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It might be that problems of vocabulary are considered to be fairly superficial within the whole issue of language and mathematics learning, but it is nevertheless critical that such problems are not ignored in the hope they will go away.

-Orton 1987, 127

Learning Mathematics Vocabulary: Potential Pitfalls and Instructional Strategies

o your students speak mathematics, or do they think that the mathematics classroom is another country where they must use a foreign language? Are they sometimes confused or overwhelmed by new vocabulary? Do they misuse words, forget key terms, or ignore important distinctions between words? Do they ask, "Where did anyone ever get a strange word like *asymptote?*" or "I forget, is twelve a factor or a multiple of twenty-four?"

The language of mathematics is an important component of our instruction. In fact, it plays at least three crucial roles in our classrooms:

- We teach through the medium of language. It is our major means of communication.
- Students build understanding as they process ideas through language.
- We diagnose and assess students' understanding by listening to their oral communication and by reading their mathematical writings.

Yet language is often a challenge for students. As teachers, we sometimes forget that the words and phrases that are familiar to us are foreign to our students. Students need to master this language if they are to read, understand, and discuss mathematical ideas. Unlike common English—which students hear, see, and use daily in reading, watching television, conversation, and elsewhere—the language of mathematics is limited largely to school. Consequently, we need to be sensitive to many issues related to the language of mathematics and students' growing fluency with it.

This article focuses on just one aspect of language, vocabulary, with the belief that the fluent use of terminology is a necessary, albeit not sufficient, condition for overall mathematics achievement. First, we analyze some difficulties that students encounter in learning mathematical language. This analysis attempts to clarify why some terms and phrases are hurdles for our students. Then we suggest teaching strategies that address these difficulties.

ANALYSIS OF MATHEMATICAL LANGUAGE

Kane, Byrne, and Hater (1974) and Shuard and Rothery (1984) highlight a number of vocabulary issues to consider when comparing the languages of mathematics and everyday English. **Table 1** summarizes those issues, together with others that we have identified. We give sample illustrations for most categories from five content strands: number, algebra, geometry, statistics and probability, and discrete mathematics. The categories are meant to elucidate students' difficulties but are not necessarily mutually exclusive.

The language issues identified in the table do not occur in isolation. Many, if not all, of these issues occur simultaneously in a single class period. Skemp, as noted in Orton (1987, p. 125), identifies two levels of language: deep structures and surface structures. The vocabulary issues that we have discussed are the "surface structures" used to transmit ideas as we engage students in discussions that lead to the "deep structures" of mathematical concepts. The purpose of **table 1** is to sensitize teachers to potential vocabulary problems because language difficulties are easily overlooked in trying to understand the conceptual difficulties that students

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Unlike common English, the language of mathematics is limited largely to school

