Dear students,

I was asked: how can we simplify expression using Euler’s Identity?

Recall that Euler’s Identity says that

\[ e^{i\theta} = \cos(\theta) + i\sin(\theta). \]

There are two main ways that we want to use this. We learn from the identity that

\[ e^{i\theta} + e^{-i\theta} = 2\cos(\theta), \]

because \( \cos \) is an even function, while \( \sin \) is odd. We also learn that

\[ e^{i\theta} - e^{-i\theta} = 2i\sin(\theta), \]

for the same reason.

**Example:**

In class, I mentioned that we wanted to simplify

\[ \frac{1}{4i} 2(e^{(3+2i)t} - e^{(3-2i)t}). \]

This looks complicated, but it’s actually totally real.

We simplify using the identities above as follows:

\[
\frac{1}{4i} 2(e^{(3+2i)t} - e^{(3-2i)t}) = \frac{e^{3t}}{2i} (e^{2it} - e^{-2it}) \\
= \frac{e^{3t}}{2i} (2i\sin(2t)) = e^{3t} \sin(2t)
\]