

Homework Day 11 Solutions - ECON 186

Problem 1.

$$E(X) = \sum_{x=1}^{\infty} x \frac{1}{x(x+1)} = \sum_{x=1}^{\infty} \frac{1}{x+1} = \infty$$

Problem 2.

$$E(X) = 18 \left(\frac{20}{50}\right) + 19 \left(\frac{22}{50}\right) + 20 \left(\frac{4}{50}\right) + 21 \left(\frac{3}{50}\right) + 25 \left(\frac{1}{50}\right) = 18.92$$

Problem 3.

a)

$$E(X) = \int_0^1 x (x^{\frac{1}{2}}) dx = \int_0^1 x^{\frac{3}{2}} dx = \frac{2}{5} x^{\frac{5}{2}} \Big|_0^1 = \frac{2}{5}$$

$$E(X^2) = \int_0^1 x^2 (x^{\frac{1}{2}}) dx = \int_0^1 x^{\frac{5}{2}} dx = \frac{2}{7} x^{\frac{7}{2}} \Big|_0^1 = \frac{2}{7}$$

$$\text{Var}(X) = E(X^2) - (E(X))^2 = \frac{2}{7} - \left(\frac{2}{5}\right)^2 = \frac{2}{7} - \frac{4}{25} = 0.12571428571$$

b)

Problem 4.

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = 10 - 9 = 1$$

$$\text{Var}(Y) = E(Y^2) - [E(Y)]^2 = 29 - 4 = 25$$

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y) = 0 - 6 = -6$$

$$\rho(X, Y) = \frac{-6}{(1)(5)} = -\frac{6}{5}$$

which is not possible since $-1 < \rho(X, Y) < 1$

Problem 5.

Recall that the Bernoulli distribution is given by the pmf

$$f(x) = Pr(X = x) = \begin{cases} p & \text{for } x = 1 \\ q = 1 - p & \text{for } x = 0 \\ 0 & \text{otherwise} \end{cases}$$

Then,

$$\psi(t) = E(e^{tx}) = pe^{t*1} + qe^{t*0} = pe^t + q$$

$$\psi'(t) = pe^t$$

$$\psi''(t) = pe^t$$

$$E(X) = \psi'(0) = p$$

$$E(X^2) = \psi''(0) = p$$

$$Var(X) = E(X^2) - [E(X)]^2 = p - p^2 = p(1 - p)$$

Interestingly, every moment of the Bernoulli distribution is equal to p .

Problem 6.

$E(X) = 5$, $E(Y) = 3$, $Var(X) = 6$, $Var(Y) = 2$, $Cov(X, Y) = 10$.

$$E(3Y - 2X + 7) = 3E(Y) - 2E(X) + E(7) = 3(3) - 2(5) + 7 = 9 - 10 + 7 = 6$$

$$Var(5X - Y + 2) = 25Var(X) + Var(Y) + Var(2) + 2(5)(-1)Cov(X, Y)$$

$$= 25(6) + 2 - 10(10) = 52$$

Problem 7.

a)

$$Pr(X \leq 3) = Pr\left(\frac{X-1}{2} \leq \frac{3-1}{2}\right) = Pr(Z \leq 1) = 0.8413$$

b)

$$\begin{aligned} Pr(2 < X < 5) &= Pr\left(\frac{2-1}{2} < \frac{X-1}{2} < \frac{5-1}{2}\right) = Pr\left(\frac{1}{2} < Z < 2\right) \\ &= Pr(Z < 2) - Pr(Z < \frac{1}{2}) = 0.9772 - 0.6915 = .2857 \end{aligned}$$

c)

$$\begin{aligned} Pr(1 \leq -2X + 3 \leq 8) &= Pr(-2 < -2X < 5) = Pr\left(-\frac{5}{2} < X < 1\right) \\ &= Pr\left(\frac{-\frac{5}{2}-1}{2} < \frac{X-1}{2} < \frac{1-1}{2}\right) = Pr(-1.75 < Z < 0) \\ &= Pr(Z < 0) - (1 - Pr(Z < 1.75)) = 0.5 - 1 + .9599 = .4599 \end{aligned}$$