Dear students,

Some of you have asked: what skills do I need to have to succeed on the final? In terms of core skills, I would suggest that you feel you can answer the following questions:

**Questions about systems:**

- How do I compute the matrix exponential of a $2 \times 2$ or $3 \times 3$ matrix? (It is unlikely I will ask about the $3 \times 3$ case, for time reasons.)
- What does the matrix exponential have to do with linear systems?
- How do I use the method of undetermined coefficients to solve a non-homogeneous linear system?
- How do I use variation of parameters to solve a nonhomogeneous linear system?
- What are the zeros of a system, and why are they significant? How do I linearize a nonlinear system at its zeros?
- What are the most common types of zeros? What are node sources? Node sinks? Saddles? Spirals? How can I tell from the linearization what kind of type I have?
- How do I write the general solution of a $2 \times 2$ linear homogeneous system, assuming the matrix for the system is diagonalizable?
- How do I write particular solutions for $2 \times 2$ (and maybe $3 \times 3$ systems), satisfying a given initial condition?
- How do I plot solutions to a $2 \times 2$ system on a plane?
- What does the “uniqueness” part from the “existence and uniqueness” theorem say about the plots of solutions?
- What does it mean for a system to be linear, versus nonlinear?
- What does it mean for a system to be autonomous?
- How do I write a single higher order equation as a system?
• How do I write a nonautonomous system as an autonomous one?

• What is the relationship between autonomous systems and vector fields? How do I sketch vector fields? How do I sketch a solution of a system from a vector field?

Questions about a single equation:

• What is Euler’s method, and midpoint Euler, and how do I use them?

• How do I use separation of variables, the power series method, the method of undetermined coefficients, and variation of parameters? What are exact equations and how do I solve them explicitly?

• What is an example of a model that uses a differential equation?

• What are fences? What do we learn from the existence of a fence? How can I show that a given curve is a fence?

• What are funnels? Anti-funnels? What do they look like?

• What are isoclines? How do I find them?

• What is the slope field, and how can I sketch it? How can I sketch a solution from the slope field?

• What is a vertical asymptote?

• What does fundamental existence and uniqueness say, at least generally, and when can uniqueness fail?

\[1\text{If you are confused about this, I encourage you to play with the tool on the website to explore this.}\]