Vocabulary Issues and Examples	
Category of Potential Pitfall	Examples
Some words are shared by mathematics and everyday English, but they have distinct meanings.	number: prime, power, factor algebra: origin, function, domain, radical, imaginary geometry: volume, leg, right statistics/probability: mode, event, combination discrete mathematics: tree
Some mathematics words are shared with English and have comparable meanings, but the mathematical meaning is more precise.	number: divide, equivalent, even, difference algebra: continuous, limit, amplitude, slope geometry: similar, reflection statistics / probability: average discrete mathematics: array, edge, and, or
Some mathematical terms are found only in a mathematical context.	number: quotient, decimal, denominator, algorithm algebra: asymptote, integer, hyperbola geometry: quadrilateral, parallelogram, isosceles, hypotenuse statistics / probability: outlier, permutation discrete mathematics: contrapositive
Some words have more than one mathematical meaning.	number: inverse, round algebra: square, range, base, inverse, degree geometry: square, round, dimensions, median, base, degree, vertex statistics / probability: median, range discrete mathematics: dimensions, inverse, vertex
Modifiers may change mathematical meanings in important ways.	number: value or absolute value, prime or relatively prime algebra: root or square root, inverse or inverse function geometry: polygon or regular polygon, bisector or perpendicular bisector statistics/probability: number or random number, probability or conditional probabilit discrete mathematics: sequence or arithmetic sequence, reasoning or circular reasoning
Some mathematical phrases must be learned and understood in their entirety.	number: at most, at least algebra: one-to-one geometry: if-then, if-and-only-if statistics / probability: stem-and-leaf discrete mathematics: if-then, if-and-only-if
Some words shared with science have different technical meanings in the two disciplines	number: divide, density algebra: solution, radical, variable geometry: prism, degree, image, radian statistics / probability: simulation, experiment discrete mathematics: matrix, element, cell, tree
Some mathematical terms sound like everyday English words.	number: sum or some algebra: sine or sign, cosine or cosign geometry: pi or pie, dual or duel, plane or plain statistics/probability: leaf, as in stem-and-leaf, or leave discrete mathematics: complement or compliment, graph or graft
Some mathematical words are related, but students confuse their distinct meanings.	number: factor and multiple, hundreds and hundredths, numerator and denominator algebra: equation and expression, solve and simplify geometry: theorem and theory statistics/probability: dependent events and independent events discrete mathematics: converse, inverse, and contrapositive
Technology may use language in special ways.	number: EXP (for scientific notation and not for exponentiation), iPart or INT algebra: LOG (for base-ten logarithms, not any logarithm), scale geometry: DrawInv (with the inverse not a function) statistics/probability: LinReg or LnReg
A single English word may translate into Spanish or another language in two different ways.	round (redondear), as in "round off," or round (redondo), as in "circular"
Shorthand or abbreviations are often used in place of the complete word or phrase, even if students must pro- nounce the entire word when verbalizing the shorthand.	inverse for inverse function; sin for sine, cos for cosine, and tan for tangent

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may have. In the next section, we suggest strategies to help resolve some difficulties inherent in students' learning mathematical vocabulary and to help students use that vocabulary to develop deeper meanings.

STRATEGIES TO PROMOTE VOCABULARY DEVELOPMENT

Although a number of potential pitfalls occur in using language, a few healthy practices, when used consistently, can effectively prevent or resolve many language difficulties. Although certain strategies may be best for certain pitfalls, we believe that teachers need to regularly use a variety of strategies for best results. Recognizing that students think and learn in many ways, we propose a spectrum of approaches, including oral, written, visual, and kinesthetic modes (Borasi et al. 1998; Gardner 1983). We close with a final strategy, using word origins, that involves several modes. Throughout, educators need to remember that vocabulary learning and mathematical understanding are intertwined.

General strategies

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One of the simplest teaching ideas is to build concepts first, then attach vocabulary to established ideas. For example, teachers can ask geometry students to sort several quadrilaterals, then identify categories, including, for example, those with exactly two pairs of parallel sides. When students can identify these shapes, the formal name, *parallelogram*, can be attached. At that point, another very simple step should be taken: to say, write, and spell the word clearly. Students can then record the new term and its meaning with a diagram in a personal glossary.

Oral strategies

Students' use of mathematical language, however, must go much deeper than saying and spelling. They need to own the language and use it comfortably. Such language fluency requires intensive language use. Working orally in groups on problemsolving tasks offers opportunities for students to "talk mathematics." Further, teachers can listen in, reinforce correct usage, and help students rephrase mathematical ideas as needed. Small groups furnish an unintimidating format. Oral work can also help in reading mathematics. Borasi et al. (1998) offer a strategy called *say something*, in which partners reading mathematics exposition stop intermittently to share aloud emerging understandings, comments, and questions.

The "silent teacher" is another device that puts more students' voices in the room and helps the teacher hear how students "talk mathematics." One of the authors stumbled on this idea when she had laryngitis. Students had to read for themselves teacher-prepared overhead transparencies on properties of logarithms. The teacher discovered that students stumbled over notation that should have been familiar. Some did not know that they should read $\log_2 8$ as "the log base 2 of 8" and instead read it as "log of two to the eighth." This discovery was an eye-opener for the instructor, who decided that she had too often read the symbols for the students. The silent teacher listened better to students and could coach them into correctly vocalizing the symbols. Rather than wait for a sore throat, teachers can use this strategy regularly and have students "talk mathematics" for themselves. As Usiskin (1996, p. 236) notes, "If a student does not know how to read mathematics out loud, it is difficult to register the mathematics...."

