Homework Day 12 Solutions - ECON 186

Problem 1.

a)

$$H_0 : \widehat{\beta} = 0$$
$$H_1 : \widehat{\beta} \neq 0$$

b)

$$t = \frac{\sqrt{n}\left(\widehat{\beta} - \beta\right)}{s} = \frac{10\,(5.67 - 0)}{29} = 1.96$$

Now, we want to check whether this is significant at the 5% level, which with 99 degrees of freedom gives us $t_c = 1.984$.

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$$t_c > t \rightarrow 1.984 > 1.96$$

So, the t-statistic is not significant at the 5% level! Although just barely.

c)

To find an approximation for the p-value, all we must do is look up the t-statistic and degrees of freedom on the t-table. It is clear to see that our t-statistic is just barely above the 5% significance level for 100 degrees of freedom (although the actual degrees of freedom is 99). The p-value looks to be about 0.055.

Using a p-value calculator, we can find that the p-value is 0.0528.

d) A p-value of 0.0528 means that if it was true that air pollution has no effect on the number of asthma hospitalizations, if we took an infinite number of samples from the population, the effect size we would get would be bigger than the one we found in our one sample 5.28% of the time. So, in conventional significance level terms, we can say that our result is significant at the 10% level, but not at 5%.

Problem 2.

a)

The law of large numbers tells us that the sample mean converges in probability to the population mean of the distribution X. The formula for the sample mean of X would be

$$\overline{X} = \frac{\overline{X}_1 + \overline{X}_2 + ... + \overline{X}_{1000}}{1000}$$

The number of trials is very likely large enough that the sample mean $\overline{X}$ would be quite close to $E(X) = \frac{1}{2}$

b)
The Central Limit Theorem tells us that if the sample size is large enough (we perform enough trials), the distribution formed by the $X_i's$ should be approximately normal. This is very likely to be enough trials that the distribution would look close to normal.

Problem 3.

a)

$$\overline{X} = \frac{\sum X_i}{n} = \frac{4 + 4.19 + 4.28 + 4.93 + 5.07 + 5.64 + 6.67 + 8.41 + 9.43 + 11.43 + 13.92 + 7.09 + 6.57}{23}$$

$$\frac{+6.44 + 6.35 + 5.26 + 5.65 + 6.03 + 3.26 + 3.22 + 2.78 + 1.8 + 2.35}{23} = 5.86$$

$$s_x = \sqrt{\frac{\sum \left(X_i - \overline{X}\right)^2}{n-1}} = \sqrt{\frac{(4-5.86)^2 + (4.19-5.86)^2 + (4.28-5.86)^2 + (4.93-5.86)^2 +}{22}}$$

$$\sqrt{\frac{+(5.07-5.86)^2 + (5.64-5.86)^2 + (6.67-5.86)^2 + (8.41-5.86)^2}{22}}$$

$$\sqrt{\frac{(9.43-5.86)^2 + (11.43-5.86)^2 + (13.92-5.86)^2 + (7.09-5.86)^2 +}{22}}$$

$$\sqrt{\frac{(6.57-5.86)^2 + (6.44-5.86)^2 + (6.35-5.86)^2 + (5.26-5.86)^2 +}{22}}$$

$$\sqrt{\frac{(5.65-5.86)^2 + (6.03-5.86)^2 + (3.26-5.86)^2 + (3.22-5.86)^2 +}{22}}$$

$$\sqrt{\frac{+(2.78-5.86)^2 + (1.8-5.86)^2 + (2.35-5.86)^2}{22}} = 2.87$$

b)

$$\overline{X} = \frac{\sum X_i}{n} = \frac{7.35 + 6.16 + 6.21 + 6.85 + 7.56 + 7.99 + 7.61 + 7.42 + 13.01+}{28}$$

$$\frac{+11.10 + 12.46 + 10.62 + 7.67 + 8.39 + 8.85 + 8.49 + 8.55 + 7.86+}{28}$$

$$\frac{+7.01 + 5.87 + 5.02 + 4.61 + 4.01 + 4.27 + 4.29 + 4.80 + 4.63 + 3.66}{28} = 7.23$$

$$s_x = \sqrt{\frac{\sum \left(X_i - \overline{X}\right)^2}{n-1}} = \sqrt{\frac{(7.35-7.23)^2 + (6.16-7.23)^2 + (6.21-7.23)^2 + (6.85-7.23)^2}{27}}$$

$$\sqrt{\frac{+(7.56-7.23)^2 + (7.99-7.23)^2 + (7.61-7.23)^2 + (7.42-7.23)^2 +}{27}}$$

$$\sqrt{\frac{(13.01-7.23)^2+(11.10-7.23)^2+(12.46-7.23)^2+(10.62-7.23)^2+}{27}}$$

$$\sqrt{\frac{(7.67-7.23)^2+(8.39-7.23)^2+(8.85-7.23)^2+(8.49-7.23)^2+}{27}}$$

$$\sqrt{\frac{(8.55-7.23)^2+(7.86-7.23)^2+(7.01-7.23)^2+(5.87-7.23)^2+}{27}}$$

$$\sqrt{\frac{+(5.02-7.23)^2+(4.61-7.23)^2+(4.01-7.23)^2+}{27}}$$

$$\sqrt{\frac{+(4.27-7.23)^2+(4.29-7.23)^2+(4.80-7.23)^2+(4.63-7.23)^2+(3.66-7.23)^2}{27}}=2.47$$

c)
Well, it looks like the mean for the Republicans is higher. This does not tell us anything causal about the relationship between political party and interest rates for two interlocking reasons. First, interest rates are controlled by the Federal Reserve and the president has no direct control over them whatsoever. Second, events such as the oil crisis in the 1970's and the Great Recession of 2008 gave rise to very high and low interest rates, respectively, which really had nothing to do with the political party of the president at the time.

d)
Squaring the standard deviations, we can find that $var_D = 8.24$ and $var_R = 6.1$.

$$t=\frac{\overline{X}_D-\overline{X}_R}{\sqrt{\frac{var_D}{n_D}+\frac{var_R}{n_R}}}=\frac{5.86-7.23}{\sqrt{\frac{8.24}{23}+\frac{6.1}{28}}}=\frac{-1.37}{0.759}=-1.81$$

At the 95% confidence level with $23+28-2=49$ degrees of freedom, $t_c \approx 2.01$.

$$2.01 > |-1.81|$$

So, the means are actually not statistically different at the 95% confidence level (5% significance level).