ECON 186 Class Notes: Linear Algebra

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Introduction to Matrices

- A very useful and compact way of writing systems of equations is to use matrices.
- Suppose we have the system of equations

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = d_1$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = d_2$$

$$..$$

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = d_m$$

Introduction to Matrices

- We may write these as a set of three matrices, where A is the set of coefficients, x is the set of variables, and d is the set of constants.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad d = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_m \end{bmatrix}$$

- Matrix A contains m rows and n columns, so we say that it has dimension $m \times n$.
- Notationally, the row number always comes before the column number, as in the above notation.

Introduction to Matrices

- Note: Matrices can only be used to represent linear equation systems. How well linear equations actually describe economic relationships depends on the specific situation, but linearity is often assumed in theoretical and empirical work, and is often shown to be a reasonable approximation.
- As a specific example, Ordinary Least Squares, the most basic and widely used tool in econometrics assumes linearity, but is quite uncommon that it would not be a reasonable and more intuitive approximation to any non-linear approach.

Introduction to Matrices Example

- Suppose we have the linear system

$$6x_1 + 3x_2 + x_3 = 22$$

$$x_1 + 4x_2 - 2x_3 = 12$$

$$4x_1 - x_2 + 5x_3 = 10$$

$$A = \begin{bmatrix} 6 & 3 & 1 \\ 1 & 4 & -2 \\ 4 & -1 & 5 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad d = \begin{bmatrix} 22 \\ 12 \\ 10 \end{bmatrix}$$

- We can then express the entire system of equations simply as $Ax = d$.

National Income Model Example

- Consider the national income model:

$$Y = C + I_0 + G_0$$

$$C = a + b(Y - T)$$

$$T = d + tY$$

- Express the national income model in the form $Ax = d$.

National Income Model Example

- First, rewrite the model in a form where the endogenous variables are separated from the exogenous and the endogenous variables are lined up.

$$\begin{array}{rrrcrcr} Y & & -C & = & I_0 & + & G_0 \\ -bY & +bT & +C & = & a & & \\ -tY & +T & & = & d & & \end{array}$$

- Now, it is clear to see that

$$A = \begin{bmatrix} 1 & 0 & -1 \\ -b & b & 1 \\ -t & 1 & 0 \end{bmatrix}, \qquad x = \begin{bmatrix} Y \\ T \\ C \end{bmatrix}, \qquad d = \begin{bmatrix} I_0 + G_0 \\ a \\ d \end{bmatrix}$$

Introduction to Vectors and Matrices

- A vector is simply a special case of a matrix.
  - A column vector is an $m \times 1$ matrix while a row vector is a $1 \times n$ matrix.
    - ★ Column vector: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$
    - ★ Row vector: $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$
- Geometrically, vectors can be interpreted as n-tuples, that is, a point in n-dimensional space.
  - As an example, the column vector $u = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ is interpreted as an arrow from the origin, the ordered pair $(0,0)$ to the ordered pair $(3,2)$.
  - This geometric interpretation is rarely used in economic applications, so we will not focus on this. Read pages 60-62 for discussion.