Some teachers have also found choral response to be a helpful verbal or rhythmic strategy for some simple but problematic areas. For example, one of our colleagues asks, "What is a logarithm?" and requires students to chant in unison, "A logarithm is an exponent." Similarly, in algebra he asks, "What does a negative exponent mean?" to which students respond, "A negative exponent means take the reciprocal."

At times, having students invent mathematical language may be beneficial. Rubenstein (1996) describes a situation in which a geometry student proposed that an easier term for *angle bisector* would be *midray*. The class later extended the concept to develop *midline* and *midplane*. Student invention cannot always be planned, but it can be a potent strategy for helping students own concepts and language.

These oral strategies allow teachers to gain understanding of students' thinking. Silver, Kilpatrick, and Schlesinger (1990, p. 16) note that the oral recitations in classrooms two generations ago gave teachers "a window through which to view Grandma's thoughts, her reasoning, and her grasp of terms, or notations, or fundamental ideas. That view made it possible to diagnose and evaluate Grandma's understanding." Used regularly, oral strategies encourage student discourse and can help educators modify instruction in important ways.

Writing strategies

Numerous authors—for example, Azzolino (1990); Chapman (1996); Dougherty (1996); and Mayer and Hillman (1996)—have described the power and value of students' writing about mathematics. Student journals offer just one way of listening to students communicate mathematics and can often be used to capture ideas recently addressed in class. For example, class discussions can conclude with such journal-writing stems as the following:

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- I thought that a function was _____. Now I know that a function is _____. (Dougherty 1996)
- Compare what *similar* means in everyday English with what it means in mathematics.
- Complete the following analogy, and explain your thinking: Prism is to pyramid as cylinder is to
- Tell the meaning of *range* in statistics, in studying functions, and in everyday English. What idea do all the meanings share? What is special about each of the mathematical meanings?
- Square and cube have geometric meanings and are also used for second and third powers, respectively. How are the geometry and powers related?
- What is the difference between the square of a number and the square root of a number?

As these examples illustrate, a common confusion can be the source for a writing entry. This strategy is part of a more general teaching philosophy: If you cannot fix it, feature it!

Students need to see and discuss writing samples of varying quality to learn what constitutes clear, valid mathematical communication. Teachers may initially create such samples or use student papers from previous years. Within a supportive learning atmosphere, students can peer-edit and evaluate one another's writing. Checking another student's writing to determine its validity and clarity requires a different level of thinking than that required in writing for oneself. Another student may have used a different valid strategy, and the peer reviewer has to evaluate a response from another perspective. Examples of peer review in regard to proof-writing can be found in Thompson and Senk (1993).

Another simple strategy integrates writing and problem solving. On take-home problem-solving assignments, students fold their paper vertically down the middle. On the left, they record their problem-solving work. On the right, they write explanations of their thinking. Although students initially remark that these assignments take more time to complete, they also note that the writing strengthens their understanding. In particular, over a semester, our students for whom English is a second language have shown marked improvement with this strategy in both their mathematics and their English.

Toumasis (1995), Shield and Swinson (1996), and McCoy, Baker, and Little (1996) modify the idea of journal writing to blend written descriptions with visual images by having students write definitions and draw or identify examples and nonexamples. Exemplary responses can be posted on bulletin boards, published in school newspapers, and shared on the overhead projector. Students need to discuss exemplary responses and less-than-exemplary responses.

