 13$$

So, the particular solution to the initial value problem is

$$y(t) = -10 + 13e^{2t}$$

Problem 3.

$$\frac{dy}{dt} + \frac{1}{t}y - 2 = 3t + t^2$$

First, put the differential equation in the proper form

$$\frac{dy}{dt} + \frac{1}{t}y = t^2 + 3t + 2$$

Apply the formulas

$$\mu(t) = e^{\int \frac{1}{t}dt = e^{ln|t|}} = |t|$$

Since we are looking only at values of t greater than 0, we can drop the absolute value sign. Then,

$$y(t) = \frac{\int \mu(t)g(t)dt + c}{\mu(t)} = \frac{\int t\left(t^2 + 3t + 2\right)dt + c}{t} = \frac{\int \left(t^3 + 3t^2 + 2t\right)dt + c}{t}$$

$$= \frac{\frac{t^4}{4} + t^3 + t^2 + c}{t} = \frac{t^3}{4} + t^2 + t + ct^{-1}$$

Problem 4.

First, notice that this is a separable differential equation, so put it in the proper form

$$e^y dy = (2x - 4)\, dx$$

Next, integrate both sides

$$\int e^y dy = \int (2x - 4)\, dx$$

$$e^y = x^2 - 4x + c$$

Then, we can solve for an explicit solution:

$$y(x) = ln\left(x^2 - 4x + c\right)$$

Applying the initial condition, we get

$$0 = ln\,(25 - 20 + c) \to e^0 = 1 = 5 + c \to c = -4$$

So the particular solution is

$$y(x) = ln\left(x^2 - 4x - 4\right)$$