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# Teaching $E = mc^2$ : Mass Without Mass



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**T**he mass-energy relation is, according to Einstein, the most important result of the special theory of relativity.<sup>1</sup> Many educators have called for the inclusion of more such “modern” (post-1900) physics in our introductory courses.<sup>2</sup> Although we frequently treat  $E = mc^2$  as a simple numerical relation, useful for solving nuclear physics problems, it is more important to discuss what this equation means. What can it tell us about the nature of energy and matter? Does rest mass arise purely from the energy of force fields? It seems not to be widely recognized that the answer to this last question is now known, at least for 90% of the mass of ordinary matter, to be “yes.”

## The meaning of $E = mc^2$

The mass-energy relation is much more than a formula. Like all the equations of physics, it stands for an idea. And in this case, the conceptual content is especially significant.<sup>3</sup>

Begin the conceptual explanation with everyday examples. Throw a ball, stretch a rubber band, lift a physics book. In each case, the system’s mass increases (in the case of the lifted book, the “system” is the book and Earth). But the energy increase is only a few joules, so the mass increase is a few joules divided by  $c^2$ , a tiny number that would be difficult or impossible to observe. Nevertheless, the theory says that the mass of each system increases. This is surprising. When you heat a pot of soup, you give it on the order of  $10^5$  joules of energy, increasing its mass by a billionth of a gram. Surprising, but still undetectable. These simple examples show why we don’t notice,

in everyday life, that mass changes whenever energy changes. They also should dispel the common misconception that

$E = mc^2$  is uniquely applicable to nuclear processes.

For a revealing everyday example, use a pair of magnets and a few interactive Concept Test<sup>4</sup> questions. Beginning with the magnets clinging together, pull them apart and hold them, separated, at rest. Ask, “Did the energy of the magnet system increase, decrease, or remain unchanged, and how do you know?” It increased, because you did work in separating them. Ask, “Where is that extra energy located?” It’s in the space (which could be vacuum) between the magnets, i.e., it’s in the magnetic field. Ask, “Did the mass of the magnet system increase, decrease, or remain unchanged?” It increased, because the energy increased and  $m = E / c^2$ . Finally, “Where is that mass increase?” It’s in the field, because that’s where the energy is! Fields, even fields in empty space, have *mass*. This is surprising. As an aside, you might mention that this also demonstrates the physical reality of electromagnetic fields.<sup>5</sup>

The formula can be checked in nuclear reactions, because here the energy changes are large. For example, if uranium is fissioned and the thermal energy removed, the fractional mass loss is about 0.1% and easily detectable. Similarly, when two deuterium nuclei are fused to make helium, the mass loss is about 0.6%.

But the meaning of  $E = mc^2$  is more clearly conveyed by the example of the magnets: *Any system having energy has mass*. Furthermore, *any system having*

*mass has energy.* This latter statement is best illustrated by matter-antimatter annihilation. For example, when an electron and positron, each of mass  $M$ , annihilate, the two particles vanish. Question: Is it possible that nothing appears in their place? If  $E = mc^2$ , then some form of energy must appear because energy is conserved. In fact, radiation appears, and measurements show that it has energy  $2Mc^2$  (plus the initial kinetic energy of the particles). Even if they're at rest, there are  $2Mc^2$  joules of "stored work" in the positron-electron pair!

This example can be used to dispel another common misconception. It is sometimes said that "mass is converted to energy" in experiments like this one. But mass is never converted into energy, and energy is never converted into mass, because energy (and hence mass) can never be created or destroyed. In pair annihilation, we have precisely the same amount of energy, and mass, before and after the annihilation. However, it is correct to say that *matter* (which has rest mass) is converted into *radiation* (which does not).

## Mass Without Mass<sup>6</sup>

John Wheeler coined the phrase "mass without mass" to indicate the possibility of removing any mention of mass from the fundamentals of physics.<sup>7</sup> The idea is to account for the fundamental particle masses entirely in terms of fields. Early in the 20th century, Hendrik A. Lorentz<sup>8</sup> and others pursued the dream of explaining the electron's mass entirely in terms of its electromagnetic field, but Lorentz's classical theory was superseded by quantum physics. Today, we still do not know what gives the electron its mass, but we have nearly achieved Lorentz's dream in a more dramatic context: The masses of protons and neutrons ("nucleons"), which constitute nearly the entire mass of ordinary matter, appear to arise almost entirely from the "color" fields of their constituent quarks.