Activities that emphasize awareness and usage activities can also be interwoven throughout instructional programs. For example, students can collect and share examples of mathematical terminology, graphs, or symbols from the media; write about what they have learned; and critique the presentations. They can also be asked to write about their own experiences using mathematical terms. We call one helpful assignment "think twice mentally." We ask students to identify from their own lives a situation in which they used or could have used mental math, detail the story, pose a mathematics question, and write two different mentalmath approaches that answer the question. They then share these stories orally and in writing, and the teacher uses them to learn about both the mental strategies that the students are using and their ability to communicate mathematically. As in foreignlanguage acquisition, immersion in language usage is necessary to develop fluency.

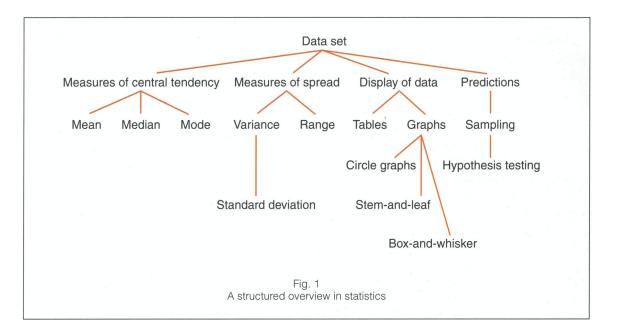
Visual strategies

In today's classroom, many students are strong visual learners, and they benefit when we support verbal learning with visual strategies. As students learn vocabulary and associated relationships, a structured overview (Smith and Kepner 1981) gives a visual picture of the many interrelationships. **Figure 1** shows a sample structured overview related to data analysis. Such an image can be used as an advance organizer by having students brainstorm aspects of the topic that they already know. It can also be used as a summary tool in which students take all the related concepts from a unit and place them in appropriate relationships.

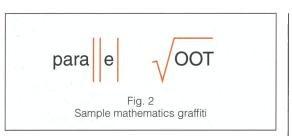
Another visual strategy uses picture dictionaries in which pictures are connected with written descriptions in the students' own words. Thiry (1990) describes a charadeslike contest in which teams try to guess mathematics terms or concepts from pictures drawn by their teammates.

Mathematical graffiti is another visual tool to aid students in thinking about aspects or characteristics of the language. **Figure 2** contains sample graffiti that illustrate plays on terminology. After students have been shown examples, they can create their own, which can then be shared on a bulletin board so that they benefit from one another's thinking.

Having students create mathematical cartoons enables teachers to assess and diagnose mathematical understanding at the same time. For students who struggle with written or verbal strategies, the visual and artistic opportunities offer another means for students to be successful. If you cannot fix it, feature it!



Rehearsals can polish language usage before the final performance



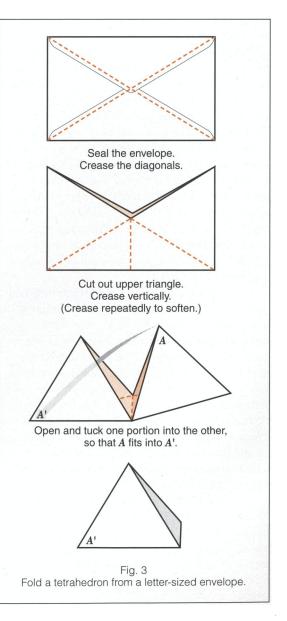
Kinesthetic strategies

Many students benefit from more kinesthetic approaches. For instance, such commercially available manipulatives as algebra tiles make factors and products concrete for students. In geometry, students can build their own three-dimensional figures to help learn the names of shapes, their parts, and their properties. Students can make a tetrahedron from a letter-sized envelope, as shown in **figure 3.** This model allows students to see and feel vertices, edges, and faces and comprehend why height and slant height are different. Similarly, constructing a circle or an ellipse directly from its locus definition makes its meaning concrete.

Group or individual projects culminating in public presentations furnish forums for greater use of mathematical language. Students may want to dramatize their learning through the medium of theater. Rehearsals can polish language usage before the final performance. McConnell (1995) makes many excellent suggestions in this area.

