Homework Day 4 Solutions - ECON 186

Problem 1. Chiang and Wainwright 6.2 #1

1)

a)

$$\frac{\triangle y}{\triangle x} = \frac{4\left(x + \triangle x\right)^2 + 9 - \left(4x^2 + 9\right)}{\triangle x} = 8x + 4\triangle x$$

b)

$$\frac{dy}{dx} = f^{'}(x) = 8x$$

c)

$$f^{'}(3) = 24$$

$$f^{'}(4) = 32$$

Problem 2. Chiang and Wainwright 6.4 #1,2,3

1)

$$q = \frac{v^2 + v - 56}{v - 7} = \frac{(v-7)(v+8)}{v-7} = v + 8 \qquad (v \neq 1)$$

Then,

$$\underset{v \to 7}{lim} \; v + 8 = 7 + 8 = 15$$

So, the right-hand side limit = left-hand side limit = 15.

Another way of seeing this is to not reduce the equation and plug in numbers to the left and right of 7. For example, plug in the values $v = 6.9, 6.999, 7.001, 7.1$

$$q(6.9) = 14.9,\; q(6.999) = 14.999$$

$$q(7.001) = 15.001,\ q(7.1) = 15.1$$

2) The function can be rewritten as

$$q = \left(v^3 + 6v^2 + 12v\right)/v = v^2 + 6v + 12 \qquad (v \neq 0)$$

a)

$$\lim_{v \to 0} q = 12$$

b)

$$\lim_{v \to 2} q = 2^2 + 6(2) + 12 = 4 + 12 + 12 = 28$$

c)

$$\lim_{v \to a} q = a^2 + 6a + 12$$

3)

a)

$$\lim_{v \to +\infty} q = \lim_{v \to +\infty} 5 - \lim_{v \to +\infty} \frac{1}{v} = 5 - 0 = 5$$

b)

$$\lim_{v \to -\infty} q = \lim_{v \to -\infty} 5 - \lim_{v \to -\infty} \frac{1}{v} = 5 - 0 = 5$$

Problem 3. Chiang and Wainwright 6.6 #3(a)

3)

a)

$$\lim_{v \to 0} = \frac{\lim_{v \to 0} (3v + 5)}{\lim_{v \to 0} (v + 2)} = \frac{5}{2}$$

Problem 4. Chiang and Wainwright 6.7 #2(a),3(a)

2)

a)

$$\underset{v \to N}{lim}\ q = N^2 - 5N - 2$$

3)

a)

$$\underset{v \to N}{lim}\ q = \frac{N+2}{N^2+2}$$

Problem 5. Chiang and Wainwright 7.2 #3(a, b, d, f), 7(a)

3)

a)

$$\left(9x^2 - 2\right)3 + (3x+1)\,18x = 27x^2 - 6 + 54x^2 + 18x = 3\left(27x^2 + 6x - 2\right)$$

b)

$$(3x+10)\,(12x-7) + \left(6x^2 - 7x\right)3 = 36x^2 - 21x + 120x - 70 + 18x^2 - 21x$$

$$= 54x^2 + 78x - 70$$

d)

$$(ax-b)\,(2cx) + cx^2a = 2acx^2 - 2bcx + acx^2 = cx\,(3ax - 2b)$$

f)

$$\left(x^2+3\right)\left(-1x^{-2}\right) + x^{-1}\,(2x) = -1 - 3x^{-2} + 2 = 1 - 3x^{-2}$$

7)

a)

$$\frac{x(2x)-(x^2+3)(1)}{x^2}=\frac{2x^2-x^2-3}{x^2}=1-3x^{-2}$$

Problem 6. Chiang and Wainwright 7.3#1, 3(a)

1)

$$\frac{dy}{dx}=\left(\frac{dy}{du}\right)\left(\frac{du}{dx}\right)=\left(3u^2+2\right)(-2x)=-6xu^2-4x=-6x\left[5-x^2\right]^2-4x$$

$$=-6x\left(25-10x^2+x^4\right)-4x=-6x^5+60x^3-154x$$

3)

a) Let $w=3x^2-13$, this implies that $\frac{dw}{dx}=6x$. Since $y=w^3$, we have

$$\frac{dy}{dx}=\frac{dy}{dw}\frac{dw}{dx}=3w^2(6x)=18x\left(3x^2-13\right)^2$$

Problem 7. Chiang and Wainwright 10.3 #3(c,d,e)

3)

c)

$$ln\left(\frac{3}{B}\right)=ln\,(3)-ln\,(B)$$

d)

$$ln\left(Ae^2\right)=ln\,(A)+ln\left(e^2\right)=ln\,(A)+2ln(e)=ln\,(A)+2$$

e)

$$ln\left(ABe^{-4}\right)=ln\,(A)+ln\,(B)+ln\left(e^{-4}\right)=ln\,(A)+ln\,(B)-4$$

Problem 8. Chiang and Wainwright 10.5 #1(e, f), 3 (d, f), 4(c)

1)

e)

$$\frac{dy}{dx} = (2ax + b)\, e^{ax^2+bx+c}$$

f)

$$\frac{dy}{dx} = xe^x + e^x = e^x(x+1)$$

3)

d)

$$\frac{dy}{dx} = 5\frac{2\,(t+1) * 1}{(t+1)^2} = \frac{10}{t+1}$$

f)

$$\frac{dy}{dx} = \frac{d}{dx}\left[ln\ \ x + 8ln\,(1-x)\right] = \frac{1}{x} + \frac{-8}{1-x} = \frac{1-9x}{x\,(1-x)}$$

4)

c)

$$\frac{dy}{dt} = 2\,(13)^{2t+3}\, ln\, 13$$