1. For each of the twelve questions below, state whether the assertion is true or false. (You do not need to justify your answer.)

(a) Every vector field is the gradient of some function.
(b) Define variables $u$ and $v$ so that $x = 2u + 3v$, $y = 2u - 3v$. Then

$$\left| \frac{\partial(x, y)}{\partial(u, v)} \right| = 12$$

(You do not need to check that our change of variables transformation is one to one and onto.)
(c) Every one-to-one transformation is necessarily onto.
(d) The transformation $T(u) = (u^3)$ is one-to-one.
(e) The vector field $F(x, y) = (x^2, y^2)$ on the $xy$ plane is conservative.
(f) Let $F$ be a vector field. Then, no matter what $F$ is, we always have

$$\text{div(curl}(F)) = 0.$$ 

(g) Let $(x_1, x_2, x_3)$ and $(y_1, y_2, y_3)$ be two points in $xyz$ space, and let $C_1$ and $C_2$ be two different curves from $(x_1, x_2, x_3)$ to $(y_1, y_2, y_3)$. Let $F$ be any vector field. Then, no matter what $F$ is,

$$\int_{C_1} F \, dS = \int_{C_2} F \, dS.$$ 

(h) Let $(\rho, \theta, \varphi)$ be the usual spherical coordinates. Then, the set in $xyz$-space described in spherical coordinates by $\theta = \pi/4$ can be correctly described as a half-plane.

(i) Let $F$ be a vector field. Then, no matter what $F$ is, the integral $\iint_S F \cdot dS$ is multiplied by negative one when the orientation of $S$ is changed.
(j) Let \( S \) be the boundary of a solid, and let \( F \) be any vector field. Then, no matter what \( F \) is,
\[
\int \int_S \text{curl}(F) \, dS = 0.
\]

(k) Let \( f \) be a function on \( xyz \)-space. Then, no matter what \( f \) is,
\[
\text{div}(\nabla f) = 0.
\]

(l) Let \( S \) be the unit sphere centered at the origin. Then
\[
\int \int_S e^{x^2} \, dS = 0.
\]

2. Evaluate \( \int_1^2 \int_2^3 \int_0^1 8xyz \, dz \, dx \, dy \).

3. Evaluate the integral \( \iint_D e^{x/y} \, dA \), where \( D \) is the region in the \( xy \) plane described by the inequalities
\[
1 \leq y \leq 2, \quad y \leq x \leq y^3.
\]

4. Is the vector field
\[
F(x, y) = (2x^3y^4 + x) \hat{i} + (2x^4y^3 + y) \hat{j}
\]
conservative? If it is, find a function \( f \) such that \( \nabla f = F \). If it is not conservative, why not?

5. Integrate \( \int \int_W (x^2 + y^2 + 2z) \, dV \), where \( W \) is the region in \( xyz \) space that lies below the paraboloid \( z = 25 - x^2 - y^2 \), inside the cylinder \( x^2 + y^2 = 4 \), and above the \( xy \)-plane.

6. Compute the length of the spiral in the \( xy \) plane parametrized by \( \psi(t) = (e^t \cos(t), e^t \sin(t)) \), for \( 0 \leq t \leq 2\pi \).

7. Let \( a > 0 \) and \( R > a \) be two constants. Compute the surface area of the part of the sphere \( x^2 + y^2 + z^2 = R^2 \) with \( z > a \).

8. Use the Divergence Theorem to evaluate \( \int \int_S F \, dS \), where \( F \) is the vector field given by \( F(x, y, z) = (y^2z, xz^3, 4z^2x) \), and \( S \) is the boundary of the cube defined by the inequalities
\[
-1 \leq x \leq 1, \quad -1 \leq y \leq 1, \quad 0 \leq z \leq 2.
\]

9. Use Green’s theorem to calculate the area of the region \( D \) bounded by the curve parametrized by \( \sigma(t) = (\sin(2t), 2\sin(t)) \) for \( 0 \leq t \leq \pi \).

10. Let \( S \) be defined by \( z = 4 - x^2 - y^2 \) for \( 0 \leq z \leq 4 \) and \( z = \sqrt{x^2 + y^2} - 2 \) for \(-1 \leq z \leq 0 \). Let \( F \) be the vector field \( F(x, y, z) = (x^2, xz, 4z^2x) \).
(a) Is $F$ conservative?
(b) What is the flux of the curl of $F$ across $S$?

11. Compute:

(a) $\int_1^4 \int_1^{\sqrt{x}} \frac{e^x y^2}{y^2} dy \, dx$

(b) $\int \int_S F \, dS$, where $F$ is the vector field defined by $F(x, y, z) = (x^4, 2y^2, z)$, and $S$ is the half of the surface

$$\frac{1}{4} x^2 + \frac{1}{9} y^2 + z^2 = 1$$

with $z \geq 0$, oriented downward.

**A fun extra practice challenge!** The question below is not quite the kind of question I would ask you, but it’s a good question to think about, and you might enjoy it.

Let $S$ be the sphere $x^2 + y^2 + z^2 = 1$. Show that the area of the part of $S$ between two parallel planes separated by distance $d$ only depends on $d$ (and not on the choice of planes) by computing this area.