Word origins

Etymologies, or origins of words, are another source for building bridges between everyday language and mathematical language. Students are often unaware that words have evolved over time and



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have histories and roots. When students know these roots, they can make connections between common English words with which they are familiar and mathematics terms. For example, *perpendicular* comes from a root, *pend*, meaning to hang, because when a weight hangs freely on a string, it forms a perpendicular to the ground. Related English words include *pendant* and *pendulum*. Similarly, *product* and *factor*, mentioned previously, can also be clarified through their etymologies. A *product* is something that is produced, or is a result; and a *factor* is an ingredient or contributor, just as 6×9 has the factors, or contributors, of 6 and 9 that produce the result, or product, 54.

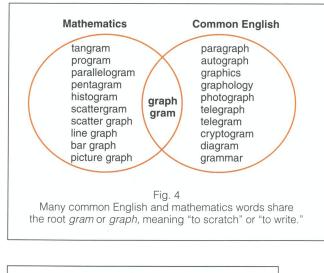
A more challenging concept, *asymptote*, is easier to assimilate when it is compared with a related common word, *symptom*. The root *sym* means *together* and *ptote* means *to fall*. So a *symptom* is a condition that falls together with an illness. By negating the idea with the prefix "a," an *asymptote* to a curve is a line that "does not fall together" with the curve. Such analogies and word origins are a strong approach in helping students form a connected view of language. *The Words of Mathematics* (Schwartzman 1994) is an excellent source for learning the origins of mathematics words. Other resources are McIntosh (1994) and the *Study Skills Handbook* of the University of Chicago School Mathematics Project (1995).

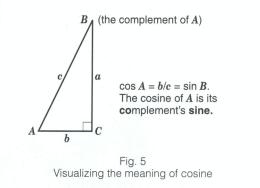
Teachers who are familiar with a few word origins can introduce them when new terminology is encountered. They can also invite students to research word histories for themselves. Diagrams are useful tools that can help make sense of mathematics terms and build general language development by starting with a root and finding related terms. The example in **figure 4** begins with the intersection with graph or gram, meaning to scratch or to write, and uses the crescents to identify mathematics and common English words stemming from those roots.

Occasionally, word origins, pictures, and mathematics concepts merge. For example, as when students learn that *cosine* comes from the *complement's sine*, they can use the word itself to identify a property (see **fig. 5**).

SUMMARY

The language of mathematics is a vital tool for student learning. Therefore, enculturating students to the vocabulary, phrasings, and meanings of mathematical language is a dimension of instruction that needs specific attention. This article analyzes some of the difficulties that students face and suggests several strategies to address these challenges. We hope that teachers will use and extend these ideas to help students own and command the language of mathematics.

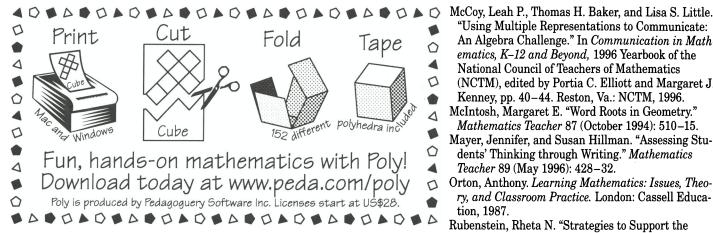




REFERENCES

- Azzolino, Aggie. "Writing as a Tool for Teaching Mathematics: The Silent Revolution." In *Teaching and Learning Mathematics in the 1990s*, 1990 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Thomas J. Cooney and Christian R. Hirsch, pp. 92–100. Reston, Va.: NCTM, 1990.
- Borasi, Raffaella, Marjorie Siegel, Judith Fonzi, and Constance F. Smith. "Using Transactional Reading Strategies to Support Sense-Making and Discussion in Mathematics Classrooms: An Exploratory Study." *Journal for Research in Mathematics Education* 29 (May 1998): 275–305.
- Chapman, Kathleen P. "Journals: Pathways to Thinking in Second-Year Algebra." *Mathematics Teacher* 89 (October 1996): 588–90.
- Dougherty, Barbara J. "The Write Way: A Look at Journal Writing in First-Year Algebra." *Mathematics Teacher* 89 (October 1996): 556–60.
- Gardner, Howard. Frames of Mind: The Theory of Multiple Intelligences. New York: Basic Books, 1983.
- Kane, Robert B., Mary Ann Byrne, and Mary Ann Hater. *Helping Children Read Mathematics*. New York: American Book Co., 1974.
- McConnell, John. "Forging Links with Projects in Mathematics." In *Connecting Mathematics across the Curriculum*, 1995 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Peggy House and Arthur F. Coxford, pp. 198–209. Reston, Va.: NCTM, 1995. →

