1. Explain how to construct the following:

(a) An equilateral triangle on a line segment $AB$, given a straightedge and compass.
(b) The perpendicular to a line, through a given point off that line, given a straightedge and compass.
(c) A perspective view of the plane filled with quadrilaterals, given a straightedge and an initial quadrilateral.
(d) A line segment of length $\sqrt{2}/3$, given a line segment of length 1. Please explain why your line segment has the claimed length.

2. (a) How do I write $f(x) = \frac{2x+3}{x+1}$ as a composition of the generating transformations $x \to 1/x$, $x \to x + \ell$, and $x \to kx$?
(b) As a function on $\mathbb{RP}^1$, what is $f(0)$? What is $f(-1)$? What is $f(\infty)$?
(c) How do I write $z \to 4z$, viewed as a transformation of the upper-half plane, as a composition of reflections about hyperbolic lines?

3. (a) What is the vertical line $x = 3$, when written in terms of $z$ and $\bar{z}$?
(b) What is the circle with center 5 and radius 2 when written in terms of $z$ and $\bar{z}$?
(c) Find a Mobius transformation taking the line $x = 3$ to the circle with center 5 and radius 2.

4. (a) What is the hyperbolic distance between $2i$ and $7i$?
(b) Give an explicit formula for a sequence of points $s_n$ in the hyperbolic plane such that the hyperbolic distance between $s_n$ and $s_{n-1}$ is $n$.

5. (a) Prove that the diagonals of a parallelogram bisect each other.
(b) Prove that the angle sum of any quadrilateral is $2\pi$.
(c) Let $A$ and $B$ be two points on a circle. Prove that for all points $C$ on one of the arcs connecting $A$ and $B$, the angle $ACB$ is constant.

6. (a) Show that any two points in the hyperbolic plane are connected by a unique hyperbolic line.
(b) Show using the definitions that any two lines in \( \mathbb{R}P^2 \) intersect at a unique point.

(c) Show using the definitions that there are four lines in \( \mathbb{R}P^2 \), no three of which have a common point. You should give an explicit description of these lines.

7. (a) What is the formula for the cross-ratio?
   
   (b) Show that the cross-ratio is preserved by the transformation \( x \to 1/x \).
   
   (c) Let \( L_1 \) be the line in the \( xy \)-plane defined by the equation \( y = 1 \), and let \( L_2 \) be the line in the \( xy \)-plane defined by the equation \( x = 1 \). If \( (x_0, 1) \) is a point on \( L_1 \), what is its image on \( L_2 \) under projection from the origin?

8. (a) Give an explicit formula for (Euclidean) reflection about the line \( y = 2x \). (In other words, if the reflection is denoted by \( f \), for every \( (x, y) \) tell me what \( f(x, y) \) is.)
   
   (b) Give an explicit formula for rotation about the point \((1, 0)\) by 45-degrees.
   
   (c) Write the transformation \( z \to z + 4 \) as a composition of reflections about hyperbolic lines.

9. (a) What is the hyperbolic distance between \( 2 + i \) and \( 2 + 2i \)?
   
   (b) What is the hyperbolic distance between \( i \) and \( i + 1 \)?

10. (a) Give the definition of the medians of a triangle.
    
    (b) Prove using vectors that the medians of a triangle always intersect, and give a formula for where they intersect.

**Extra credit:** Let \( \mathbb{Z}_3 \) be the field with 3-elements. In other words, \( \mathbb{Z}_3 \) has elements 0, 1, 2 and the way you add or multiply these elements is to add/multiply them as integers, and then take the remainder when you divide by 3. (So \( 2 \cdot 2 = 1 \), for example, in \( \mathbb{Z}_3 \)). How many elements are there in the projective plane over \( \mathbb{Z}_3 \)?