Homework Day 10 Solutions - ECON 186

Problem 1. Chiang and Wainwright 14.2 #1(a, b, c, e), 2(a,f)

1)

a)

$$\int 16x^{-3}dx = -8x^{-2} + c$$

b)

$$\int 9x^8 dx = x^9 + c$$

c)

$$\int \left(x^5 - 3x\right) dx = \int x^5 dx - 3\int x dx = \frac{x^6}{6} - \frac{3}{2}x^2 + c$$

e) Let $u = x^2 + 1$, then $du = 2xdx$, so $dx = \frac{du}{2x}$.

$$\int \frac{4x}{x^2+1}dx = 2\int \frac{1}{u}du = 2ln\, u + c = 2ln\left(x^2+1\right) + c$$

2)

a)

$$\int 13e^x dx = 13e^x + c$$

f) Let $u = x^2 + 9$, then $du = 2xdx$, so $dx = \frac{du}{2x}$. Then,

$$\int xe^{x^2+9}dx = \frac{1}{2}\int e^u du = \frac{1}{2}e^u + c = \frac{1}{2}e^{x^2+9} + c$$

Problem 2. Chiang and Wainwright 14.3 #1(a, d), 2(b)

1)

a)

$$\int_1^3 \frac{1}{2}x^2 dx = \frac{1}{6}x^3\Big|_1^3 = \frac{1}{6}\left(3^3 - 1^3\right) = \frac{1}{6}(26) = \frac{13}{3}$$

d)

$$\int_{2}^{4}\left(x^3-6x^2\right)dx=\left(\frac{x^4}{4}-2x^3\right)\Big|_2^4=\frac{4^4}{4}-2\left(4^3\right)-\frac{2^4}{4}+2\left(2^3\right)$$
$$=\frac{256}{4}-2(64)-\frac{16}{4}+2(8)=64-128-4+16=-52$$

2)

b) Let $u=x+2$, then $du=dx$. Then,

$$\int_{-1}^{e-2}\frac{dx}{x+2}=\int_{*}^{*}\frac{du}{u}=ln\,(x+2)|_{-1}^{e-2}=ln\,e-ln\,1=1-0=1$$

Problem 3. Chiang and Wainwright 14.4 #3(a, d)

3)

a)

$$\int_{0}^{\infty}e^{-rt}dt=\lim_{b\to\infty}\int_{0}^{b}e^{-rt}dt=\lim_{b\to\infty}-\frac{e^{-rt}}{r}\Big|_0^b$$
$$=\lim_{b\to\infty}-\frac{e^{-rb}}{r}+\frac{e^{-r0}}{r}=0+\frac{1}{r}=\frac{1}{r}$$

d)

$$\int_{-\infty}^{0}e^{rt}dt=\lim_{a\to-\infty}\int_{a}^{0}e^{rt}dt=\lim_{a\to-\infty}\frac{e^{rt}}{r}\Big|_a^0$$
$$=\lim_{a\to-\infty}\frac{e^{ra}}{r}+\frac{e^{-r0}}{r}=0+\frac{1}{r}=\frac{1}{r}$$

Problem 4.

Let $c=0$ and then evaluate the first integral. By letting $u=x^2+1$ so that $du=2xdx$, we find that

$$\int_{-\infty}^{0}\frac{2x}{\left(x^2+1\right)^2}dx=\lim_{a\to-\infty}\int_{a}^{0}\frac{2x}{\left(x^2+1\right)^2}dx=\lim_{a\to-\infty}\int_{*}^{*}\frac{1}{u^2}du=$$
$$\lim_{a\to-\infty}-\frac{1}{u}\Big|_*^*=\lim_{a\to-\infty}-\frac{1}{x^2+1}\Big|_a^0=\lim_{a\to-\infty}\left(-1+\frac{1}{a^2+1}\right)=-1$$

Then, evaluate the second integral.

$$\int_{0}^{\infty}\frac{2x}{\left(x^2+1\right)^2}dx=\lim_{b\to\infty}\int_{0}^{b}\frac{2x}{\left(x^2+1\right)^2}dx=\lim_{b\to\infty}\int_{*}^{*}\frac{1}{u^2}du$$
$$=\lim_{b\to\infty}-\frac{1}{u}\Big|_*^*=\lim_{b\to\infty}-\frac{1}{x^2+1}\Big|_0^b=\lim_{b\to\infty}\left(-\frac{1}{b^2+1}+1\right)=1$$

So, both parts of the integral are convergent, so the original integral is convergent and its value is equal to

$$\int_{-\infty}^{\infty} \frac{2x}{\left(x^2+1\right)^2} dx = \int_{-\infty}^{0} \frac{2x}{\left(x^2+1\right)^2} dx + \int_{0}^{\infty} \frac{2x}{\left(x^2+1\right)^2} dx = -1 + 1 = 0$$

Problem 5.

In order for this pmf to satisfy the axioms of probability, it must be that $\sum_{x \in X} f(x) = 1$, so using the infinite geometric series formula,

$$\sum_{x=0}^{\infty} f(x) = \sum_{x=0}^{\infty} \frac{c}{2^x} = 2c = 1 \rightarrow c = \frac{1}{2}$$

Problem 6.

a)

$$Pr\left(X < \frac{1}{2}\right) = \frac{4}{3} \int_{0}^{\frac{1}{2}} \left(1 - x^3\right) dx = \frac{4}{3} \left(x - \frac{x^4}{4}\right)\Bigg|_{0}^{\frac{1}{2}} = \frac{4}{3} \left(\frac{1}{2} - \frac{\left(\frac{1}{2}\right)^4}{4}\right) = 0.6458$$

b)

$$Pr\left(\frac{1}{4} < X < \frac{3}{4}\right) = \frac{4}{3} \int_{\frac{1}{4}}^{\frac{3}{4}} \left(1 - x^3\right) dx = \frac{4}{3} \left(x - \frac{x^4}{4}\right)\Bigg|_{\frac{1}{4}}^{\frac{3}{4}} = \frac{4}{3} \left(\frac{3}{4} - \frac{\left(\frac{3}{4}\right)^4}{4} - \frac{1}{4} + \frac{\left(\frac{1}{4}\right)^4}{4}\right) = 0.5625$$

c)

$$Pr\left(X > \frac{1}{3}\right) = \frac{4}{3} \int_{\frac{1}{3}}^{1} \left(1 - x^3\right) dx = \frac{4}{3} \left(x - \frac{x^4}{4}\right)\Bigg|_{\frac{1}{3}}^{1} = \frac{4}{3} \left(1 - \frac{1^4}{4} - \frac{1}{3} + \frac{\left(\frac{1}{3}\right)^4}{4}\right) = 0.5597$$