Homework Day 6 Solutions - ECON 186

Problem 1. Chiang and Wainwright 9.5 #1(a, e), 3(a) (Hint: Use equation (9.14) and find the first five terms of the Taylor Series)

1)

a)

$$5! = 1 * 2 * 3 * 4 * 5 = 120$$

e)

$$\frac{(n+2)!}{n!} = \frac{1 * 2 * ... * n * n + 1 * n + 2}{1 * 2 * ... * n} = (n+1)(n+2)$$

3)

a)

$$\phi(-2) = \frac{1}{3}, \phi^{'}(-2) = \frac{1}{9}, \phi^{''}(-2) = \frac{2}{27}, \phi^{'''}(-2) = \frac{6}{81}, \phi^{(4)}(-2) = \frac{24}{243}$$

So, by (9.14),

$$\phi(x) = \frac{1}{3} + \frac{1}{9}(x+2) + \frac{1}{27}(x+2)^2 + \frac{1}{81}(x+2)^3 + \frac{1}{243}(x+2)^4 + R_4$$

$$= \frac{1}{243}\left(211 + 131x + 51x^2 + 11x^3 + x^4\right) + R_4$$

Problem 2.

a) First, take logs of both sides of the production function

$$ln\ y_t = ln\ a_t + \alpha ln\ k_t + (1-\alpha)\, ln\ n_t$$

Next, find the first order Taylor series expansion around the steady state values

$$ln\ y^* + \frac{1}{y^*}(y_t - y^*)$$

$$= ln\ a^* + \frac{1}{a^*}(a_t - a^*) + \alpha ln\ k^* + \frac{\alpha}{k^*}(k_t - k^*) + (1-\alpha)\, ln\ n^* + \frac{(1-\alpha)}{n^*}(n_t - n^*)$$

Note that $ln\ y^* = ln\ a^* + \alpha ln\ k^* + (1-\alpha)\, ln\ n^*$, so these terms cancel, and we are left with

$$\frac{1}{y^*}(y_t - y^*) = \frac{1}{a^*}(a_t - a^*) + \frac{\alpha}{k^*}(k_t - k^*) + \frac{(1-\alpha)}{n^*}(n_t - n^*)$$

b) Define the "tilde" variables as percent deviations from the steady state values. Specifically,

$$\widetilde{y}_t = \frac{y_t - y^*}{y^*}$$

$$\widetilde{a}_t = \frac{a_t - a^*}{a^*}$$

$$\widetilde{k}_t = \frac{k_t - k^*}{k^*}$$

$$\widetilde{n}_t = \frac{n_t - n^*}{n^*}$$

Then, we can write the log-linearization as

$$\widetilde{y}_t = \widetilde{a}_t + \alpha \widetilde{k}_t + (1-\alpha)\, \widetilde{n}_t$$