Homework Day 7 - ECON 186

Problem 1. Chiang and Wainwright 9.2 #1

#1
Find the stationary values of the following (check whether they are relative maxima or minima or inflection points), assuming the domain to be the set of all real numbers:
(a)$y = -2x^2+8x+7$ (b) $y = 5x^2+x$ (c)$y = 3x^2+3$ (d)$y = 3x^2-6x+2$

Problem 2. For problem 1, parts (a) and (b), determine whether each function is concave or convex (strictly?)

Problem 3. Chiang and Wainwright 9.3 #1, 6

#1
Find the second and third derivatives of the following functions:
(a)$ax^2 + bx + c$ (c)$\frac{3x}{1-x}(x \neq 1)$
(b)$7x^4 - 3x - 4$ (d)$\frac{1+x}{1-x}$
#6
A person who is neither risk-averse nor risk-loving (indifferent toward a fiar game) is said to be "risk-neutral".
(a)What kind of utility function would you use to characterize such a person?
(b)Using the die-throwing game detailed in the text, describe the relationship between $U(\$15)$ and EU for the risk-neutral person.

Problem 4. Chiang and Wainwright 9.4 #1, 3(b, c, d, e), 6

#1
Find the relative maxima and minima of y by the second-derivative test:
(a)$y = -2x^2 + 8x + 25$ (c)$y = \frac{1}{3}x^3 - 3x^2 + 5x + 3$
(b)$y = x^3 + 6x^2 + 9$ (d)$y = \frac{2x}{1-2x}(x \neq \frac{1}{2})$
#3
A firm has the following total-cost and demand functions:
$C = \frac{1}{3}Q^3 - 7Q^2 + 111Q + 50$
$Q = 100 - P$
(b)Write out the total-revenue function R in terms of Q.
(c)Formulate the total-profit function π in terms of Q.
(d)Find the profit-maximizing level of output Q^*.
(e)What is the maximum profit?
#6
A purely competitive firm has a single variable input L(labor), with the wage rate W_0 per period. Its fixed inputs cost the firm a total of F dollars per period. The price of the product is P_0.
(a)Write the production function, revenue function, cost function, and profit function of the firm.
(b)What is the first-order condition for profit maximization? Give this condition an economic interpretation.
(c)What economic circumstances would ensure that profit is maximized rather than minimized?

Problem 5. Chiang and Wainwright 11.2 #1, 4, 5

#1
Table 11.1

Condition	Maximum	Minimum
First-order necessary condition	$f_x = f_y = 0$	$f_x = f_y = 0$
Second-order sufficient condition	$f_{xx}, f_{yy} < 0$ and $f_{xx}f_{yy} > f_{xy}^2$	$f_{xx}, f_{yy} > 0$ and $f_{xx}f_{yy} > f_{xy}^2$

Use Table 11.1 to find the extreme value(s) of each of the following four functions, and determine whether they are maxima or minima:

1. $z = x^2 + xy + 2y^2 + 3$
4. $z = e^{2x} - 2x + 2y^2 + 3$
5. Consider the function $z = (x - 2)^4 + (y - 3)^4$

(a)Establish by intuitive reasoning that z attains a minimum ($z^* = 0$) at $x^* = 2$ and $y^* = 3$.
(b)Is the first-order necessary condition in Table 11.1 satisfied?
(c)Is the second-order sufficient condition in Table 11.1 satisfied?
(d)Find the value of d^2z. Does it satisfy the second-order necessary condition for a minimum in (11.9)?

Problem 6. Chiang and Wainwright 11.3 #4(a, d), 5(a, d)

#4
Express each of the following quadratic forms as a matrix product involving a symmetric coefficient matrix:
(a) $q = 3u^2 - 4uv + 7v^2$ (d)$q = 6xy - 5y^2 - 2x^2$
#5
From the discriminants obtained from the symmetric coefficient matrices of Prob.4, ascertain by the determinantal test which of the quadratic forms are positive definite and which are negative definite.