1. Consider\[ x' = x(2 - x). \] (1)

(a) Is this differential equation linear, or nonlinear?
(b) Sketch the slope field, by plotting at least 10 different points, representing at least 5 different slopes.
(c) Let \( x(t) \) solve (1), with initial condition \( x(3) = 1 \). Use Euler’s method, with step size .5, to estimate \( x(4.5) \).

2. (a) Verify that \( x(t) = Ce^{-t} \) is a solution to the differential equation \( x' + x = 0 \). What is \( C \) if \( x(0) = 3 \)?
(b) Convert the system
\[ x' = 4x + 7y, \quad y' = -2x - 5y \]
into matrix form. In other words, write it as a single differential equation, with matrices and vectors.

3. Solve the following single differential equations:

(a) \( x' = 6x^2t \)
(b) \( 2tx - 9t^2 + (2x + t^2 + 1)x' = 0 \), either implicitly or explicitly.

4. (a) Let \( C_1 \) and \( C_2 \) be constants. Verify that
\[ x(t) = C_1e^{-t} \begin{bmatrix} -1 \\ 1 \end{bmatrix} + C_2e^{4t} \begin{bmatrix} 2 \\ 3 \end{bmatrix} \] (2)
is a solution to the differential equation
\[ x' = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} x. \]

(b) What are \( C_1 \) and \( C_2 \) if \( x(0) = \begin{bmatrix} 0 \\ -4 \end{bmatrix} \), where \( x(t) \) is given by (2)?
5. Consider the system of equations
\[
\frac{dx}{dt} = 3x - y^2, \quad \frac{dy}{dt} = \sin(y) - x.
\]
(a) How many zeros does the equation have?
(b) What is the linearization of this system near (0, 0)?

6. (a) What is the general solution to
\[
x' = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} x?
\]
(b) What is the general solution to
\[
x' = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} x + t \begin{bmatrix} 2 \\ -4 \end{bmatrix}?
\]

7. Let
\[
A = \begin{bmatrix} 9 & -5 \\ 4 & 5 \end{bmatrix}.
\]
(a) What is $e^{tA}$? (Your answer should be a real matrix.)
(b) What is the general solution to $x' = Ax$?

8. Find a solution to
\[
x' = \begin{bmatrix} -5 & 1 \\ 4 & -2 \end{bmatrix} x
\]
that satisfies $x(1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

9. (a) Give a qualitative description of the solutions to
\[
x' = \begin{bmatrix} 3 & -9 \\ 4 & -3 \end{bmatrix} x.
\]
(b) Plot two different solutions $x(t) = (x_1(t), x_2(t))$ of
\[
x' = \begin{bmatrix} -5 & 1 \\ 4 & -2 \end{bmatrix} x
\]
in the $x_1x_2$ plane.