If you look over the standard model wall chart,<sup>9</sup> you will discover a funny thing: The masses of the up and down quark are listed, respectively, as  $3 \text{ MeV}/c^2$  and  $6 \text{ MeV}/c^2$  (you might want to discuss this mass unit with your students), but the proton, which is said to be made of two up quarks and one down quark, is stated to have a mass of  $938 \text{ MeV}/c^2$ . Something doesn't add up. The difference, amounting to nearly 99% of the proton's mass, arises from the energy of

the quarks' force fields. Similarly, a neutron's mass is far larger than the sum of the masses of its three quarks (one up and two downs). One might quickly conclude that some 99% of the mass of a nucleon arises from these fields, just as a portion of the mass of two separated magnets arises from their magnetic field. But this is too simplistic, because we can't assume that this 99% is really independent of the residual mass of the quarks; if some of the energy in the field is dependent on these masses, it is possible that a significant part of a nucleon's mass arises from quark masses.

The relatively small mass of a nucleon's quarks suggests investigating a nucleon model in which all quark rest masses are set equal to zero. Since gluons (the remaining constituents of nucleons, in addition to the quarks) have zero rest mass, this is a "pure field" model. When this model is used to calculate nucleon masses, the results are accurate to within 10%!<sup>10</sup> The mass comes, of course, from  $m = E/c^2$ , where  $E$  is the energy of the fields and motions of the massless quarks and gluons. Since better than 99% of the mass of ordinary matter arises from nucleons, this model tells us that at least 90% of the mass of ordinary matter is "mass without mass."

The remaining 10% might arise in a similar way. The standard model of particle physics predicts the existence throughout the universe of a field called the "Higgs field." Direct evidence for the Higgs field could come within a few years when and if the "Higgs boson," the quantum of the Higgs field, is discovered. If verified, the Higgs field will explain the fundamental particle masses (e.g., the rest masses of quarks and electrons) in terms of the energy of interaction between particles and the Higgs field. Thus, we will have an explanation of all of the mass of ordinary matter in terms of fields.

Thus, at least as regards ordinary matter, modern physics is on the verge of verifying the century-old vision of "mass without mass." This view, according to which material particles are only force fields in space, could also be termed a "field view of reality." It is a view that is right in tune with the thrust of the relativistic quantum field theories that underlie contemporary physics. For example, Nobel laureate Steven Weinberg, a leading quantum field theorist, stated:

[According to the physical theories developed during the 1920s] there was supposed to be one field for each type of elementary particle. The inhabitants of the universe were conceived to be a set of fields—an electron field, a proton field, an electromagnetic field—and particles were reduced to mere epiphenomena. In its essentials, this point of view has survived to the present day, and forms the central dogma of quantum field theory: *the essential reality is a set of fields* [Weinberg’s emphasis] subject to the rules of special relativity and quantum mechanics; all else is derived as a consequence of the quantum dynamics of these fields.<sup>11</sup>

In this view, “There is no ‘there’ there” (to quote the poet Gertrude Stein), no “things” at all. Electrons and other material particles are only force fields in “empty” space, like the field in the “empty” gap between two magnetic poles. This view implies that every “thing,” *everything*, is interactions and motion. It is the interactions and motion themselves that are fundamental rather than the material particles that we had always supposed were doing the interacting and the moving. In a culture still steeped in the Newtonian mechanistic tradition, this would appear to be a significant insight into the nature of physical reality. Our students deserve to know about it.<sup>12</sup>

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3. For an especially clear discussion of other aspects of the mass-energy relation, see Ralph Baierlein, “Teaching  $E=mc^2$ : An exploration of some issues,” *Phys. Teach.* **29**, 170–175 (March 1991).
4. Eric Mazur, *Peer Instruction* (Prentice Hall, Upper Saddle River, NJ, 1997).
5. Einstein insisted that fields are real. See Albert Einstein and Leopold Infeld, “The reality of the field,” in *The Evolution of Physics* (Simon and Schuster, New York, 1938), pp. 148–156; on p. 151 we find: “The electric and magnetic field, or in short, the electromagnetic field is, in Maxwell’s theory, something real. The electric field is produced by a changing magnetic field, quite independently, whether or not there is a wire to test its existence; a magnetic field is produced by a changing electric field, whether or not there is a magnetic pole to test its existence.”
6. Much of the information in this section comes from Frank Wilczek’s article “Mass without mass I: Most of matter,” *Phys. Today* **52**, 11 (Nov. 1999).
7. John A. Wheeler, *Geometrodynamics* (Academic Press, New York, 1962), p. 25.
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10. See Ref. 6.
11. Steven Weinberg, quoted in Heinz Pagels’s *The Cosmic Code* (Bantam Books, New York, 1983), p. 239.
12. Further discussion can be found in the author’s textbook, *Physics: Concepts and Connections*, 3rd ed. (Prentice Hall, Upper Saddle River, NJ, 2003).

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