Analogies and word origins can help students form a connected view of language

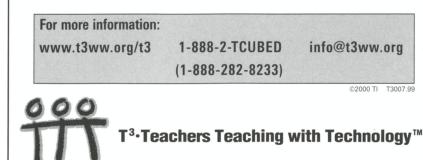




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"Using Multiple Representations to Communicate: An Algebra Challenge." In Communication in Mathematics, K-12 and Beyond, 1996 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Portia C. Elliott and Margaret J. Kenney, pp. 40-44. Reston, Va.: NCTM, 1996.

- McIntosh, Margaret E. "Word Roots in Geometry." Mathematics Teacher 87 (October 1994): 510-15.
- Mayer, Jennifer, and Susan Hillman. "Assessing Students' Thinking through Writing." Mathematics Teacher 89 (May 1996): 428-32.
- Orton, Anthony. Learning Mathematics: Issues, Theory, and Classroom Practice. London: Cassell Education. 1987.
- Rubenstein, Rheta N. "Strategies to Support the Learning of the Language of Mathematics." In Communication in Mathematics, K-12 and Beyond, 1996 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Portia C. Elliott and Margaret J. Kenney, pp. 214-18. Reston, Va.: NCTM, 1996.
- Schwartzman, Steven. The Words of Mathematics: An Etymological Dictionary of Mathematical Terms Used in English. Washington, D.C.: Mathematical Association of America, 1994.
- Shield, Mal, and Kevan Swinson. "The Link Sheet: A Communication Aid for Clarifying and Developing Mathematical Ideas and Processes." In Communication in Mathematics, K-12 and Beyond, 1996 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Portia C. Elliott and Margaret J. Kenney, pp. 35-39. Reston, Va.: NCTM, 1996.
- Shuard, Hilary, and Andrew Rothery. Children Reading Mathematics. London: John Murray Publishers, 1984.
- Silver, Edward A., Jeremy Kilpatrick, and Beth Schlesinger. Thinking through Mathematics: Fostering Inquiry and Communication in Mathematics Classrooms. New York: College Entrance Examination Board, 1990.
- Smith, Cyrus F., Jr., and Henry S. Kepner, Jr. Reading in the Mathematics Classroom. Washington, D.C.: National Education Association, 1981.
- Thiry, Jean M. "Mathematics Pictionary: The Sounds of Silence." Mathematics Teacher 83 (March 1990): 200 - 201.
- Thompson, Denisse R., and Sharon L. Senk. "Assessing Reasoning and Proof in High School." In Assessment in the Mathematics Classroom, 1993 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Norman L. Webb and Arthur F. Coxford, pp. 167-76. Reston, Va.: NCTM, 1993.
- Toumasis, Charalampos. "Concept Worksheet: An Important Tool for Learning." Mathematics Teacher 88 (February 1995): 98-100.
- University of Chicago School Mathematics Project. Study Skills Handbook. Glenview, Ill.: Scott Foresman & Co., 1995.
- Usiskin, Zalman. "Mathematics as a Language." In Communication in Mathematics, K-12 and Beyond, 1996 Yearbook of the National Council of Teachers of Mathematics (NCTM), edited by Portia C. Elliott and Margaret J. Kenney, pp. 231-43. Reston, Va.: NCTM, 1996. 🔕

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